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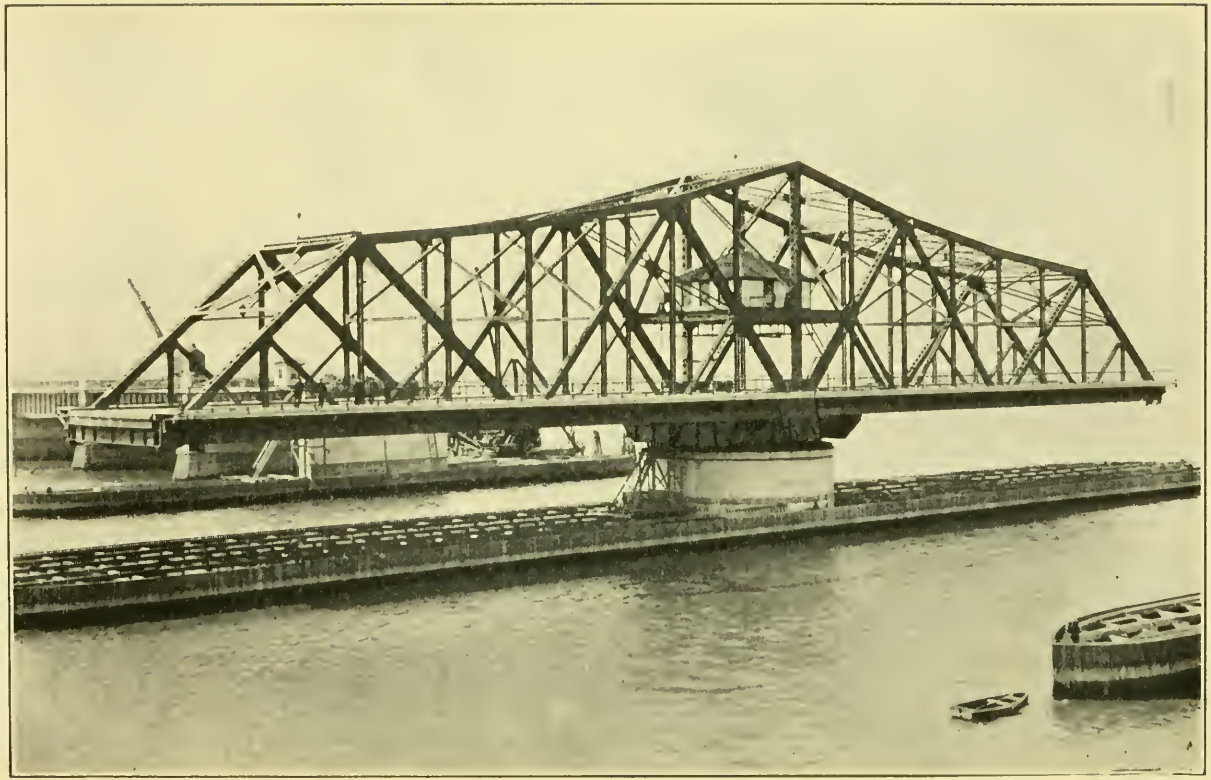
MOVABLE BRIDGES

VOLUME I.—SUPERSTRUCTURE



Frontispiece

PERTH AMBOY BRIDGE

CLARENCE W. HUDSON, M. AM. SOC. C. E., ENGINEER

Highway, center-bearing swing span.
365 ft. long., 55 ft. wide.

Swinging weight, 4,200,000 lbs.
Two 125-h.p. electric motors.

MOVABLE BRIDGES

BY

OTIS ELLIS HOVEY, B.S., C.E.

Member of the American Society of Civil Engineers, American Society of Mechanical Engineers, American Society for Testing Materials, American Iron and Steel Institute, and Associate Member of the American Railway Engineering Association

Assistant Chief Engineer of American Bridge Company

IN TWO VOLUMES

VOLUME I.—SUPERSTRUCTURE

NEW YORK

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1926

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TO MY WIFE,
WHO INSPIRED THE BEGINNING OF ITS PREPARATION
AND WHOSE ENCOURAGEMENT
HAS MADE ITS COMPLETION POSSIBLE,
THIS BOOK
IS AFFECTIONATELY DEDICATED

18361

PREFACE

This book was written because it was believed that a practical treatise on the design of movable bridges and their machinery would be of interest and value to practicing engineers and advanced students in engineering schools. Many authors have treated the design of the superstructure of movable bridges in connection with that of fixed spans, but few have discussed the special problems peculiar to the arrangement and design of the machinery. The data formerly available in this important branch of the subject were scattered through treatises on machine design, handbooks, the technical periodicals, and the publications of engineering societies; or had been developed by the engineers of manufacturing bridge companies and closely held for their own use. One of the objects of this book is to collect and arrange the best of this material in convenient form for study and use, and to put the results of original investigations and the lessons learned from long experience before fellow-engineers, in the hope of rendering a real service to the profession.

After a brief history of the development of early designs, the various types of bridges are discussed, and enough statistical information is given to assist in determining the most desirable type of bridge for particular conditions; and also to serve as a basis for preliminary calculations. The various methods of stress analysis are so well known that only a brief statement of some of the simplest and most practical is given. The chapter on the elastic deflections of structures has been made more complete. The chapter on details of design includes discussions of camber, end-lifting mechanism, disc and roller-bearing centers, and other details which apply to swing bridges. The analysis of end-lifting mechanism, and the illustrative example are typical of convenient and practical methods which may be applied to the design of similar mechanism for other purposes. End-rail joints, operators' houses, and counterweights are treated briefly in the remaining chapters.

Appendix A gives an analysis of the stresses in lenticular discs, and Appendix B proposes a new method for the design of the tread plates on the supporting and segmental girders of rolling-lift bridges.

Numerous examples are given throughout the book to illustrate the application of new or unusual formulae or processes.

The calculus has been freely used where it served to add clearness and precision to analysis.

The author acknowledges, with thanks, the permission to use many cuts, credit having been given in connection with each cut. He also is indebted to Mr. F. L. Dudley, M. Am. Soc. C. E., for most of the calculations in the example of the deflections of a truss in Chapter IX; to Mr. Dudley and Mr. C. E. Paine, for assistance in the preparation of the discussion of end-lifting mechanism in Chapter X; to Mr. F. H. Horton for help in compiling the lists of Patents in Chapters IV and V; and also to many other friends for valuable data and suggestions.

The power required to operate movable bridges, the arrangements and design of their machinery, complete specifications, and many tables giving useful data will be included in the second volume.

Otis Ellis Hovey.

January, 1926.

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69. Eq. (1). Change 1430 to 1340.
79. Seventh line. Change 3K to 3L.
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110. Twelfth line. Change similer to similar.
144. Patent number, Osborn, F. C. should be 1,158,546.
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176. Eq. (1), third line. Change P_a to Pa and P_b to Pb.
177. Third line above the cut. Change four to five.
217. Fig. 8A, left end. Change T_1 to T'.
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273. Fifth line. Insert the words, highway and, after for.
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bridges as crossing the moats around castles and fortified towns." Also, "the Egyptians built no permanent bridges across the Nile, but were familiar with framing trestle work, and with pontoon draw bridges."

About 460 B.C., Nitocris, Queen of Babylon, built a bridge across the Euphrates, after temporarily diverting the water. According to Herodotus,[2] the piers were of stone blocks bound together with iron and lead, and the spans were wooden platforms built so that they could be withdrawn at night, to prevent people passing from side to side of the river,

[1] Knight's "American Mechanical Dictionary," Boston, 1884.
[2] Herodotus, 1–186.

MOVABLE BRIDGES

CHAPTER I

HISTORICAL INTRODUCTION

1. General.—The term movable is chosen here to designate any type of bridge that can be changed in position so as to open a clear passage, or to afford an increased headway, for ships or boats in navigable channels. In general this is the purpose to be accomplished by making movable a bridge intended for railway or highway traffic.

In the ancient and mediaeval periods the object of movable bridges was usually to afford protection against military operations, and these bridges were commonly used over the protective moats around fortifications. They were intended to be opened in order to interrupt the approach roadways and also in order that the open leaf might constitute an additional door, or gate, closing the entrance to the fortification. It is difficult to determine whether the earliest movable bridges were intended for military protection or to allow the passage of boats.

2. Ancient Movable Bridges.—In an article on draw bridges, Edward H. Knight[1] states that, "the earliest mention is in the Egyptian monuments, when Rameses II celebrated his victories over fortified cities, 1355 B.C. . . . The sepulchral and palatial paintings represent the bridges as crossing the moats around castles and fortified towns." Also, "the Egyptians built no permanent bridges across the Nile, but were familiar with framing trestle work, and with pontoon draw bridges."

About 460 B.C., Nitocris, Queen of Babylon, built a bridge across the Euphrates, after temporarily diverting the water. According to Herodotus,[2] the piers were of stone blocks bound together with iron and lead, and the spans were wooden platforms built so that they could be withdrawn at night, to prevent people passing from side to side of the river,

[1] Knight's "American Mechanical Dictionary," Boston, 1884.

[2] Herodotus, 1–186.

in the dark, to commit robberies. However rudimentary in design, this was certainly a genuine movable bridge.

It is well known that the Chinese developed unusual engineering skill at an early period, as evidenced by the building of the Great Wall, which bridged several streams. It was begun during the third century, B.C., repaired during the fifteenth century, A.D., and extended about 300 miles during the sixteenth century. It seems quite probable that the Chinese may have used movable bridges across their canals at a very early date.

The Phoenicians greatly encouraged the art of civil engineering around the Mediterranean, and probably extended their practice to small movable bridges.

The earliest Roman bridge of which we have a record was the Pons Sublicius, built in 621 B.C., by Ancus Martius. It connected the Janiculum and Mons Aventinus. As indicated by its name, it was a pile bridge, and some writers state that it had a draw span. This bridge was defended by Horatius, the legendary hero, against the Etruscans under Lars Porsena, and was destroyed, during a campaign, in 510 B.C. It was rebuilt and long maintained as a wooden pile bridge.

Pontoon bridges were used in early military expeditions, and it is known that some of them had one or more boats arranged so that they could be easily swung aside to allow ships to pass. Darius built a bridge of boats across the Thracian Bósphorus. Xerxes constructed a similar bridge across the Hellespont, about 480 B.C. This bridge was between Sestos and Abydos, which was the narrowest point. Xerxes' first bridge was destroyed by a tempest almost as soon as it was finished. It was then rebuilt with 360 boats on the side toward the Euxine Sea and 313 on that of the Hellespont. The alignment was made so as to reduce the probability of its being destroyed by storms, and at three places boats were lashed together and arranged so that they could be swung aside to allow ships to pass through the openings. These appear to have been genuine pontoon draw spans.

Time and space prevent a thorough discussion of other ancient movable bridges. The Romans rapidly developed the design and construction of fixed arch bridges, many examples of which may still be seen, but it is difficult to trace the use of movable structures for several centuries after the beginning of the Christian era. Late in the twelfth century, bridge building was much stimulated by the organization of the *Fratres Pontifices*, or Brethren of the Bridge, by Johannes Benedictus. This brotherhood was formally recognized by Pope Clement III, in 1189. The founder, commonly known as St. Benezet, has been termed the Patron Saint of Bridge Engineers. He built the fixed arch

bridge at Avignon, which was finished in 1176, and at his death, in 1222, was interred in a chapel which he had provided on the bridge.

A draw bridge was maintained in a bridge over the Thames, in London, from early times. Thompson [3] states that, "it is generally believed, however, that the year following Aulaf's invasion, namely 994, there was built a low Wooden Bridge, which crossed the Thames at St. Botolph's Wharf, yonder, where the French passage vessels are now lying; and a rude thing enough it was, I'll warrant; built of thick rough-hewn timber planks, placed upon piles, with moveable platforms to allow the Saxon vessels to pass through it, Westward." This bridge is said to have been burned and rebuilt several times, in timber, and undoubtedly with a draw span in each case.

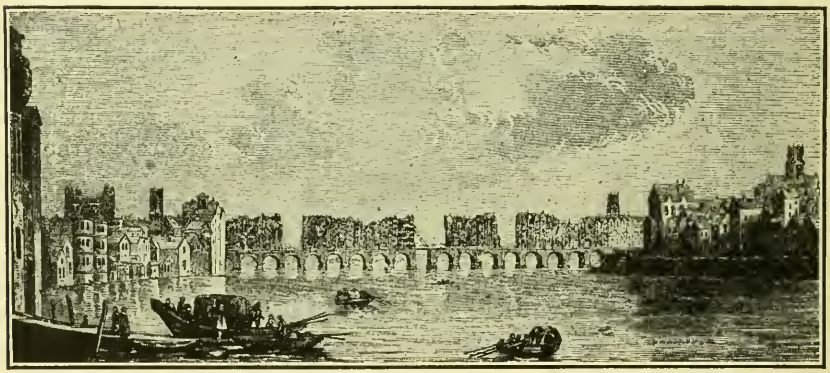
In the year 1176, which marked the completion of the Avignon bridge, Peter, Chaplain of the Church of St. Mary Cole, in London, who is called Peter of Colechurch, was commissioned by Henry III to build a permanent stone arch bridge across the Thames, in the business center of the city. It was finished in 1209. The work was so well done that the bridge was in continuous use until 1831, when the present modern arch bridge was completed. The old bridge was notable in at least two particulars. The piers were so large and the spans so short that about two-thirds of the available cross-section of the river was occupied by the piers; with the result that at each tide the water ran under the arches like a veritable millrace. There was a draw bridge in the seventh span from the Surrey, or southerly, end. A somewhat extended search fails to establish the exact design of the draw span, but a cut from an old painting plainly shows that there were two chains leading from the rising end of the span and extending diagonally upward until they entered near the top of a building upon the fixed span adjacent to the draw. As no artist would be likely to insert such unsightly lines in a painting unless they existed on the structure, it is evident that the movable span was a genuine draw bridge, in the original meaning of that term. Whether the draw was counterweighted does not appear in any available account.

Figure 1A is a reproduction of a woodcut from the painting referred to above.

3. The Renaissance.—The epoch of the Renaissance, from the latter part of the fourteenth century to the middle of the sixteenth, was marked by a rapid and remarkable development in literature, art, science, government, and religious reform. It was a time of great intellectual activity, as may be illustrated by the recital of the names of such

[3] "Chronicles of London Bridge," by Richard Thompson, 'An Antiquarian,' London, 1827.

leaders of thought as Palladio, Galileo, Kepler, Columbus, Napier, Raphael, Rubens, Dante, Shakespeare, Gutenberg, Luther, Wyclif, and Michelangelo; all of whom lived during this period. Leonardo da Vinci must also be named as one of the most remarkable men of that time. His genius was most diversified, and his work and writings included masterly investigations in the fields of natural philosophy, art, and engineering. Many of his sketches have been preserved, and have been published in *Codice Atlantico*,[4] from which two examples have been chosen. Figure 1B, from a sketch made about 1500, represents an unequal-armed, or bob-tailed, center-bearing swing bridge, operated by hand winches, by means of ropes passed through snatch-blocks. Fig. 1C represents a very well-conceived pontoon swing span, which enters a niche in the canal wall when open.



John Murray, London.

FIG. 1A.

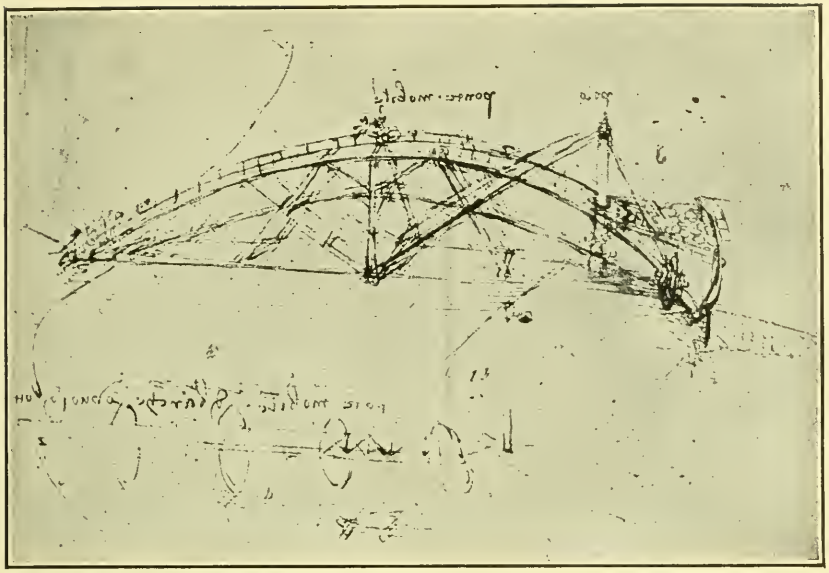
Some rough, small-scale sketches show that he had also developed the general ideas of counterweighted draw bridges turning on end trunnions, and also counterweighted vertical-lift bridges.

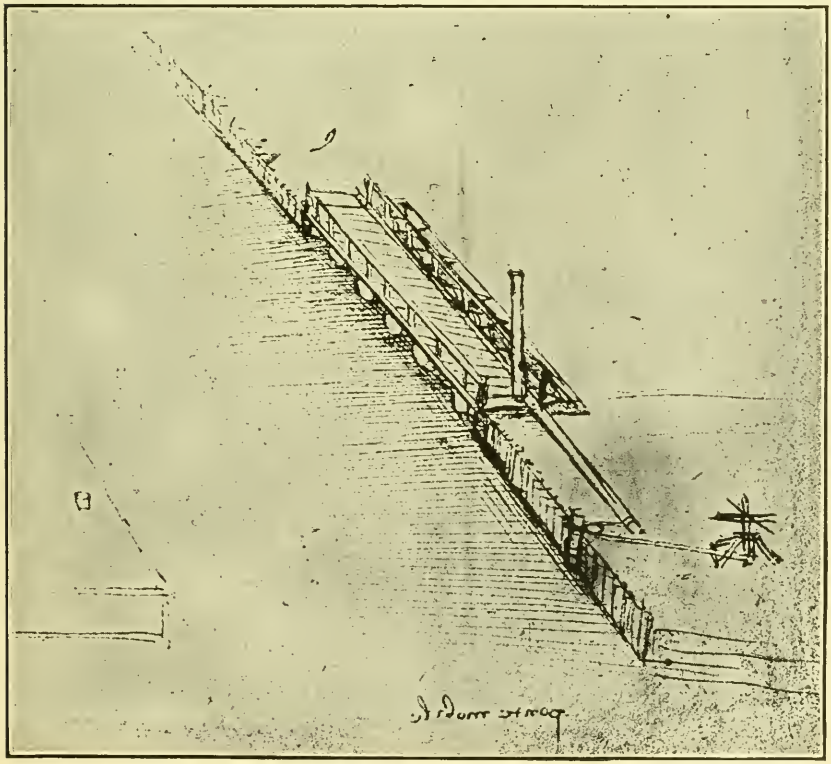
A marked improvement in the construction of fixed and movable bridges occurred during the Renaissance, and many designs were developed which have served as a basis for the modern types used in the most recent practice in Europe and America.

M. Viollet-le-Duc [5] comments that, if draw bridges were already in use toward the end of the thirteenth century, they were usually placed in the advance protective works, rather than at the portal, of

[4] *Il codice Atlantico nella Biblioteca Ambrosiana di Milano* riprodotto e publicato dalla Regia Accademia dei Lincei, Milano, 1894.

[5] "Dictionnaire Raisonné de L'Architecture," Paris, MDCCCLXIX.





Il codice Atlantico.

FIGS. 1B and 1C.

fortifications. He also states that at remote times, rolling, retractile bridges were commonly used, particularly in the southern provinces of France, and in Italy. Figure 1D shows the arrangement of some of these bridges.

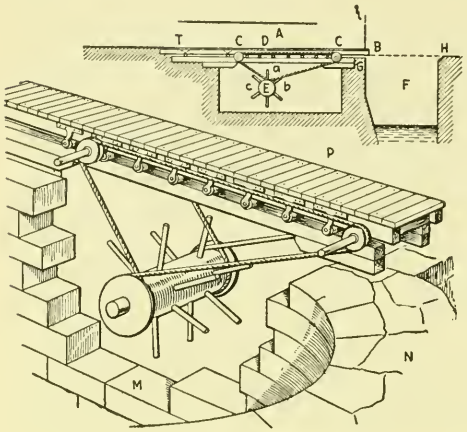


Fig. 1D.

In connection with an account of the siege of Orleans, by the English, in 1428, he shows illustrations of and describes a fortified bridge having three draw bridges. One of them was supported at one end by the wall of the fort, on a hinged pedestal. Vertically above was a pair of heavy wooden beams, parallel to the roadway, also hinged to the wall. The outer ends of these beams were attached to the rising end of the draw by vertical chains. The inner ends were floored over by a counterweight, extending the full width of the bridge, above the roadway. The draw was thus perfectly balanced. When the bridge was opened it closed the outside of the entrance to the fort, while the wide counter-weight fully closed the inside, thus forming a double protection.

4. Early modern types. — M. Gauthey[6] has described many mediaeval and early modern types of draw bridges. The oldest is that shown in Fig. 1E. The bridge is raised by a lever frame, *L*, pivoted at support *M* and attached to the end of the bridge by chains. The rear end of the frame is usually counterweighted. Equilibrium is established around the trunnion *M* when the bridge is closed. When the leaf, the frame *L*, the chains, and a line joining *M* with the heel trunnions of the leaf form a parallelogram, the bridge

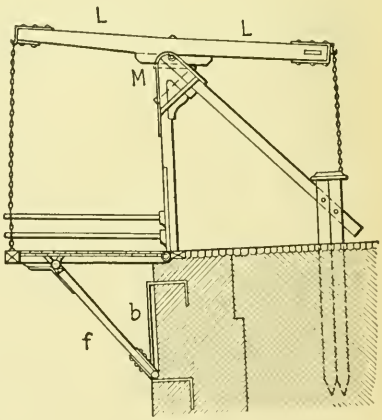


Fig. 1E.

[6] "Traité de la Construction des Ponts," Published by M. Navier, Paris, 1813.

will be balanced throughout its movement. In double-leaf bridges, struts *f*, articulated to the bridge, rest in a niche in the masonry when the bridge is closed, and slide along guides *b* when the span is moving. The two leaves thus become cantilevers extending to the middle of the waterway. Bridges of this type were built in Holland many years ago, and several of them are still in use, notably in Amsterdam.

As this type of draw bridge obstructs the tow-paths when open, M. Lamblardie devised the double-leaf rolling bascule type, one-half of which is shown in Fig. 1F, also taken from M. Gauthey's *treatise*. The heel end of the span rests on a quadrant *B*, having teeth which engage a rack on which it rolls. A pit is provided for the counter-weighted end of the bascule, and a strut supports the rear end when the

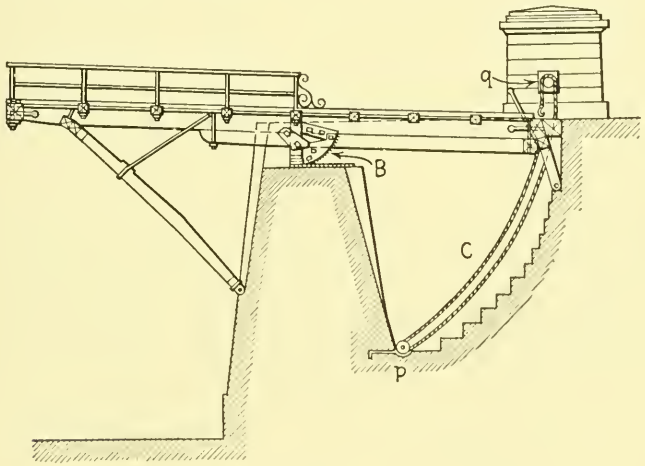


Fig. 1F.

bridge is closed. A compression strut supports the leaf when closed, as in Fig. 1E. When the bridge is opened, it rolls back away from the canal wall, and the tow-path is carried along the wall in front of the open leaf. The bridge is operated by a hand winch *q*, by means of a chain *c*, passing around a pulley at *P*. A bridge of this type was built at Havre, and others were built along the Ourcq Canal.

M. Bernard Forest de Bélidor[7] illustrated and described several types of old draw bridges. One of them was an ingenious counter-weighted bascule bridge which was built at Givet in 1716. Another was constructed at Toul. He comments that the design, which some attribute to M. Pelletier, was actually an old type which had

[7] "La Science des Ingénieurs," Paris, 1813. Original edition, 1729.

long been known in Germany, where several had been built, among others, those at Hamburg and Lubec. Figure 1G represents the essential outlines of the design. The leaf A is pivoted at G, the lever B at F, the arm C at H, the strut D at I and J, and the counterweight W, at K. The figures $LMFG$ and $FHIJ$ are parallelograms; hence, if W balances the bridge when closed, the balance is maintained throughout the movement of the span. The counterweight is much lighter than the leaf, its weight depending upon the proportions of the lengths of the various lever arms.

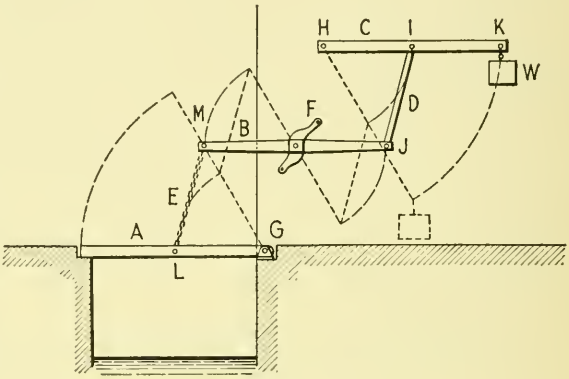


FIG. 1G.

In order to simplify the mechanism and adapt it for easy operation from within the fortifications, and to secure an exact balance and at the same time leave the passage into the forts clear of all mechanism, M. Bélidor designed a draw bridge that has been used, in modified form, in the construction of several modern bridges. Figure 1H shows the principles of his design. At one end of the span A, a tower B carries one or more sheaves F at its top. A chain or cable H is attached to the bridge at E, and to a counterweight W, which moves down a curve C, when the bridge is opened. This curve he called a sinusoid, but it is more nearly a cycloid. The counterweight W balances the bridge when closed. Let the weight of the bridge be G, and that of the counterweight be W, both acting at their respective centers of gravity. If the bridge is balanced by the counterweight in all positions, no work

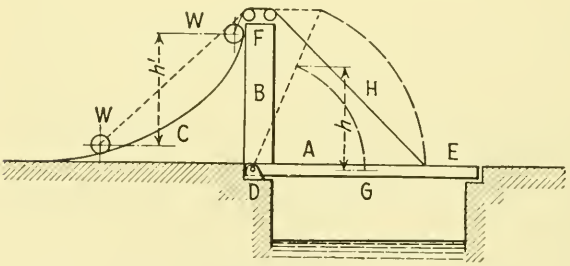


FIG. 1H.

is done during the movement of the span. Hence $Wh' = Gh$. The length of the cable H, from E to W, remains constant, whatever the position of the leaf. These two conditions serve to determine any desired number of points on the equilibrium curve C.

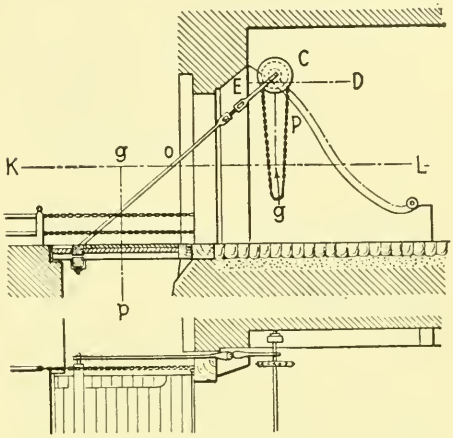


Fig. 1 I.

M. J. V. Poncelet[8] attributes a useful modification of M. Bélidor's design to M. Delile. Figure 1 I shows the general arrangement, which is quite similar to that of Fig. 1H except that a strut O connects the bridge with the counterweight. The equilibrium curve is determined in the same manner as that described above, but it is quite different in its form. This bridge is operated by driving the counterweight down the curve by man-power or otherwise.

Figure 1J shows a further modification by Lieut.-Col. P. Bergère. Here the lever BC is attached to both the bridge and the counterweight, and is pivoted to a roller at O, which moves along a horizontal track MN when the bridge is opened.

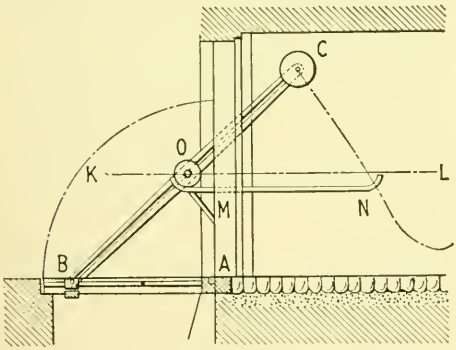


Fig. 1J.

About 1810 or 1811, Capt. Derché built bridges at Osopo and Palmanova of a design shown by Fig. 1K. The balance is maintained by a counterweight Q, which unwinds a chain from a drum with a spiral groove of varying radius, while the chain attached to the bridge winds upon a drum of constant diameter. The radius of the spiral groove is made such that an exact balance is maintained while the bridge moves.

[8] "Cours de Méchanique Appliqué aux Machines," Paris, 1876, Vol. II. The section relating to draw bridges was presented before the Academy of Science, Feb. 17, 1834, and published in 1835.

Another interesting design is shown in Fig. 1L. This appears to have been proposed by M. Poncelet, to simplify previous designs, and at the same time to secure an exact balance. The counterweight moves vertically and exerts a constant pull upon the bridge. A curved track *Cba* is attached to the moving leaf. A groove is provided on the track, around which a chain is passed. One end of the chain is anchored to the end of the curved track and the other is attached to the counterweight. The curve is proportioned so that the effective lever arm of the chain about the trunnion *A* is reduced as the bridge is opened and the chain is unwound, thus producing the required balance during the movement.

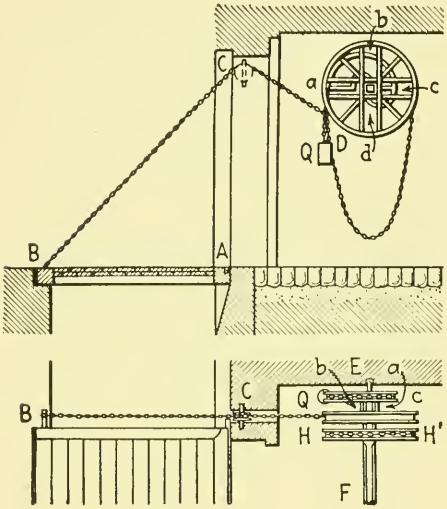


FIG. 1K.

These examples include a few of the many ingenious methods used long ago for balancing draw bridges. They are enough to indicate that some of the early engineers gave much study to the problem, and designed efficient and practical draw bridges. They also show that our more recent designers owe a heavy debt to the pioneers in these types of construction. While the early military bridges were of short span and light construction, the methods used in their design were sound, and only require such modifications as are demanded by modern heavy loads and the use of modern power-driven machinery.

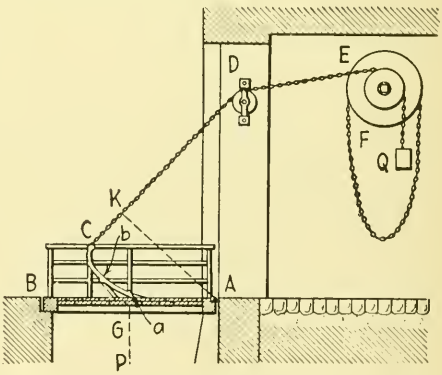


FIG. 1L.

5. Early swing bridges.—Draw bridges and bascule bridges appear to have been used much earlier than swing spans, and it is difficult to

establish the earliest use of the types revolving about vertical axes.

M. Bélidor, in "La Science des Ingénieurs," already referred to, illustrated and described a swing bridge consisting of two spans, each with two unequal arms, the longer meeting at the middle of the channel as shown in Fig. 1M. The spans were center-bearing. The center had an upper disc, with a spherical surface convex downward, resting upon a lower disc, the top of which was a spherical surface concave upward. Outside of the disc center was a metal ring with 12 balance wheels turning on axles, which served to limit the oscillations of the bridge when revolving. The long arms, meeting at the middle of the channel, were cut at an angle with the center line, and the wooden end floorbeams were finished with a tongue on one and a groove in the other. The spans opened in opposite directions, and when closed they were continuous for shears, but discontinuous for moments, at the mid-channel joint.

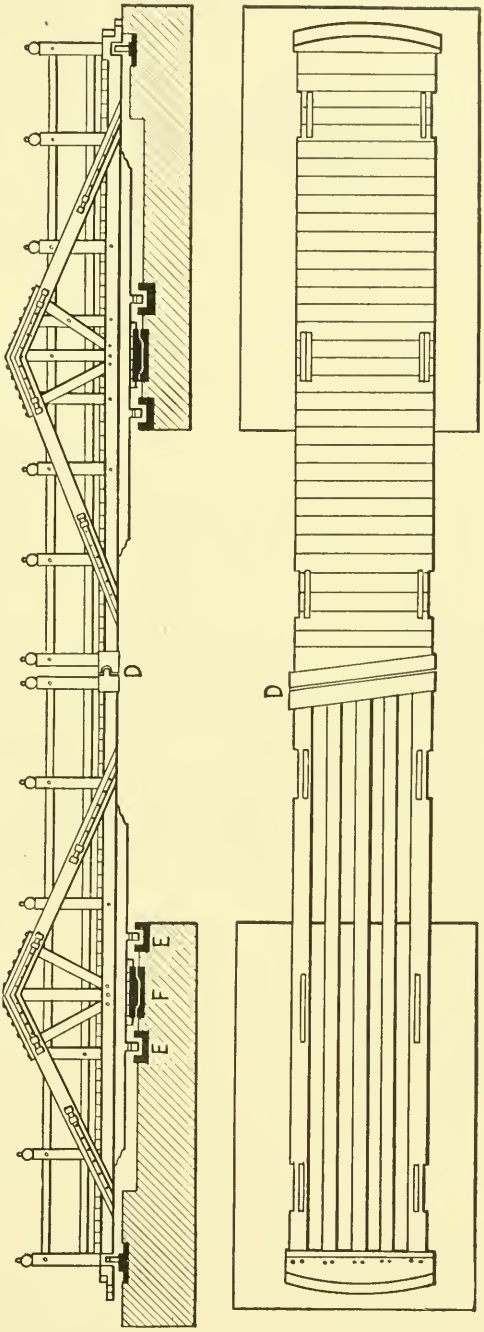


FIG. 1M.

Figure 1N shows one-half of a double-swing bridge at Cherbourg which is fully described by Bélidor.[9] He states that, " Of all the swing

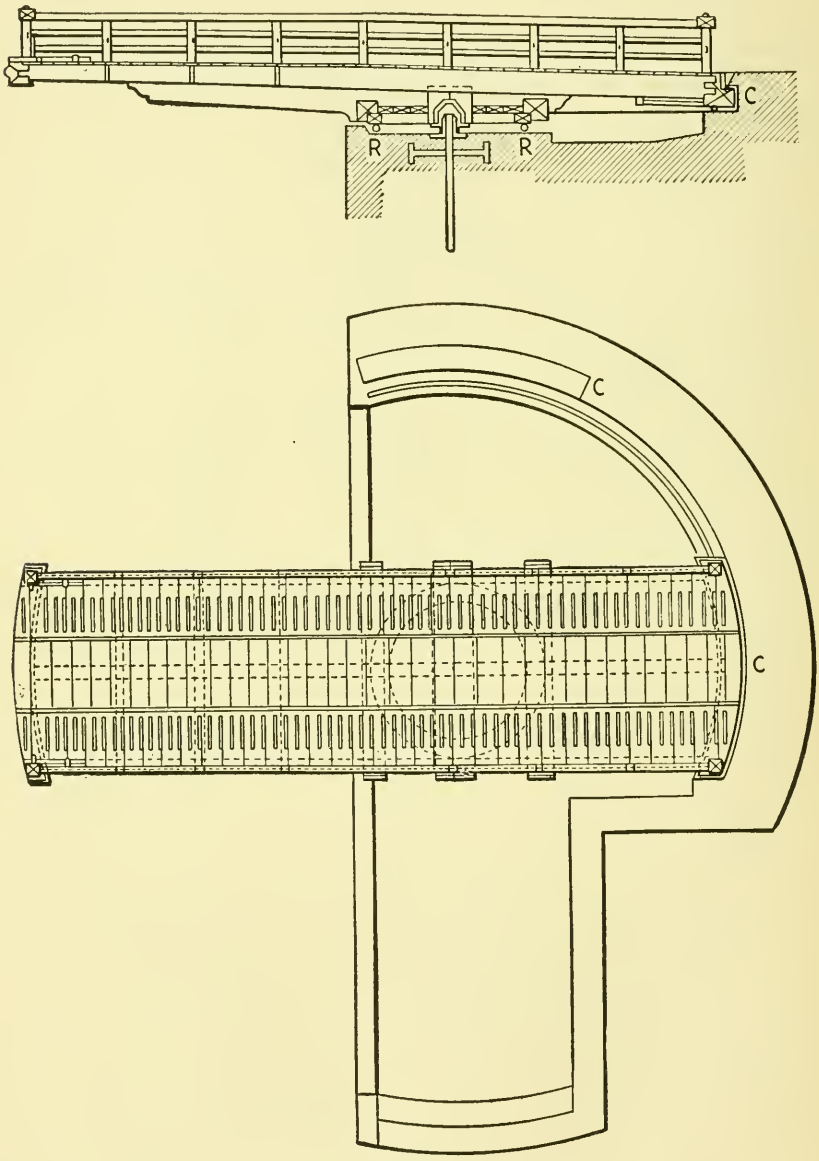


Fig. 1N.

[9] "Architecture Hydraulique," by M. Bélidor, Paris, MDCCLXX, Part II, Vol. II, p. 428.

spans which have come to my attention, none is more perfect than that for the large lock at Cherbourg." This bridge was designed by M. Salomon de Caux, the eminent engineer and architect, "Royal Engineer-in-Chief," after his return to France; and probably was built about 1625. The clear waterway was 42.65 ft., the width of the roadway was 12.5 ft., the length of the long arms was 27.6 ft., and that of the short arms 15.4 ft. The two swing spans were center-bearing, with balance wheels, consisting of balls turning on axles, to steady the bridge when turning. The balance-wheel track was 10 ft. in diameter. The short arms were not counterweighted, but were furnished with balls, turning on axles, which were attached to the top of short cantilevers at the bottom of the end floorbeams. These balls engaged an over-hanging coping, of heavy stones, cut to a segment of a circle as shown on the plan in Fig. 1N. Metal strips were embedded in the masonry to form raceways for the balance and rear anchor balls, in order to protect the masonry from wear.

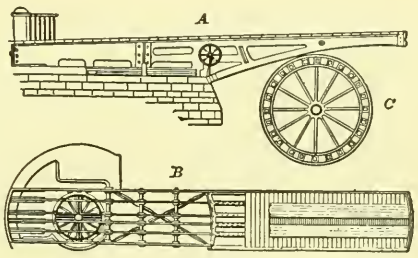


Fig. 1O.

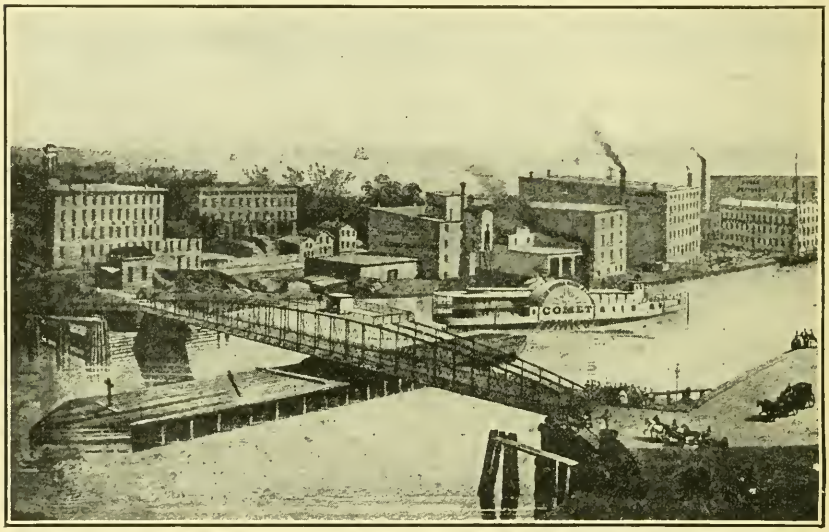
The centers were of cast iron. The lower casting was securely bedded in the cut-stone masonry, and anchored by a central pin extending several feet into it. Lead was poured into the casting around the pin. A steel disc was set into the top of the pivot, and the load was delivered to it by a similar steel disc in the inside of the upper socket-casting, which, in turn, was bedded in pitch, and securely anchored to the wooden center girders of the bridge.

When the bridge was closed the ends of the channel arms interlocked, to form a shear lock, the bridge thus becoming continuous for shears, but discontinuous for moments. An effective end-lifting device on the rear of the short arm provided means for bringing the rear anchor balls into contact with their raceways. As all clearances were taken up when the rear ends were lifted, the only deflections under live load were those due to elastic distortion. The bridge was stiff and satisfactory under the heaviest vehicles of the time.

The first iron bridge was probably the cast-iron arch bridge at Coalbrookdale, in England. It spans the river Severn and was designed by Pritchard, in 1773, and built by Abraham Darby, in 1776. The use of cast iron was soon extended to the construction of movable bridges.

Some of the earliest were built in connection with the London Docks, completed in 1805. St. Katherine's Docks, near the Tower of London, were finished in 1818. Fig. 1O shows one-half of a double-leaf swing bridge at these docks.

The longer arm extends to the middle of the channel, the shorter, or shore arm, being counterweighted. When the bridge is closed the longer arms meet at the center of the channel and also abut against the masonry, thus forming an arch for live loads. The clear span between the walls is 44 ft. 9 in. Each leaf is rim-bearing, and turns on 24 rollers, supported on a cast-iron track. The bridge is operated by hand



Chicago Historical Society.

Fig. 1P.

winches, geared to a horizontal rack. It is stated that "the bridge is well balanced and easily swung upon its vertical pivot by a single attendant."

6. First Rush Street Bridge, at Chicago.—Most of the early movable bridges in the United States were wooden structures. As there is very little early American engineering literature available, it is difficult to establish where the first iron draw bridge was built. It appears that the first bridge built over the Chicago River, at Rush Street, was the earliest all-iron movable bridge that was built in the Middle West.

Figure 1P shows the general arrangement of this bridge, which was a rim-bearing structure. "The Story of Chicago," by Kirkland, published in 1892, states: "In 1856 a fine iron bridge (the first in the West),

was built across the river at Rush Street and cost $48,000, of which the city paid $18,000, and the Galena and Illinois Central Railroads $15,000 each." The Galena Railroad is now a part of the Chicago and Northwestern Railway. The compression members of the trusses were of cast iron and the tension bars of "the very best quality of wrought iron." The bridge was probably operated by steam-power, for the records show that it was destroyed Nov. 3, 1863, when it was opened while a drove of cattle were on one end. It was rebuilt and finally destroyed in the great fire of 1871.

7. The New Epoch.—Many other designs of early draw bridges, bascules, swing spans, and retractile bridges might be cited, but enough have been mentioned to indicate the skill of some of the early engineers and to bring out the fact that the design of movable bridges has been a gradual evolution from the early types to the more refined designs demanded by modern conditions.

A new and most important epoch in this and many other branches of engineering is directly due to the invention of the steam engine by James Watt, in 1769, and the introduction of practical steam locomotives by George Stephenson, in 1829. These two inventions ushered in the new epoch, marked by the manufacture of power, and are responsible for the rapid advance in nearly all forms of engineering design that may be termed modern. The design and construction of movable bridges has kept pace with the advances in other branches of engineering, until it has become almost a specialty in the United States.

CHAPTER II

TYPES OF MOVABLE BRIDGES

1. General.—The types of movable bridges to be treated here are those made movable to accommodate navigation traffic in the waterways crossed. They may be adapted for railway or highway bridge traffic. The many kinds intended for other purposes, as for cranes in shops and yards and for handling various materials, are usually so special that they must be considered as different types. The methods used in the design of superstructure and machinery of ordinary draw bridges are applicable, in many cases, to the more specialized structures. Some, like coal- and ore-handling bridges, are complicated machines and require special consideration outside of the limits of the present volume.

The engineer should approach the design of draw bridges for railway and highway traffic with an appreciation of the fact that even these simple structures are machines, and that they require more care and thought than is given to fixed bridges of similar size and importance. They must be safe and efficient for ordinary traffic; and the arrangement, design, and construction of the machinery must be adequate for its purpose, easily maintained, and adapted for long service.

2. Classification.—Movable bridges may be classified as follows:

(1) Swing bridges, turning about a vertical axis.

(1*a*) Shear-pole bridges, turning about a vertical axis near one end.

(1*b*) Folding, or jack-knife, bridges, turning about vertical axes near one end.

(2) Bascule bridges, turning about a horizontal axis, rolling back on a circular segment, or turning about a movable horizontal axis.

(3) Lift bridges, moving vertically.

(4) Retractile, or traversing, bridges, moving horizontally.

(5) Transporter bridges, consisting of an overhead fixed span carrying a suspended moving platform.

(6) Pontoon bridges, floating, and arranged to turn about a vertical axis at one end.

Some of the types named in this classification may be subdivided with

reference to the details of design. The list includes the types most likely to be required in practice.

Brief descriptions of the various types follow.

(1) *A swing bridge* consists of a superstructure arranged to turn about the vertical axis of a pivot anchored to the center pier. In ordinary cases the pivot is at the center of a span of two equal arms, which balance each other when the bridge is open, thus providing two equal openings for navigation. It is sometimes necessary to place the pivot near one end. The shorter arm must then be counterweighted to balance the longer arm when the bridge is open. In such cases there is usually only one channel available for navigation. In both types the ends of the closed bridge must be lifted, not only to afford rigid supports for the trusses, but by an additional amount to neutralize a part of the end deflection of the arms, so that loads upon one arm will not lift the end of the other from its supports. In rare cases two adjacent swing spans meet over the center of the middle channel. The ends must then be locked together where they meet and the other ends anchored to the piers, when the bridge is closed.

There are three types of swing bridges: center-bearing, the entire weight of the open span being supported by the center pivot; rim-bearing, the weight being carried by a cylindrical drum, supported by rollers when the span is open; and a combination of these two.

(*a*) *A shear-pole bridge* is a special variety of swing bridge. The pivot is located near one end of a single arm. When open, the other end of the arm is supported from the top of a two-legged shear pole, on the abutment, by rods which are attached to a pivot on its top, directly over the pivot supporting the span below. The shear pole is anchored to its supports and stayed by guy rods extending to anchors on the approach. When the bridge is closed and the swinging end lifted, the arm is a simple span supported at both ends.

(*b*) *A folding, or jack-knife, bridge* is also a special kind of swing bridge. Jack-knife bridges usually consist of a deck girder under each rail, one or more needle beams under the free end, and a gallows frame over the pivots, braced and anchored back to the shore; with ropes, or rods, extending from the needle beams to the top of the gallows frame. Each girder has a pivot. They are of unequal length and the pivots are at different distances from the back of the abutment, so that the girders lie side by side and close together when the span is open. The girders are connected by S-shaped pivoted struts and the action is quite similar to that of an old-fashioned parallel ruler. This type can only be used for railway bridges, as the rails are directly above the girders, and ties cannot be used. While there are a few of these bridges

still in use, mainly in New England, the type is nearly obsolete and will not receive further consideration.

(2) *Bascule bridges* are, strictly speaking, those in which one end rises as the other falls, but the term is commonly applied to any type moving about a horizontal axis, either fixed or moving, as well as to those that roll back on a circular segment. They may consist of a single leaf spanning the channel or of two symmetrical leaves meeting at its center. In the latter case the ends that meet must be locked together, to insure the same end deflection of each leaf, when loaded. A great variety of designs of this general type have been made. Many have been patented, and some are vigorously promoted by the patentees and owners of patents. Some types advocated have never been built; some are represented by only one, or a very few structures; while others have been used widely. The variety of designs is so great that only a few can be considered. The types selected will be those that have been used most frequently. Other meritorious designs might have been included, had space been available.

(3) *Lift bridges moving vertically* consist of simple spans resting on piers when closed. In most cases the weight of the lifting span is counterweighted by means of ropes, or chains, attached to the ends of the span and the counterweights, which pass up and over sheaves on top of towers at the ends of the bridge. Some of the earliest types were operated by hand winches and ropes, some by hydraulic accumulators, cylinders, and plungers, and others by gears meshing on vertical racks on the posts of the towers. For many years this type seemed to be obsolete, but during the last quarter of a century many important lift bridges have been built and are in satisfactory operation, and the type seems likely to survive and give good service under proper conditions.

(4) *Retractile, or traversing, bridges moving horizontally.* When closed they form simple spans across the channels. Some telescope inside of the adjoining spans; others recede above the approaches, the rear end being tilted upward and the free end downward. In some cases the approach span is first moved aside, transversely, to permit the draw span to recede in its place. Sometimes the bridge moves back diagonally, as in the case of several bridges in New York City. There are troublesome mechanical details involved in the general arrangements. More power is required than for any other type. The power needed and the time of operation are so great that few retractile bridges are now used. Small retractile bridges were quite common in southern Europe in mediaeval times. Their use was revived in the modern period, but they now appear to be nearly obsolete.

(5) *Transporter, or ferry, bridges* are rarely used. The earliest examples were of the wire-rope suspension type. A fixed span across the channel is supported on shore towers at a sufficient height to clear navigation. A platform, or car, is suspended under the span and arranged to travel across the channel from shore to shore. The capacity for traffic is limited and this type is more nearly a substitute for a ferry than a true bridge, for it has no continuous floor available for a steady stream of traffic.

(6) *Pontoon bridges* are probably one of the earliest types of movable bridges. They are adapted for use when local conditions prevent the construction of more stable structures and when a temporary crossing must be quickly made, as in military operations. In some locations where expensive permanent bridges are not warranted, they have given good service for long periods. They may consist of small boats, or pontoons, lashed together for temporary use, or more elaborate and stable pontoons in permanent structures.

3. Selection of type for a particular location.—It is difficult to give rules for the selection of the most desirable and economical design for a movable bridge. The type best adapted to one location may be unsuited to another. There are many variables in the problem and it is one that can only be solved by an extended knowledge of successful practice, after much experience and a careful study of the local conditions at any particular site.

There are some broad general principles that assist in approaching a proper solution.

(1) When the bridge is closed and ready for traffic, it should be as nearly as possible a fixed span, with positive reactions. It should be considered primarily as a bridge to carry traffic across a waterway in the safest manner.

(2) The structural and the machinery parts should be separate and distinct. When the bridge is closed and acting as a fixed span, the mechanism should not be under stress. It should only be used to move the bridge.

(3) The operating machinery should be of simple design, easily installed and maintained. Its control apparatus should be conveniently arranged and located so that the operator can easily observe the movements of the bridge with reference to navigation. He should also be able to see the ends of the floor when it is being opened or closed.

(4) The machinery should be designed so that friction and lost work are a minimum. This should apply to the moving parts of the bridge itself, as well as to the internal friction of the operating mechanism.

When the friction and power for operation are a minimum, the first cost and maintenance will also be the least. The wear of mechanical parts, cost of maintenance, lubrication, and inspection will be found to increase almost directly with the power needed to operate the bridge.

Of the types named in Art. 2, Nos. 1*a*, 1*b*, 4, and 5 are only desirable in some special locations. Type 6, pontoon bridges, have been found economical in some places where the conditions did not require more stable structures. Under normal conditions the selection will be made from types 1, 2, and 3. Their characteristics are so different that a special statement of their relative advantages is desirable.

4. Swing bridges.—When the conditions at the site are favorable and there are no restricting circumstances, a swing bridge is the simplest, best, and most economical type in first cost and maintenance.

When the bridge is closed, the ends are raised enough to cause positive reactions under all positions of loading. The two arms then act as a fixed girder over three supports. There is no tendency of the ends to lift from their supports and no locks are needed to hold them down.

There are no mechanical joints or trunnions in the superstructure to cause friction and wear and to require consequent attention, lubrication, and maintenance.

The operating machinery is not under stress when the bridge is closed. It can be simply arranged and its only office is to transmit power to move the span.

When the ends are lowered and the bridge is swung, its entire weight is carried on the center pier. It may be supported by a simple pivot, a drum turning on rollers running on a circular track, or a combination of the two. Any of these arrangements results in small turning friction.

No counterweights are required and the only mass to be accelerated is that of the span itself. Less power is needed to turn a swing bridge than for any other type. When power is expensive this results in an important economy in operation.

The operator's house can usually be placed above the roadway, at the center of the span. It is easy for the operator to observe the ends of the bridge at the breaks in the floor, and all movements of the structure are under his observation. He also has an unobstructed view of the waterway in both channels, and approaching vessels are in full view.

When the bridge is open, it offers no more surface to the wind than when it is closed. The wind pressure is nearly balanced under normal conditions. The resistance of the wind to turning being small, little additional power is required to turn in a wind.

The center of gravity of the span is low and the center of pressure

of wind forces remains at a constant elevation during motion. The general stability is little changed when the span is open.

The distribution of the dead load to the piers is better than in counterweighted bascule types. When the ends are properly lifted, part of the dead weight is transferred from the center to the end piers. For ordinary highway spans this is from 10 to 15 per cent of the dead weight, for double-track railway bridges from 15 to 25, and for single-track spans from 25 to 35 per cent. This not only insures positive end reactions at all times when the bridge is closed, but allows some reduction in the total load upon the center pier.

The only weight carried on the piers is that due to the dead and live loads on the span. No provision need be made for heavy counterweights, as in most other types. This is a decided advantage when the foundations are in soft material requiring piles or widely spread footings.

In most locations where a swing bridge can be used, the arms are of equal length, thus providing two navigable channels. This is of material advantage when the navigation traffic is large, for vessels passing in opposite directions will naturally use the separate channels.

A swing span must be fully opened to allow any but the smallest vessels to pass, and for small vessels with low masts the time of operation is longer than with some other types of bridges. This is not true when the height of stacks and masts of large vessels require other types to be fully opened. When a single ship is passing, it is common practice to move one arm away from the approaching vessel and then to resume turning so that the other arm follows the stern of the vessel quite closely. This shortens the time that the span is closed against bridge traffic.

In some other types it is necessary to increase the size of the operating motors to make them adequate to move the structures against the wind. This is seldom needed for swing bridges. The result is that a swing span may be designed to operate in the normal time.

When a draw bridge is needed in connection with fixed spans for crossing a wide waterway, and only one channel is required, the second arm of a swing span bridges a part of the waterway, reducing the length that must be covered by fixed spans. This is an advantage not offered by the other types and should always be considered when comparing the costs of various designs.

When a swing span is open it should be protected by fenders longer than the span. These are of pile and timber. Their ends should be brought to a point and a heavy cluster of piles driven to turn aside any carelessly navigated vessel, so that the bridge superstructure shall not be damaged by collisions.

Swing spans for city highway traffic may have any desired type of pavement, for the floor remains horizontal at all times.

In locations where the climate makes it necessary to close navigation during winter, a swing draw is practically a fixed bridge during the closed season. Repairs and renewals of mechanical parts can then be made almost regardless of the bridge traffic. There is usually ample room on the center pier to allow the bridge to be jacked up for inspection, repairs, and renewals of parts. This is easily done when the draw is of the center-bearing type.

The pivot of a center-bearing bridge is more easily lubricated than the trunnions and joints of other types. It is only necessary to fill the socket, containing the center discs, with oil, which requires renewal only at long intervals. No pressure system or special lubricating devices are needed.

In addition to economy in foundations, masonry, and weight of superstructure, another important saving of the swing bridge is the reduced cost of the steelwork and erection.

More attention is now given to the appearance of movable bridges than formerly. A swing span with symmetrical arms can be designed with pleasing outlines. This can be done without sacrificing meritorious design or economy.

When a double-deck bridge is required, a swing span can be used without complicating the arrangement at the ends, where they join the fixed spans. The heaviest movable bridge in the United States is a center-bearing swing span of this design.

When local conditions at the site prevent the use of equal arms, a swing bridge with unequal arms, with the shorter end counterweighted, will be found to be economical in many cases. A larger power plant will be needed for the same weight of superstructure than when the arms are equal, for the wind pressure is unbalanced.

For unimportant crossings, a hand-operated shear-pole swing bridge is the cheapest of all types. When properly designed, the closed bridge is practically equivalent to a fixed span.

5. Bascule bridges.—When local conditions are unfavorable for the use of swing spans, the bascule type is desirable in many cases. Where broad waterways are crossed by railways with growing traffic, as many parallel bascules can be built as required, and they may be closely adjacent. A minimum space is occupied and the approaches are easily built. Bascules are also desirable in cities having narrow waterways, as in Chicago. Here the adjacent property is so valuable that space cannot be afforded for opening swing spans with the center pier on one shore, and the channel is so narrow that center piers in it

form serious obstructions to navigation. Similar conditions obtain in many cities.

When a balanced bascule bridge is closed, there is no dead-load reaction at the rising end. The moving leaf must be latched down to prevent it from being opened by accident. In some types there is a negative dead-load reaction at the trunnion end. There must be some clearance between the trunnion and its bearings to allow it to turn. Passing live loads take up this clearance and cause vertical motion and impact which are transmitted through the structure to the counterweights.

Most bascule types contain large trunnions, and several forms also have systems of articulated links connected by pins, forming mechanical joints. These are troublesome to lubricate and to maintain, and the frictional resistance is large because the trunnions and joints are always under heavy stress. In some designs the counterweights are supported on separate trunnions, which are under stress whether the leaf is closed or open. Any deflection of the trunnion supports increases the friction and wear of the parts.

The power plant and operating machinery must be ample to overcome the friction of all trunnions and joints and to accelerate the mass of the moving leaf, the counterweights, and the auxiliary structural members. As the counterweights frequently weigh from two to three times as much as the leaf, the power required is proportionately large.

When the bridge is open the entire leaf and the roadway are exposed to wind pressure. The power plant and gear train must be proportioned to hold the bridge against the wind when in any position. The center of wind pressure on the open leaf is much higher than when the bridge is closed. This affects the general stability and must be considered in the design.

The piers and foundations at the trunnion end must be designed for the weight of the bridge, its counterweights, and the overturning moment of the wind pressure. In rolling-lift types the dead weight of the bridge and counterweights moves horizontally during opening and closing. Each pier and its foundations must be proportioned for the maximum loads. This is not serious when the foundations rest on rock, but increases their size and cost when they are in soft material. There is also an added liability to unequal settlement with all its attendant complications.

A bascule bridge provides only one channel for navigation. The single channel must be wider than either of the two channels provided when a swing span is used. If vessels meet when passing through the

draw, they must both use the same channel and danger of collisions is increased.

Bascules do not need to be fully opened to allow small vessels to pass. In many places such vessels form a large part of the total and bridge traffic will be interrupted less than when a swing span is used.

The power plant, when designed to operate the bridge under the most unfavorable conditions, has a large surplus of power for normal operations when there is no wind. The time of operation will then be considerably shortened when variable-speed motors are used. This is the principal reason that bascules can be opened in less time than swing bridges with smaller motors.

The fenders to protect the piers and auxiliary spans can be shorter than for swing spans. There has been a tendency to take undue advantage of this. In one case the auxiliary span adjacent to the bascule was not protected and was wrecked by a large steamer which struck the auxiliary span in the middle, having missed the channel entirely. Adequate fenders should always be provided to prevent such accidents.

The choice of pavement for highway bascules is quite limited. It must be of a type that can be securely attached to the floor, to prevent displacement when the leaf is open. It should also be as light as feasible, for every pound must be balanced by from two to three pounds of counterweight, with the consequent increase in the mechanical parts. These conditions practically limit the pavement to some form of wooden construction.

It is more difficult to make renewals and repairs of bascules than of swing bridges. When the design includes systems of articulated links, under stress from the counterweights, it is necessary to use temporary supports for the counterweights when repairs of the trunnions, links, or joints are to be made. In several cases the segmental and supporting girders of rolling bascules have required repairs or renewals. As they carry the entire weight of the leaf and counterweight, it is difficult to devise means of making the repairs without closing the bridge to traffic. Such repairs demand engineering skill of the highest order for their successful accomplishment.

The unit cost of drawings, fabrication, and erection of bascules is more than for swing spans.

Deck bascules are often designed to imitate arches. The lines are pleasing, but the structure is not an arch, although it may look like one. Such imitative construction is not truly artistic when broadly considered. Most types of through bascules with overhead counterweights are unsightly. This is particularly true of single-leaf bridges. In some cases their awkwardness is partly obscured by extraneous construction,

like ornamental panels above the piers, to hide the counterweights. These screens are not essential to the structure and serve no useful purpose. It is a difficult matter to design a through bascule that is attractive in appearance.

Sometimes it is necessary to replace an old swing span by a new structure. When conditions at the site are favorable, a bascule can be built in open position adjacent to one end of the old span without interference with traffic. The old span can then be removed by floating it away on barges and the new bascule closed, the new piers having been built previously.

6. Vertical-lift bridges.—There are some locations where the vertical-lift type is desirable and economical. When the floor is high above the water, a comparatively small lift will allow all vessels to pass. When the grade line is low, many small vessels can be passed by partly raising the draw. When the bridge crosses a wide alluvial stream and the channel is likely to shift its position, the fixed spans may be made alike and each adapted to carry towers. If the channel changes, the towers, counterweights, and machinery can be moved to other spans adjacent to the new channel span. No other type offers such an economical method of adjusting to fit shifting channels.

It is sometimes necessary to change an existing fixed span to a movable one, when a stream is made navigable. Towers, counterweights, and machinery can be built at the ends of two adjacent fixed spans and the span between them transformed to a movable one.

When the ends of a draw span must be skewed, this type offers advantages over the others, for the ends may be skewed to almost any degree with relatively small complications.

On railways with growing traffic, parallel bridges can be added as readily as when bascules are used. This type is also excellent when future changes of grade are contemplated. The towers and approaches can be built so that it is easy to change the elevation of the draw to fit the new grades when in its closed position.

When the lower floor of a double-deck bridge is well above the water, the lower deck is sometimes arranged to be lifted to contact with the upper deck to allow vessels to pass underneath. In some other locations similar construction is used, and after the lower deck has been raised the entire structure is lifted an additional distance to clear the tallest masts. In one such bridge, it is necessary to lift the upper deck only about six times a day, the lifting of the lower deck being sufficient for all the other openings. The upper deck is undisturbed when the lower is lifted to allow the smaller vessels to pass.

Closed vertical-lift spans have no positive reactions at either end

from dead load. Both ends must be latched down to prevent accidental displacement. When the counterweights are connected directly to the ends of the lift span by wire ropes or chains passing over sheaves on the towers, the weight required at each end is one-half that of the span. The total weight is thus much less than for bascules. The mass to be accelerated is more than for swing spans and less than for bascules. The friction of the mechanical parts is in about the same proportion. The wind pressure increases the resistance to motion less than in either of the other types. It increases the side pressure upon the guides in the towers, but rollers and similar devices are used to diminish the friction. The power required is intermediate between that for swing and bascule spans.

When a vertical lift is fully open the center of the wind pressure is higher than for the other types, and care must be exercised to secure general stability.

While there are no positive dead-load reactions at the ends of the span itself, the weight of the span and counterweights is delivered to the piers through the towers. Sometimes the towers are mounted upon the ends of the adjacent fixed spans. With equal approach spans, the loads on the two rest piers are equal. The distribution of loads on the foundations is more uniform than for bascules, which is an advantage when they are in soft material.

Like bascules, this type affords only one channel for navigation, with the resulting danger of collisions in the channel. The time required for small vessels to pass is less than for swing spans and about the same as for bascules. When the span must be fully raised the time will be about the same as for the swing type.

The floor is always horizontal and any design of pavement may be used on highway structures.

Renewal and repairs of mechanical parts are difficult. The counterweight chains or cables can be renewed one at a time. When the sheaves and trunnions must be repaired, one encounters an engineering problem that is troublesome and expensive to solve. It is even more difficult to renew the systems of links that are substituted for cables by some designers.

The unit cost of the superstructure is greater than for swings but less than for bascules. When the lift is not more than about 50 or 60 ft., the foundations and superstructure, complete, will cost about the same as for a swing, but the length bridged is less. When the customary under-clearance of from 135 to 150 ft. is required, a swing span is more economical.

Vertical lifts are usually symmetrical and the lines of span and tower

can be made pleasing, although the great height of the towers detracts from the architectural effect. Sometimes a light fixed span joins the tower-tops and is used to carry pipes and wires. Such an upper span improves the appearance of the bridge as a whole, but should be used only when needed. This type is convenient for double-deck bridges. The floor joints can be arranged as readily as when swing spans are used.

7. Statistics of draw bridges.—The preceding articles have enumerated some of the more salient advantages of the three most prominent types of modern movable bridges. An engineer, when discussing with another the question of the selection of the most suitable type, expressed his views quite simply. His first choice was a bridge that remained on the ground when open; his second was one that had only one end lifted in the air; and his third, one in which both ends were lifted. That is, first, swing; second, bascule; and third, vertical-lift.

It is interesting to ascertain the actual practice of designers in recent years and their present tendency. For this purpose, a study was made of the movable bridges built by the largest fabricator in the United States during the period from 1903 to 1923 inclusive. This time was divided into four intervals of seven, five, five, and four years, respectively. Table 2A gives the statistics in numbers and percentages. There has been a large reduction in the construction of railway bridges since the World War; but this is not so marked in highway bridges. In both railway and highway bridges there is a proportionately increased use of the bascule types toward the end of the period; and this tendency is more marked in highway than in railway structures. Natural conditions at the sites explain much of this change in practice. There are more highway than railway bridges in cities, where property is expensive and waterways are crowded.

Table 2B gives a summary of the three types by linear feet, percentages, and weights, also, in another column, the weights per linear foot bridged. The length covered by swing spans is much greater than for the two other types combined, but the weight per linear foot is less than for either. The difference in weight per linear foot is larger for highway than for railway structures. This is because in situations where swing spans could not be used, many city bridges have been of the bascule type; moreover, most of the highway swing spans are small.

The selection of the most suitable type of design for any site should be made after a thorough investigation of all economic and local conditions, and the engineer should approach the problem with an open mind and make his choice for the best interests of his clients and the Public.

8. Cost of maintenance, repairs, and operation of draw bridges.—The cost of maintenance and repairs is an important consideration in

determining the best design for a draw bridge. These items are continuing charges against the structure, and increase with its age. Simple structures, with a minimum of machinery, well arranged, liberally proportioned, built from the best material, and of first-class workmanship,

TABLE 2A

MOVABLE BRIDGES—1903 TO 1923

Description	1903 to 9		1910 to 14		1915 to 19		1920 to 23		Total	
	Number	Per Cent	Number	Per Cent	Number	Per Cent	Number	Per Cent	Number	Per Cent
Railway Bridges										
Swing spans	97	89.8	42	79.3	25	73.6	10	71.4	174	83.3
Bascule spans	11	10.2	5	9.4	8	23.5	4	28.6	28	13.4
Vertical-lift spans	.....	.....	6	11.3	1	2.9	.....	.....	7	3.3
Totals	108	100.0	53	100.0	34	100.0	14	100.0	209	100.0
Per Cent	51.7	.....	25.3	.....	16.3	.....	6.7	.....	100.0	
Highway Bridges										
Swing spans	41	67.2	15	68.2	1	7.7	1	9.1	58	54.2
Bascule spans	17	27.9	3	13.6	11	84.6	9	81.8	40	37.4
Vertical-lift spans	3	4.9	4	18.2	1	7.7	1	9.1	9	8.4
Totals	61	100.0	22	100.0	13	100.0	11	100.0	107	100.0
Per Cent	57.0	.....	20.6	.....	12.1	.....	10.3	.....	100.0	

TABLE 2B

MOVABLE BRIDGES—1903 TO 1923

Type	Number	Per Cent	Length	Per Cent	Weight	Per Cent	Weight per Foot
Railway Bridges							
Swing	174	83.25	36,382	86.30	126,122,000	70.96	3467
Bascule	28	13.40	4,327	10.25	38,186,000	21.48	8825
Lift	7	3.35	1,457	3.45	13,441,000	7.56	9227
Totals	209	100.00	42,166	100.00	177,749,000	100.00	4216
Highway Bridges							
Swing	58	54.21	10,014	59.51	18,205,000	25.36	1818
Bascule	40	37.38	5,527	32.85	45,645,000	63.61	8260
Lift	9	8.41	1,286	7.64	7,913,000	11.03	6154
Totals	107	100.00	16,827	100.00	71,763,000	100.00	4265

will be the most economical over a long term of years. The interest on the increased first cost will be less than the larger upkeep of an inferior structure.

Data concerning the cost of maintenance, repairs, and operation are seldom available. This is particularly true of railway bridges, for such costs are not made public. The *Annual Reports of the Department of Bridges,* City of New York, to the Mayor; supplemented by details published in *The City Record,* contain some interesting and valuable information. Fortunately, the data are given for a variety of movable bridge types. This information is given in condensed form in Tables 2C to 2J, inclusive. The costs, in each case, include painting, renewals and repairs of the structure, roadways, and walks, as well as operation. The costs for the approaches, which in some cases are very long, are also included. It is unfortunate that the cost for the draw spans was not kept separate, for the totals indicate disproportionately large costs for the large bridges with long approaches, shown in Tables 2C and 2D. These are large City swing bridges. In the case of Bridge No. 36, Tables 2G and 2H, most of the cost for structural repairs was caused by necessary reconstruction of parts of the bascules. Bridge No. 37, Tables 2E and 2F, is one of the busiest known. It was opened 90,131 times in the four years. This corresponds to an average of 1877 openings per month, or over 62 per day. The total cost of maintenance and repairs for four years was $4687, or 5.2 cents per opening. Operation cost $38,572, or 42.8 cents per opening, a total of only 48 cents per opening. This bridge is now inadequate for the heavy street traffic and must soon be replaced by a much wider structure.

TABLE 2C

Large City Swing Bridges

General Data

Bridge Number	Location	Type	Length of Draw, Feet	Length of Approaches, Feet	Roadway, Feet	Sidewalks, Feet
5	Willis Avenue	Rim-bearing	304	2167	1@42	2@ 9
6	Third Avenue	Rim-bearing	300	2428	3@17	2@ 9
7	Madison Avenue	Rim-bearing	300	1572	2@27	2@ 9.33
8	145th Street	Rim-bearing	300	1294	2@27	2@ 9
9	155th Street	Rim-bearing	408.5	2126	1@40	2@ 9
12	University Heights	Rim-bearing	270	820	1@33.5	2@ 7.5
13	Ship Canal	Center-bearing	270	276	1@32.8	2@10.83

TABLE 2D

LARGE CITY SWING BRIDGES—COST OF MAINTENANCE AND OPERATION

1913 to 1916, Inclusive

Bridge Number	Number of Openings	Painting	Structural Repairs	Walks and Roadways	Operation	Total	Age in 1916, Years
5	22,734	$8,763	$9,036	$14,526	$113,310	$145,635	15
6	10,692	10,812	6,265	2,454	98,793	118,324	18
7	8,884	496	1,043	142	76,328	78,009	6
8	7,204	5,359	2,741	190	81,815	90,105	11
9	4,085	857	7,313	5,570	80,485	94,225	21
12	4,190	3,755	1,640	523	55,878	61,796	21
13	3,759	364	2,111	2,435	62,334	67,244	8
Totals.....	61,548	$30,406	$30,149	$25,840	$568,943	$655,338	14.3
Cost per opening.....		0.49	0.49	0.42	9.25	10.65	
Per bridge per year	2,198	1,089	1,077	923	20,320	23,409	

TABLE 2E

SMALL CITY SWING BRIDGES

General Data

Bridge No.	Location	Type	Length of Draw, Feet	Length of Approaches, Feet	Roadway, Feet	Sidewalks, Feet
19	Unionport	Rim-bearing	137	111	1@20	
21	City Island	Rim-bearing	168	2000	1@32.5	2@7
22	Eastchester	Center-bearing	105	.....	1@19	
37	Greenpoint Ave.	Rim-bearing	206.67	36	1@20.67	2@5
38	Meeker Avenue	Center-bearing	200	85	1@20	2@5
39	Grand Street	Rim-bearing	229.58	.....	1@19.5	2@6

TABLE 2F

SMALL CITY SWING BRIDGES—COST OF MAINTENANCE AND OPERATION

1913 to 1916, Inclusive

Bridge Number	Number of Openings	Painting	Structural Repairs	Walks and Roadways	Operation	Total	Age in 1916, Years
19	5,741	$663	$2,528	$1,595	$16,266	$21,052	26 (?)
21	407	4155	5,196	5,404	13,685	28,440	15
22	3,060	97	4,170	1,637	15,506	21,410	26 (?)
37	90,131	1423	1,830	1,434	38,572	43,259	17
38	74,208	1231	1,868	1,478	37,517	42,094	26
39	13,716	1154	1,921	164	30,486	33,725	13
Totals.....	187,263	$8723	$17,513	$11,712	$152,032	$189,980	20.5
Cost per opening.....		0.05	0.09	0.06	0.81	1.01	
Per bridge per year	7,802	363	730	488	6,335	7,916	

TABLE 2G

CITY BASCULE BRIDGES

General Data

Bridge No.	Location	Type	Length of Draw, Feet	Length of Approaches, Feet	Roadway, Feet	Sidewalk, Feet
17	135th Street	Trunnion	1 leaf 37	19	1 @ 42	2@11.5
20	Pelham	Scherzer	2 leaves @ 60	1880	1 @ 34	2@ 7.5
23	Hamilton Avenue	Scherzer	2 leaves @ 36	130	1 @ 35	2@ 6
24	9th Street	Scherzer	2 leaves @ 28	49	1 @ 35	2@ 6
25	3rd Street	Scherzer	2 leaves @ 28	49	1 @ 35	2@ 6
27	Union Street	Scherzer	2 leaves @ 28	49	1 @ 35	2@ 6
30	Harway Avenue	Bélidor	1 leaf @ 55	245	1 @ 20	2@ 5.33
36	Vernon Avenue	Scherzer	2 leaves @ 81	1527	1 @ 40	2@ 8
42	Flushing Avenue	Scherzer	2 leaves @ 36	250	1 @ 36	2@ 8
45	Hunter's Point	Scherzer	2 leaves @ 36	44	1 @ 36	2@ 6

TABLE 2H

City Bascule Bridges—Cost of Maintenance and Operation

1913 to 1916, Inclusive

Bridge Number	Number of Openings	Painting	Structural Repairs	Walks and Roadways	Operation	Total	Age in 1916, Years
17	7,410	$31	$759	$835	$8,993	$10,618	14
20	5,433	205	784	592	27,442	29,023	8
23	26,426	140	2,902	2,410	29,362	34,814	11
24	45,368	183	3,289	1,053	29,357	33,882	11
25	19,461	702	3,173	3,668	27,889	35,432	11
27	10,210	241	2,700	356	20,701	23,998	11
30	2,327	623	573	1,335	16,498	19,029	18
36	74,927	10,511	21,084	10,580	105,750	147,925	11
42	6,738	341	1,201	822	28,825	31,189	10
45	8,581	4	485	485	11,695	12,669	10
Totals.....	206,881	$12,981	$36,950	$22,136	$306,512	$378,579	11.5
Cost per opening.		0.06	0.18	0.11	1.48	1.83	
Per bridge per year	5,173	325	924	553	7,665	9,467	

TABLE 2I

City Retractile Bridges

General Data

Bridge No.	Location	Type	Clear Waterway, Feet	Length of Draw, Feet	Length of Approaches, Feet	Roadway, Feet	Sidewalk, Feet
18	Westchester Ave.	Diagonal	40	105	9	1 @ 40	2 @ 10
26	Carroll Street	Diagonal	36	107	33	1 @ 17	2 @ 5.25
28	Washington Ave.	Diagonal	36.5	123	.....	2 @ 15.58	2 @ 5
41	Borden Avenue	Diagonal	50	169	.....	1 @ 34	2 @ 8

TABLE 2J

CITY RETRACTILE BRIDGES—COST OF MAINTENANCE AND OPERATION

1913 to 1916, Inclusive

Bridge Number	Number of Openings	Painting	Structural Repairs	Walks and Roadways	Operation	Total	Age in 1916, Years
18	4,502	$241	$505	$1,230	$17,906	$19,882	17
26	14,074	361	4,285	1,579	20,509	26,734	27
28	14,674	548	4,707	1,077	29,288	35,620	23
41	7,822	474	1,069	171	18,812	20,526	26
Totals.....	41,072	$1,624	$10,566	$4,057	$86,515	$102,762	23.3
Cost per opening.....		0.04	0.26	0.10	2.10	2.50	
Per bridge per year	2,567	102	660	254	5,407	6,423	

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CHAPTER III

SWING BRIDGES

1. Center-bearing and rim-bearing swing bridges.—The earliest swing bridges were center-bearing wooden structures. The centers were usually of cast iron and in some cases were fitted with steel discs. The entire dead and live load at the center pier was carried by the center pivot. A ring with a few balance wheels was used to maintain stability when the span was moving, or open.

Early in the nineteenth century, English engineers built several rim-bearing bridges. These were mostly double-swing spans, meeting at the middle of the channel. The arms were of unequal length, the shorter shore ends being counterweighted. In the United States, the earliest records found of iron bridges show the rim-bearing type. One example is the original Rush Street Bridge, across the Chicago River, in Chicago; this was built in 1856. In the following decade several similar iron bridges were built, mainly across streams in the Mississippi Valley. Some of the best were those designed by the late C. Shaler Smith.[1] In 1871, Charles Macdonald designed and built the Point Street Bridge, over the Providence River, in Providence, R. I.[2] It is still in service in 1926, although much overloaded by modern traffic. The center pivot is a large iron casting with a Sellers loose cone roller center mounted in its top. The weight of the bridge is suspended on the center. The loading girders are placed at 45° with the center line of the bridge and are made so that they can be adjusted to deliver all dead load to the center. A light I-beam drum, running on a live ring of small coned rollers, spaced far apart, serves to balance the bridge when moving, and also carries a part of the live load when it is closed. This bridge is a combined center- and rim-bearing structure, with the center carrying the major part of all loads.

The use of the center-bearing type has gradually increased until it has nearly superseded the rim-bearing type except for very wide city highway bridges. Designs were much improved by the late C. C.

[1] Draw Spans and Their Turntables, by C. Shaler Smith, M. Am. Soc. C. E., *Trans. Am. Soc. C. E.*, Vol. III, p. 129, 1874.

[2] The Point Street Bridge, Providence, R. I. *Engineering*, London, Feb., 1873, p. 117; and March, 1873, p. 202.

Schneider, Engineer of the Pencoyd Iron Works, in the period from 1887 to 1900. Later, while he was Consulting Engineer of the American Bridge Company, his strong advocacy of the type, as the superior design, influenced the opinions of many engineers and firmly established the center-bearing design in American practice.[3]

2. Superiority of center-bearing swing bridges.—The center-bearing type is superior to the rim-bearing in several respects.

(1) It more closely approximates a fixed span. No live load is carried by mechanical parts at the center. When the bridge is closed, wedges are mechanically driven under the center cross girders, thus delivering practically all of the live load on the center pier directly to the masonry. A part of the live load on the middle floor panels is taken by the center pivot, but this is only a small part of the total. All dead load is supported by the pivot.

In the rim-bearing type, both the dead and live loads at the center are carried by the drum, the rollers upon which it turns, the track on which the rollers run, and finally, by the masonry near its outer circumference. The primary mechanical parts thus carry a much larger proportion of the total load than the pivot of a center-bearing bridge.

(2) The main trusses, or girders, are ordinary two-span continuous structures. They are readily arranged so that there are no redundant members. In the case of rim-bearing designs this is difficult to accomplish when the middle panel over the drum contains complete diagonals.

(3) It is a much simpler machine. When the span is moving, the entire dead load is carried by the center pivot. A few balance wheels are necessary to secure stability; but the only load upon them is any small unbalanced load due to accidental minor differences in the weight of the two arms, and loads from the overturning moment of the wind pressure upon the structure. In the rim-bearing type, the dead load is carried by a large number of rollers. The only parts to wear out under load in a center-bearing span are the discs in the pivot. In the rim-bearing type, the tread plates, rollers, circular roller track, the spider; and in combined center- and rim-bearing bridges, the pivot, are all subject to wear from vertical loads.

(4) The distance from base of rail to masonry at the center is less than for a well-designed rim-bearing span. This is often a determining consideration in heavy bridges.

(5) The center pier may be smaller. In wide bridges the balance wheels and center wedges are often placed well inside of the planes of the trusses.

[3] Movable Bridges, by C. C. Schneider, Past Pres. Am. Soc. C. E., *Trans. Am. Soc. C. E.*, Vol. LX, p. 258. 1908.

(6) The shop work is simpler. The pivot and discs are the only parts requiring extreme accuracy of workmanship. The balance wheels and their track must be finished, but as the wheels only carry small loads, slight inaccuracy in the workmanship does not seriously affect operation. In a rim-bearing bridge, the drum and lower track must be truly circular and of the correct diameter. The tread plates on each must be accurately finished to exact diameter and coning. The rollers must be exactly alike and correctly coned. The circular frame in which the rollers run must be circular and in a plane. The roller axles must be radial. The struts from the roller ring to the spider must be of exactly the right length. The spider, turning around the pivot, must be fitted to turn freely, and the pivot accurately finished. When a combined center and rim-bearing design is used, the discs within the pivot must be as carefully made as those for a center-bearing span.

(7) A center-bearing bridge is easier to erect and adjust than one of the rim-bearing type. The center pivot, the balance wheels and their track, and the wedges are the only primary mechanical parts that must be accurately adjusted at the center. In a rim-bearing span the distributing girders must be adjusted to the drum. The drum and lower roller track, and the tread plates on each, must be truly horizontal, circular, and concentric with the pivot. The many rollers must be radial and at exactly the correct distance from the center of the pivot. Their axles must be truly radial. The roller ring must be circular and concentric with the pivot. The struts must be radial and connected to the spider so that it will turn freely around the pivot. The whole device is a complicated mechanism, difficult to install accurately. When a new bridge replaces an old one and traffic must be maintained during erection, the difficulties are much increased. The proper installation of such a bridge turntable is one of the most troublesome of erection problems.

(8) There is no tendency to displace the pivot of a center-bearing bridge when turning. Any inaccuracy in workmanship or erection of a rim-bearing turntable results in a tendency of the rollers to crowd inward or outward, causing large horizontal stresses on the pivot. When the radials are too weak, the spider binds and lags behind the rollers, causing large stresses and rapid wear of the spider and pivot. It is quite common for the center castings to become loosened by these forces, when anchored by bolts. They are sometimes embedded in the masonry, to prevent this. While this may secure the pivot, the destructive wear goes on and the resistance to turning is increased.

(9) A center-bearing bridge turns more easily than a rim-bearing one. While the frictional resistance is less, the difference only becomes sig-

nificant when the rim-bearing turntable is not properly made and installed.

(10) When the center pier settles, a center-bearing bridge can easily be repaired. It is quite a simple matter to re-level the center pivot and balance-wheel track. The span can be lifted bodily by jacks under the center cross girders, and the pivot restored to proper line and level. The balance-wheel track can then be leveled by shims. A similar pier settlement under a rim-bearing bridge is a more serious matter. The drum carries all of the load and is near the circumference of the pier, and there is no convenient way of placing jacks to lift the span. Repairs are troublesome and expensive, and unless extreme care is taken the results are uncertain.

(11) The center-bearing type affords a better distribution of loads upon the center pier than the rim-bearing design. With a rim bearing, the entire dead and live load is applied near the circumference of the pier and none at its center. The pier must be large and there is a waste of masonry if it is built solid. When it is built hollow, with a central shaft for the pivot, there is danger of unequal settlement, which is likely to cause cracks. In the center-bearing type, all of the dead and a part of the live load is carried at the center of the pier and the major part of the live load is delivered to two points symmetrical with respect to the center, by the center wedges and their pedestals. The load is thus well distributed over a smaller pier than would be needed for a rim-bearing draw, and it may be made solid without waste of masonry.

(12) When the bridge is not too wide, a center-bearing draw weighs less than a rim-bearing span of the same length and capacity.

(13) The unit cost of fabricating and erecting the superstructure of a center-bearing draw is less than that of a rim-bearing.

(14) The maintenance and repairs are less than for the rim-bearing type.

3. Statistics of swing bridges.—A study has been made of 174 railway and 43 highway bridges, built in the period from 1903 to 1923. Table 3A gives the results. It was found that over 73 per cent of the railway and 60 per cent of the highway bridges were center-bearing. A comparison of the weights of the superstructures proved that while 57 per cent of railway tonnage was center-bearing, this type comprised only 31 per cent of the highway tonnage. This is largely due to a preference on the part of engineers for the rim-bearing type when the bridges have wide roadways and are of heavy construction.

While the data were taken from the records of only one manufacturer, it is believed that they fairly represent the average practice in the United States for the past twenty years.

TABLE 3A

Swing Bridges—1903 to 1923

Type	Number	Per Cent	Length	Per Cent	Weight	Per Cent	Weight per Foot
Railway Bridges							
Center-bearing...	128	73.58	24,059	66.13	72,238,000	57.28	3,002
Rim-bearing.....	46	26.42	12,323	33.87	53,884,000	42.72	4,373
Totals.....	174	100.00	36,382	100.00	126,122,000	100.00	3,467
Highway Bridges							
Center-bearing...	26	60.46	3,410	46.07	4,706,000	31.31	1,379
Rim-bearing.....	17	39.54	3,992	53.93	10,323,000	68.69	2,586
Totals.....	43	100.00	7,402	100.00	15,029,000	100.00	2,030

4. Long and heavy swing bridges.—For many years it was thought that long and heavy swing bridges should always be rim-bearing. The practice has gradually changed, and the choice is now made with regard to other considerations than mere size and weight, as a few examples will indicate.

The heaviest swing bridge in the United States is the double-track, double-deck, center-bearing span of the Southern Pacific Company across the Sacramento River, at Sacramento, California. There is a double-track railway on the lower deck and an 18-foot roadway and two 5-foot sidewalks on the upper. The bridge is only 390 ft. 3 in. long, from center to center of end supports, but weighs 3374 tons. An attempt was first made to design a rim-bearing turntable to carry this great weight, but the available vertical distance of 8 ft. $4\frac{3}{4}$ in., from base of rail to masonry at the center pier was so small that no method of designing a satisfactory distributing system and drum was found. There was no difficulty in arranging the center girders within the available height. Three cross girders were used, resting directly upon the center pivot which was set in a depression in the pier.

The Emergency Dams at the locks of the Panama Canal are also center-bearing, and those at Gatun weigh only a little less than the Sacramento Bridge. The arms are 164 ft. 3 in. and 98 ft. long, a total of 262 ft. 3 in. from center to center of end supports. The short end is heavily counterweighted. When fully equipped, the turning weight is 3351 tons.

There is a four-track, center-bearing bridge on the Bessemer and Lake Erie Railroad at Conneaut Harbor, Ohio.[4] This bridge is notable because it has four tracks, each carrying E60 loading, while the distance from base of rail to the center masonry is only 5 ft. 8 in. The trusses have an under clearance of only 3 ft. 3 in., below the base of rail. There are two main trusses, 32 ft. 7 in. center to center, and two outside trusses the centers of which are 16 ft. outside of the main trusses. Overhead transverse trusses serve to support the outer longitudinal trusses. The bridge is 235 ft. long and weighs 1400 tons.

The Coos Bay single-track, center-bearing draw span of the Willamette Pacific Railroad is 458 ft. long, and is probably the longest center-bearing span built up to the present time. There is no reason why still longer spans should not be built when required.

The longest draw bridge in the United States is the double-track, rim-and-center-bearing swing span across the Willamette River, near Portland, Oregon. It is 521 ft. center to center of end supports, and 524 ft. 2 in. out to out.[5] The distance from base of rail to the masonry of the center pier is 21 ft. $10\frac{9}{16}$ in. The weight, fully equipped, is 2750 tons.

The longest four-track swing span in the United States is the Harlem River Bridge, of the New York Central Railroad, in New York City.[6] It is 389 ft. long, between centers of end supports. There are three main trusses and a solid plate-and-angle trough floor. The superstructure is carried by a double drum, turning on two sets of 72-coned rollers. The outer rollers are 24 in. and the inner $20\frac{7}{16}$ in. in diameter, both with 11-in. face. The turning weight is 2500 tons.

5. General.—A few diagrams of various types of swing draw bridges follow. No details of the design of the structural steelwork will be given. Simple methods of making stress calculations are developed in Chapter VII. After the stresses are known and properly combined, the methods of proportioning the steelwork outlined in any good specification for fixed spans apply to similar members of draw bridges. Care must be taken to make adequate provision for stress reversals, which occur in more members than in fixed spans of similar size. Attention must also be given to the lateral and transverse bracing, particularly at the center of the bridge. The stability of the open bridge should also receive special consideration. In center-bearing bridges the design

[4] A Four-Track, Center-bearing, Railroad Draw Span, by Louis H. Shoemaker, M. Am. Soc. C. E. *Trans. Am. Soc. C. E.*, Vol. LXXV, p. 711, 1912.

[5] The Vancouver-Portland Bridges, a Report by Ralph Modjeski, M. Am. Soc. C. E., Chief Engineer, 1910.

[6] *Engineering News*, 1893, I, p. 559; II, pp. 167, 285; 1896, I, p. 293.

of the center cross girders of heavy spans must be carefully studied. Suggestions relative to the design of the drums of rim-bearing bridges will be given in the Specifications for Movable Bridges, in Appendix A of Vol. II.

A few typical truss outlines of several classes of swing spans are given in Articles 6, 7, 8, 9, 10, 11 and 12. Combined center sections and end elevations of typical designs are also shown. The distances from base of rail, or top of floor, to masonry at the ends and centers of the spans, are shown by dimensions below the diagrams, near the left ends and the centers. No odd or unusual designs have been selected.

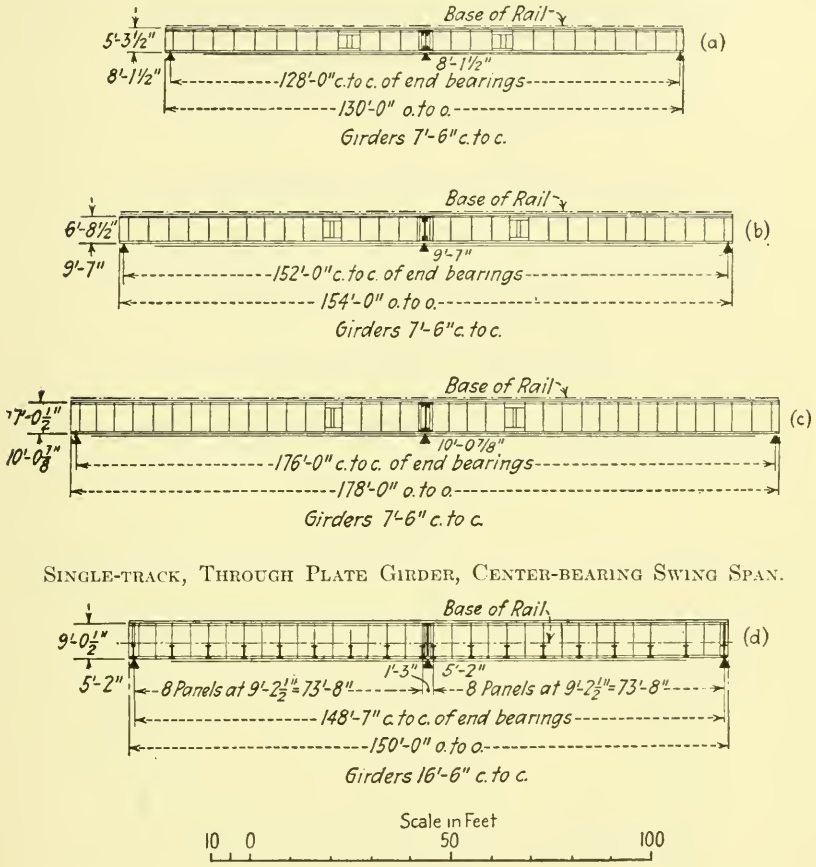
Approximate weights of the steelwork are given to assist in determining dead-load assumptions for new designs. As the dead weights depend upon the unit stresses and methods of design, as well as the load capacity of the bridge, the live loads and specifications for each example have been noted. The probable weights for new designs for other loads and specification requirements may be closely estimated from the data given.

6. Single-track, deck plate girder, center-bearing swing spans.— The general arrangement and principal dimensions of a series of single-track, deck plate girder, center-bearing swing spans are shown in Fig. 3A, (*a*), (*b*), and (*c*). These bridges were designed for Cooper's Class E50 live loads, and in accordance with the 1908 Specifications of the Seaboard Air Line Railway. Single center cross girders were used, superposed on the center pivots. The turning and wedging machinery is operated by man-power, applied by an ordinary capstan on the deck, near the middle of the span.

A combined center section and end elevation of bridge (*c*) is shown in Fig. 3B (*a*). The center pivot turns on two discs 12 in. in diameter. The upper is of bronze, convex downward, and the lower of steel, concave upward. The center and end wedges move longitudinally. Counterweighted latches were used at both ends of the span. The two transverse balance wheels and the track are outside of the main girders, and the two longitudinal wheels are attached to transverse girders. Vertical adjustment was provided by shims between the center cross girders and the pivot, and between the main girders and the upper wedge seats.

7. Single-track, through plate girder, center-bearing swing spans.— The general arrangement and principal dimensions of a single track, through plate girder, center-bearing swing span are shown in Fig. 3A, (*d*). This bridge was designed for a live load of about Cooper's Class E55, followed by 5000 lbs. per linear foot, and in accordance with the Common Standard Specifications of the Southern Pacific Company. Two center cross girders were used, as the floor was shallow. The

SINGLE-TRACK, DECK PLATE GIRDER, CENTER-BEARING SWING SPANS.



SINGLE-TRACK, THROUGH PLATE GIRDER, CENTER-BEARING SWING SPAN.

FIG. 3A.

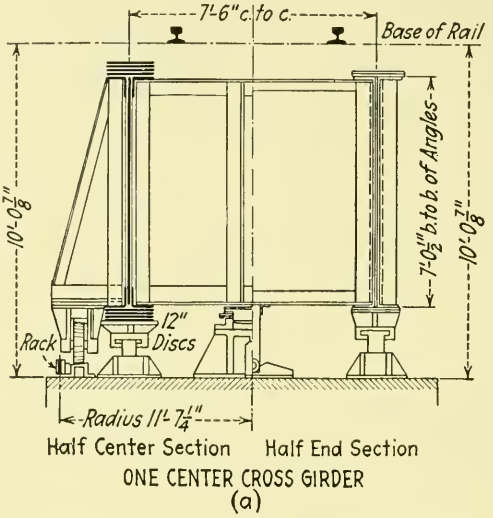
TABLE 3B

REACTIONS AND WEIGHTS

Figure	Span, Feet	End Wedge Reaction, Pounds	Center Wedge Reaction, Pounds	Maximum Load on Center Pounds	Weight of Steelwork, Pounds
(*a*)	128	237,000	549,000	221,000	195,000
(*b*)	152	268,000	614,000	284,000	245,000
(*c*)	176	343,000	685,000	347,000	316,000

The machinery of the 176 ft. span weighed 35,500 lbs. These bridges were built in 1915.

SINGLE-TRACK, DECK PLATE GIRDER, CENTER-BEARING SWING SPAN.



SINGLE-TRACK, THROUGH PLATE GIRDER, CENTER-BEARING SWING SPAN.

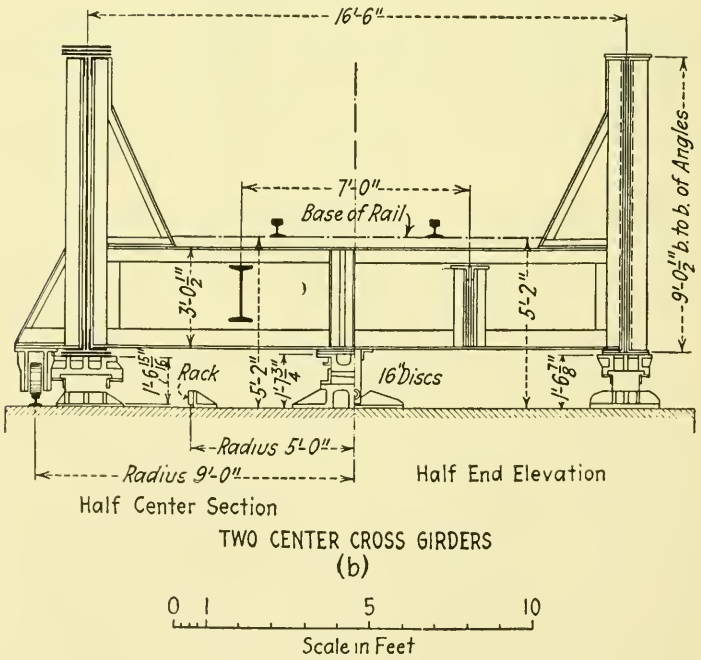


FIG. 3B.

center discs are 16 in. in diameter. The machinery is operated by man-power, applied by a capstan. Fig. 3B, (*b*), is a combined center

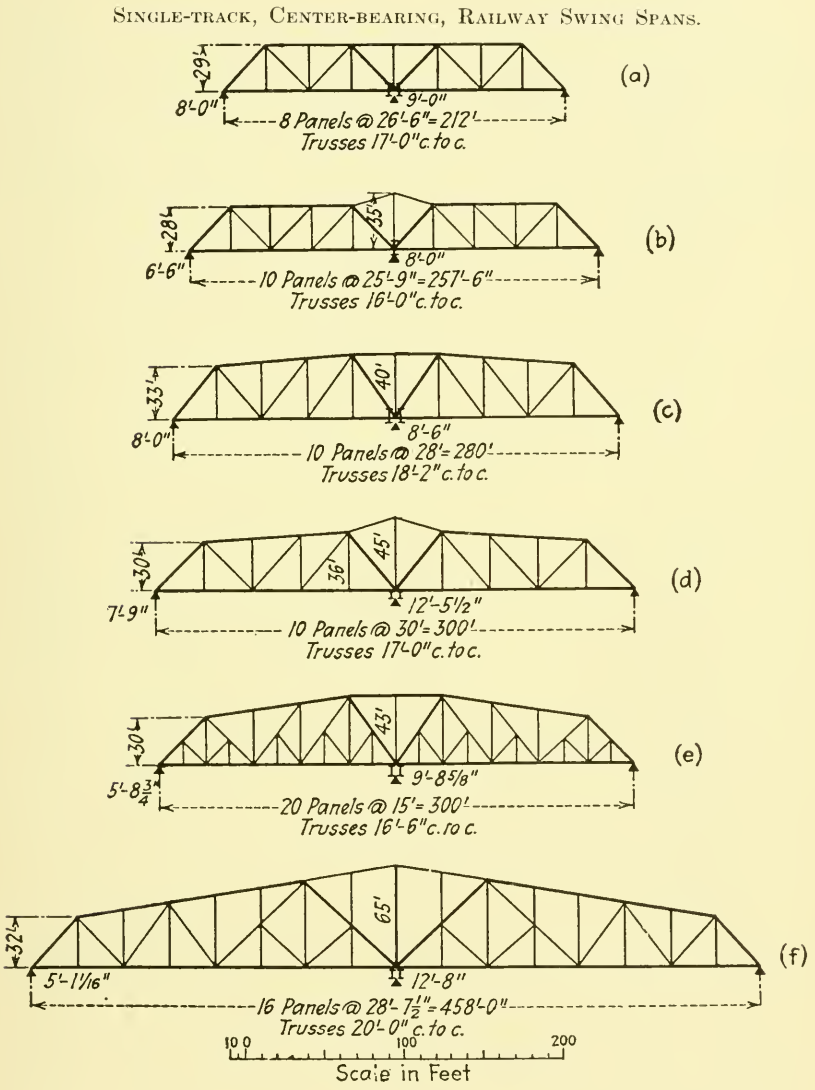


Fig. 3C.

section and end elevation. The balance wheels are outside and the rack is inside of the main girders. Vertical adjustment was made by shims. End wedge reaction, 167,000 lbs.; center wedge reaction,

287,000 lbs.; and maximum load on the center, 419,000 lbs. The weight of the steelwork was 322,000 lbs., of which 34,000 lbs. was machinery.

This bridge was built in 1906.

8. Single-track, center-bearing, railway swing spans.—The general arrangement and dimensions of the trusses of six single-track, center-bearing, railway swing spans are shown in Fig. 3C. Bridges (*a*), (*c*), and (*e*) are riveted throughout; (*b*), (*d*), and (*f*) are riveted, except that eyebars, with pin connections, are used in the middle panels of the upper chord. The live loads and specifications for which these bridges were designed and the reactions and weights are given in Tables 3C and 3D.

TABLE 3C

Live Loads and Specifications

Figure	Live Loads	Specifications	Built
(*a*)	E50	Isthmian Canal Commission and C. C. Schneider.	1914
(*b*)	E50	Seaboard Air Line Ry. of 1908 and J. E. Greiner's Spec. for Movable Bridges.	1916
(*c*)	E55 with 5,000 lbs. per foot	Southern Pacific Co.'s Common Standard 1006, revised, 1921.	1922
(*d*)	E60	Boller, Hodge & Baird, Consulting Engineers.	Designed 1912
(*e*)	E60	N. Y. N. H. & H. R. R. of 1908.	1910
(*f*)	E55 with 5,000 lbs. per foot	Southern Pacific Co.'s Common Standard 1006, revised, 1921.	1914

TABLE 3D

Reactions and Weights

Figure	Span, Feet	End Wedge Reaction, Pounds	Center Wedge Reaction, Pounds	Maximum Load on Center, Pounds	Weight of Steelwork, Pounds	Weight of Machinery, Pounds	Total Weight, Pounds
(*a*)	212	360,000*	550,000*	1,056,000*	.....	.....	511,000
(*b*)	257–6	390,000*	770,000*	932,000*	517,000	49,000	566,000
(*c*)	280	315,000	463,000	1,197,000	867,000	113,000	989,000
(*d*)	300	393,000	661,000	900,000	755,000	94,000	849,000
(*e*)	300	550,000*	921,000*	1,615,000	1,200,000	106,000	1,306,000
(*f*)	458	413,000	738,000	2,176,000	1,727,000	149,000	1,876,000

* Impacts are included in these reactions.

SINGLE-TRACK, CENTER-BEARING, RAILWAY SWING SPANS.

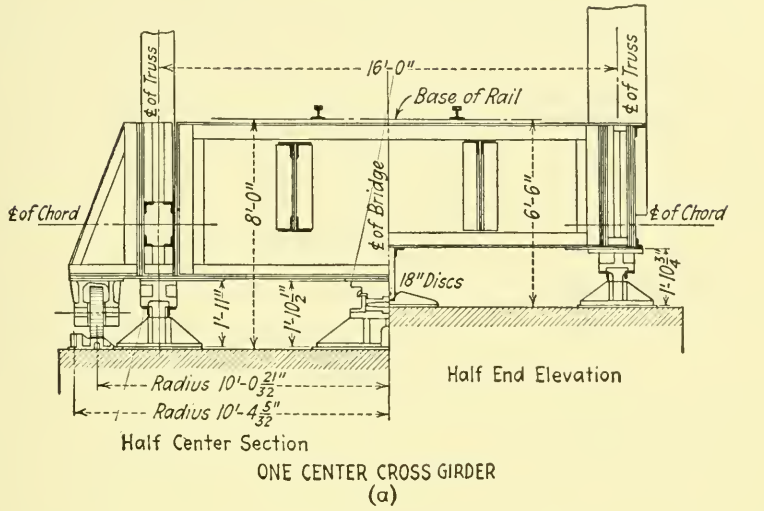


FIG. 3D.

A combined center section and end elevation of the 257-ft. 6-in. span, Fig. (*b*), is shown in Fig. 3D, at (*a*). A single center cross girder was used, superposed on the pivot. This bridge is operated by manpower, applied by a capstan on the floor. The pivot has three discs 18

Double-track, Center-bearing, Railway Swing Spans.

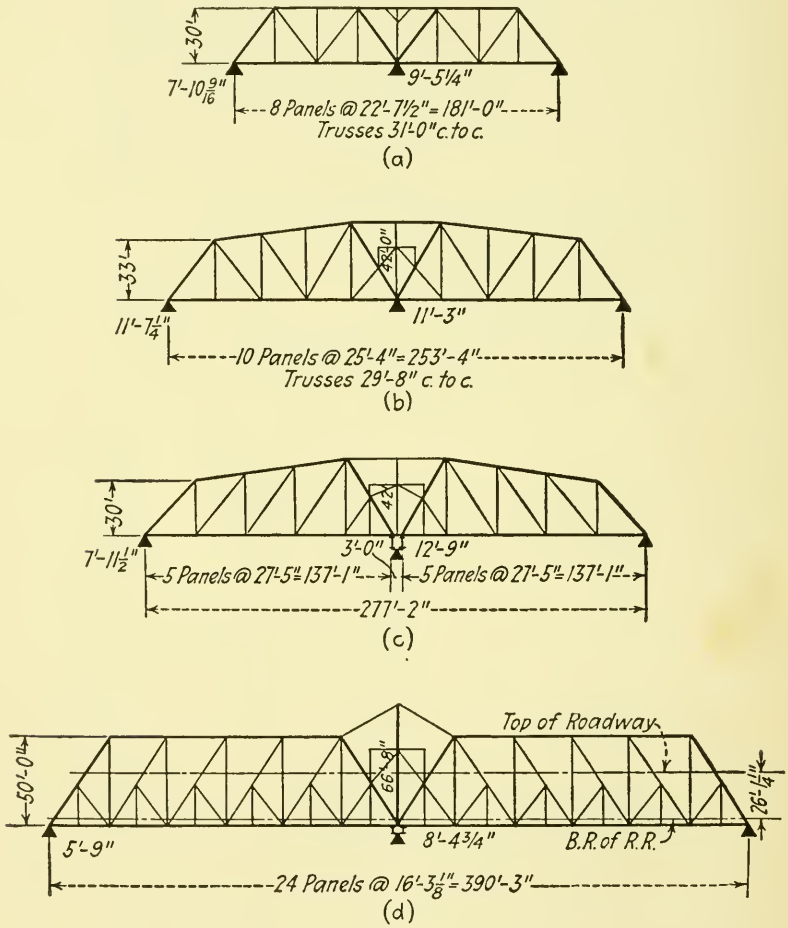


Fig. 3E.

ins. in diameter, the top and bottom discs of tempered steel and the middle one of hard bronze. The center and end wedges move longitudinally and are operated simultaneously, as is the usual practice with this design. There are four balance wheels, the transverse pair mounted on cantilever brackets opposite the center cross girder and the longitudinal

pair on girders between the trusses. Height adjustment is made by shims.

Fig. 3D, at (*b*), shows similar details of the 280-ft. span (*c*), of Fig. 3C. The arrangement is similar to that in Fig. 3D, (*a*). In this bridge two center cross girders were necessary; and eight balance wheels were required; the four transverse ones were mounted on cantilever brackets, opposite the center cross girders, and the four longitudinal ones were placed directly under the stringers. The pivot turns on two discs 24 in. in diameter. The upper is of tempered steel and the lower of hard bronze. This bridge is operated by a heavy, truck-type, high-speed gasoline engine. Details of the machinery are given in Volume II, Chapter III, Art. 4. As usual, the latches are of the automatic, balanced type and the height adjustment is by shims.

9. Double-track, center-bearing, railway swing spans.—A group of diagrams of four double-track, center-bearing, railway swing spans is shown in Fig. 3E. Bridges (*a*) and (*b*) are riveted throughout, and (*c*) and (*d*) are of the same construction except that eyebars and pins were used in the middle panels of the upper chord. The live loads and specifications for which these bridges were designed and the reactions and weights are given in Tables 3E and 3F.

TABLE 3E

Live Loads and Specifications

Figure	Live Loads	Specifications	Built
(*a*)	E60	N. Y. N. H. & H. R. R. of 1912.	1917
(*b*)	E60	C. R. R. of N. J., which was based on Am. Ry. Engrg. and M. of W. Assn. of 1910.	1912
(*c*)	Two 187-ton engines with 5,000 lbs. per linear foot	Pennsylvania R. R. of 1905	1905
(*d*)	E55 with 5,000 lbs. per linear foot...	Southern Pacific Co.'s Common Standard 1006.	1911

Figure 3F shows a combined center section and end elevation of the 253-ft. 4-in. span in Fig. 3E, (*b*). Two center cross girders and eight balance wheels were required. There was ample distance from base of rail to masonry, so that it was easy to provide adequate stiffness and to work out a good arrangement. The wedge reactions are heavy, and grillages of I-beams, with top and bottom plates, were embedded in

the masonry to distribute the loads. The end wedges move longitudinally, and those at the center move transversely. The pairs of transverse balance wheels are carried by cantilever brackets, in line with the cross girders and the longitudinal pairs on short transverse girders. The pivot turns on three discs, 30 in. in diameter. The top and bottom discs are of soft steel, case-hardened and accurately ground to spherical bearing surfaces. The middle disc is of hard bronze.

TABLE 3F

Reactions and Weights

Figure	Span, Feet	End Wedge Reaction, Pounds	Center Wedge Reaction, Pounds	Maximum Load on Center, Pounds	Weight of Steelwork, Pounds	Weight of Machinery, Pounds	Total Weight, Pounds
(*a*)	181	667,000*	964,000*	1,418,000*	886,000	111,000	997,000
(*b*)	253–4	.....	.....	.....	1,383,000	192,000	1,575,000
(*c*)	277–2	780,000	1,290,000	2,132,000	.....	.....	1,497,000
(*d*)	390–3	868,000	1,528,000	6,748,000	4,420,000	342,000	4,762,000

* Impacts are included in these reactions.

This bridge is operated by a duplex horizontal steam engine, specially designed for draw-bridge service. There are two duplicate boilers, each large enough to supply steam for the engine. One is held in reserve, to avoid delays in case the other should be out of service. A band brake, of large capacity, is mounted on a transverse counter shaft near the engine. The operator's house is provided with coal and water tanks, a coal hoist, an ash chute, and toilet facilities.

This bridge is used as an example of the application of the analysis of power and the time, space and velocity relations, given in Volume II, Chapter II.

Two duplicate bridges were built from this design, across the Hackensack and Passaic Rivers, on the line of the Central Railroad of New Jersey, between Jersey City and Newark, N. J.

Figure 3G shows the general arrangement of the Sacramento River Bridge, to which reference was made in Art. 4. The center was set in a depression in the pier, and the lower wedge pedestals were embedded in the concrete, to save height. Heavy grillages distributed the great weight to the masonry. There are three center discs, 52 in. in diameter. They are the largest that have been built. The upper and lower are of soft steel, deeply case-hardened, with the concave spherical sur-

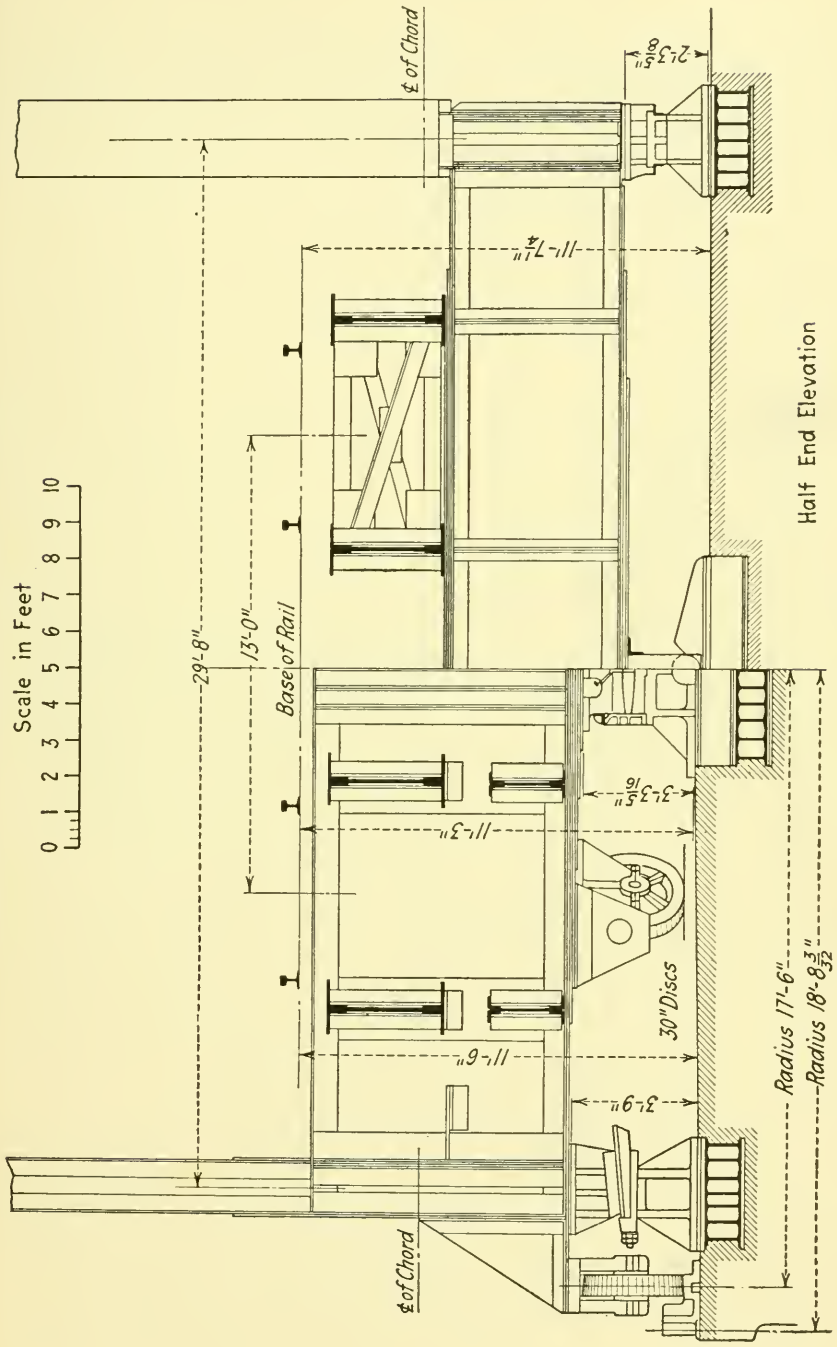


FIG. 3F.

DOUBLE-TRACK, CENTER-BEARING, RAILWAY SWING SPAN.

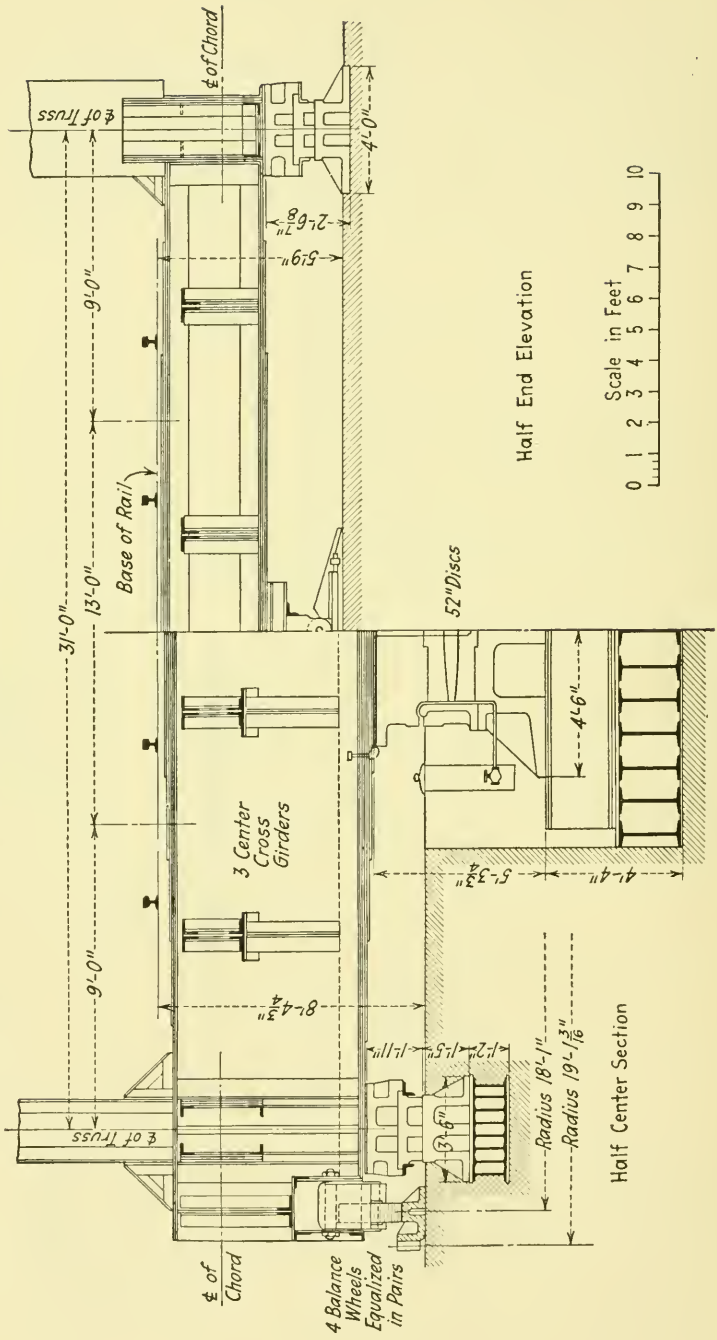

Double-track, Double-deck, Center-bearing, Railway and Highway Swing Span.
℄ of Truss
℄ of Chord
4'-0"
2'-6⅞"
5'-9"
Base of Rail
31'-0"
13'-0"
9'-0"
9'-0"
3 Center Cross Girders
8'-4¾"
52" Discs
4'-6"
5'-3¾"
4'-4"
Half End Elevation
Scale in Feet
0 1 2 3 4 5 6 7 8 9 10
1'-2"
1'-5"
1'-11"
3'-6"
Radius 18'-1"
Radius 19'-1 3/16"
Half Center Section
℄ of Chord
4 Balance Wheels Equalized in Pairs

Fig. 3G.

faces accurately ground. The middle disc is of hard bronze. Special provision is made for lubrication and cleaning. As this bridge is a double-deck structure, the surface exposed to wind pressure is large, with the result that there are heavy pressures on the transverse balance wheels. The reaction is 378,000 lbs. Four wheels were used on each

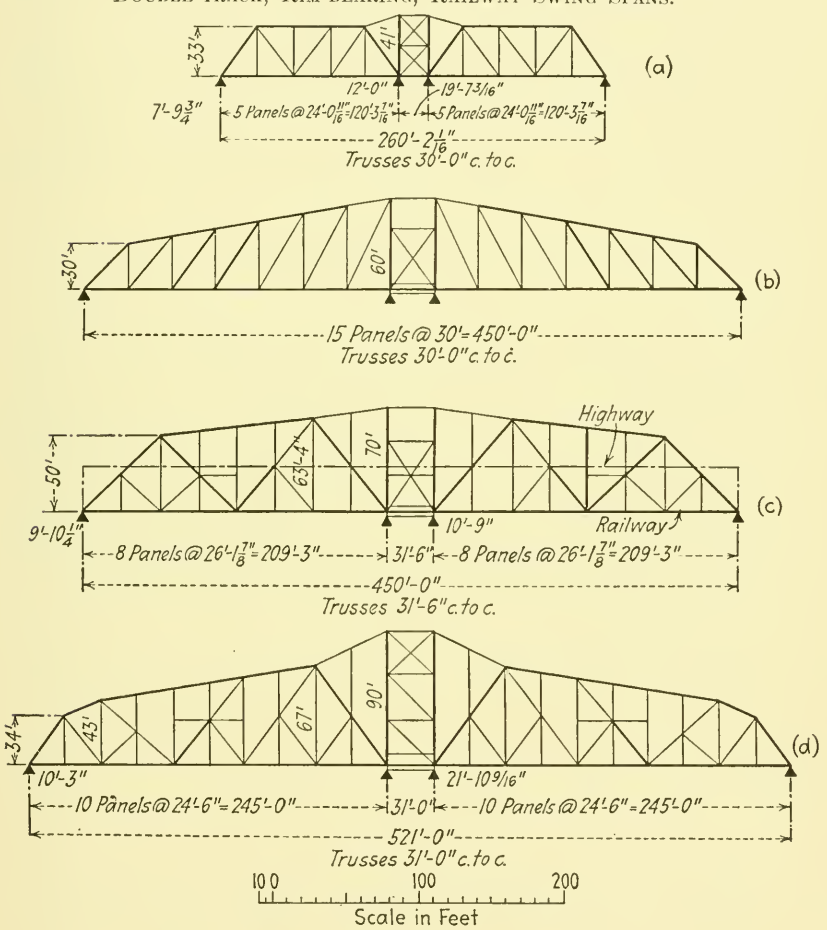


Fig. 3H.

side, equalized in pairs. Two pairs were used longitudinally, making a total of twelve wheels for the bridge. The wedges all move longitudinally and are the largest yet built.

The mechanism is operated by electric motors. Four main pinions engage the rack on the center pier. Details of the machinery are given in Volume II, Chapter III, Art. 6.

10. Double-track, rim-bearing, railway swing spans.—The truss outlines of four double-track, rim-bearing, railway swing spans are shown in Fig. 3H. Bridge (*a*) is riveted, except that eyebars and pins are used in the three middle panels of the upper chord. Bridge (*b*) is pin-connected throughout, and eyebars are used in all purely tension members. Bridge (*c*) is riveted, except for the use of eyebars and pins in the three middle panels of the upper chord and the upper half of the adjacent diagonal web members. Bridge (*d*) is pin-connected except for the small vertical web members and the bracing between the middle posts. This is the span mentioned in Art. 4 as the longest swing span in the United States. The live loads and the specifications for which they were designed and the reactions and weights are given in Tables 3G and 3H.

TABLE 3G

Live Loads and Specifications

Figure	Live Loads	Specifications	Built
(*a*)	E35 with 4,000 lbs. per linear foot	Erie R. R. of 1900 as revised June, 1905.	1907
(*b*)	Trusses, 3,000 and floor 5,000 lbs. per linear foot, basis.	George S. Morison, Consulting Engineer.	1893
(*c*)	E90 with 7,500 lbs. per lin. ft. Hy., 100 lbs. per square foot, floor, 50 lbs. per square foot on the trusses.	C. B. & Q. R. R., special for silicon steel and heat-treated eyebars.	1915
(*d*)	About E52 with 5,000 lbs. per per linear foot.	Ralph Modjeski, Consulting Engineer.	1907

TABLE 3H

Reaction and Weights

Figure	Span, Feet	End Reaction, Pounds	Maximum Load on Center Pivot, Pounds	Maximum Load on Drum, Pounds	Weight of Steelwork, Pounds	Weight of Machinery, Pounds	Total Weight, Pounds
(*a*)	260–2 1/16	328,000	792,000	2,064,000	1,341,000	229,000	1,570,000
(*b*)	450	315,000	None	2,836,000	1,890,000	240,000	2,130,000
(*c*)	450						
(*d*)	521	1,071,000	1,126,000	7,882,000	4,188,000	817,000	5,005,000

A combined center section and end elevation of bridge (*a*) is shown on Fig. 3I. This is a typical illustration of the method of building a combined center-and-rim-bearing draw. This bridge carries a light load, for a double-track railway structure, and the depth from base of rail to masonry was made as small as would allow satisfactory details and a good drum. About 29 per cent of the total load is carried by the center pivot and 71 per cent by the drum. The radial distributing girders are 45 in. deep, and the drum is 48 in., from back to back of angles. The drum turns on 48 cast-steel coned rollers, 18 in. in diameter, with 10-in. faces. There are three center discs, 20 in. in diameter, of the usual materials. The provision for draining the area within the roller track should be noted.

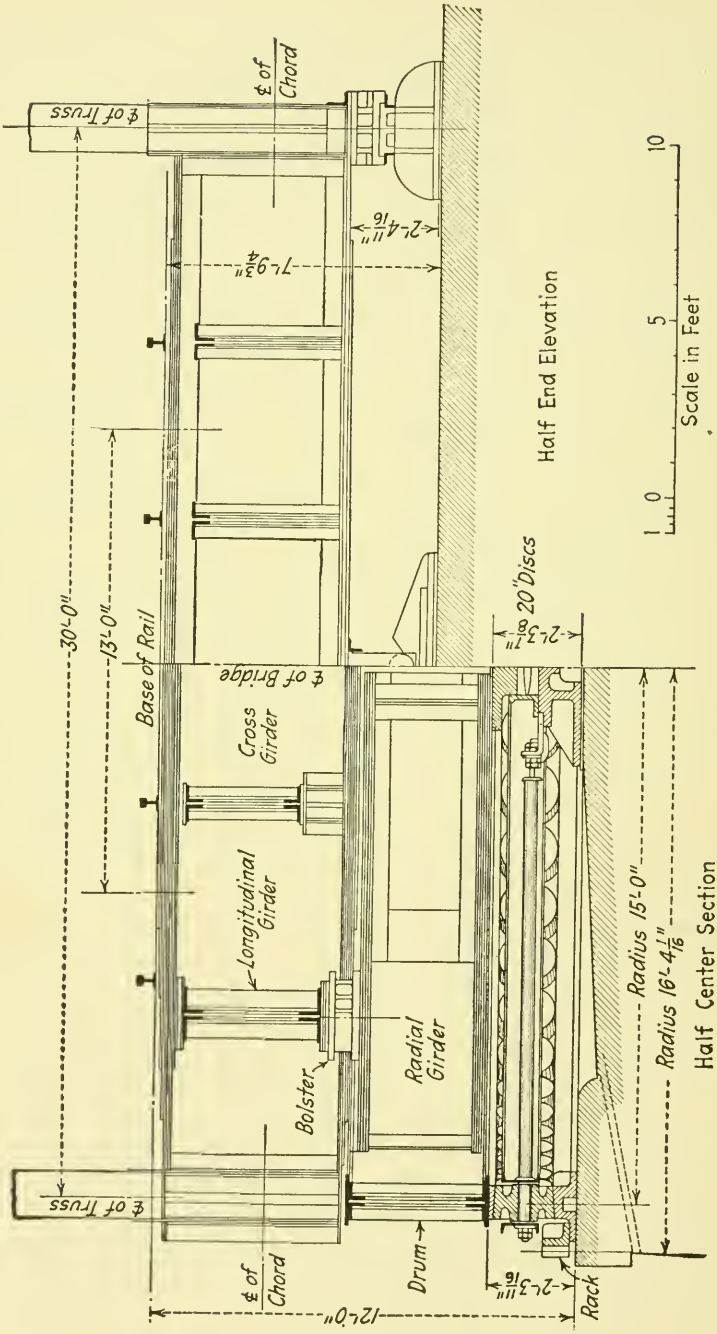
The end wedges are of the ordinary type, and move longitudinally. Sliding end rail latches, which fit the rail contours, are operated from the bridge end-lifting mechanism. These latches are withdrawn before the end wedges begin to move, and are driven after the ends have been lifted by an ingenious special device.

11. Center-bearing, highway, swing spans.—The truss outlines of four center-bearing, highway, swing spans are shown in Fig. 3J. The live loads and the specifications for which they were designed and the reactions and weights are given in Tables 3I and 3J.

Bridge (*a*) has a 24-ft. roadway and one 5-ft. sidewalk. Two electric railway tracks are spaced symmetrically about the center line. As this is a temporary structure on a very busy highway with heavy traffic, the roadway is paved with creosoted wooden blocks. The pavement rests on 2-in. transverse planks, laid on 3-in. longitudinals supported by transverse ties, spaced 16 in. between centers. The bridge is operated by electric motors, controlled from a small cabinet in the plane of the truss adjacent to the sidewalk.

Bridge (*b*) has a 19-ft. 8-in. roadway, with a single, standard-gauge railway track on the center line. There is one sidewalk 6 ft. wide. The entire floor is covered with buckle plates, filled with concrete, which forms the roadway and sidewalk surfaces. The bridge is operated by electric motors, controlled from a house above the clearance line, in the middle panel.

Bridge (*c*) has a 21-ft. roadway, with one standard-gauge railway track on the center line. The pavement is of 3-in. creosoted wooden blocks, laid on 3 by 6-in. transverse planks, fastened to spiking strips on the stringers. The bridge is operated by electric motors, controlled from a small operator's house on cantilever brackets in the middle panel, and outside of one of the trusses. The arrangement of the machinery is shown in Volume II, Chapter III, Art. 3. Two duplicate bridges were built from these plans.



DOUBLE-TRACK, RAILWAY SWING SPANS, RIM-AND-CENTER-BEARING.

FIG. 31.

Bridge (*d*) is an important city structure. The roadway is 23 ft. 4 in. between curbs and has two electric railway tracks, spaced symmetrically about the center line. A wood-block pavement was laid on

Center-bearing Highway Swing Spans.

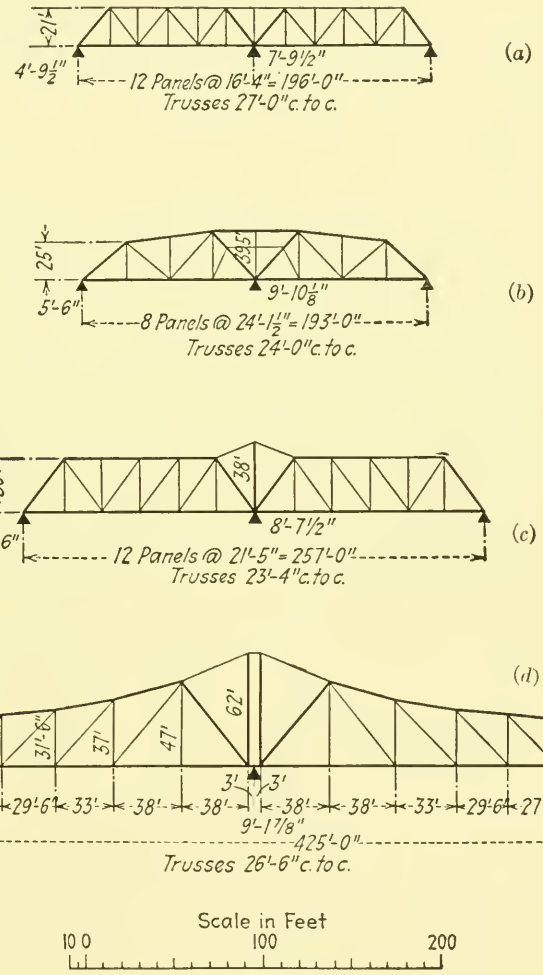


Fig. 3J

heavy transverse planks. There are two sidewalks 6 ft. wide. The bridge is operated by electric motors below the floor and controlled from an operator's house in the middle panel, above the roadway clear-

ance line. The house also contains an auxiliary gasoline engine for emergency operation.

TABLE 3I

Live Loads and Specifications

Figure	Loads	Specifications	Built
(*a*)	Electric car tracks, 2–40-ton cars. Roadway, 20-ton truck or 125 lbs. per square foot. Sidewalk, 80 lbs. per square foot.	A. R. E. A., 1910, revised, for unit stresses, material, and workmanship.	1922
(*b*)	Railway track, E39.6 with 3630 lbs. per linear foot, with impact. Highway-floor, 31.8-ton truck. Sidewalk, 92 lbs. per square foot, reduced to 61 lbs. per square foot for truss loads.	Special. Unit tension stress basis, 15,600 lbs. per square inch.	1922
(*c*)	Railway tracks, 50-ton coal cars. Highway floor, 15-ton road roller or 100 lbs. per square foot. Impact, 25 per cent for stringers, floor-beams, and trusses.	A. R. E. A., 1910, revised, for stresses, material and workmanship.	1915
(*d*)	Electric car tracks, 2–50-ton cars. Highway floor, 25-ton truck. Trusses, 80 lbs. per square foot of roadway and 2–27-ton cars, also 60 lbs. per square foot on sidewalk. Snow load, 20 lbs. per square foot.	Boller, Hodge, & Baird, Consulting Engineers.	1915

TABLE 3J

Reactions and Weights

Figure	Span, Feet	End Wedge Reaction, Pounds	Center Wedge Reaction, Pounds	Maximum Load on Center, Pounds	Weight of Steelwork, Pounds	Weight of Machinery, Pounds	Total Weight,* Pounds
(*a*)	196	109,000	302,000	554,000	356,500	53,500	410,000
(*b*)	193	242,000*	366,000*	1,306,000*	581,000	90,000	671,000
(*c*)	257	195,000*	291,000*	820,000*	478,500	49,500	528,000
(*d*)	425	254,000	269,000	2,130,000	1,501,000	184,000	1,685,000

* Impacts are included in these reactions.

A combined center section and end elevation of bridge (*a*) is shown in Fig. 3K. The center wedges are placed well inside of the planes of the trusses, in order to reduce the diameter of the center pier. The center pivot turns on two discs 15 in. in diameter, the upper of hard bronze and the lower of tempered steel. Both the end and center wedges move longitudinally. Four balance wheels were used. The rack and balance-wheel track were combined in units and cast in segments. The entire structure is about as simple as it can be made.

Figure 3L gives similar details for bridge (*d*). Two center cross girders were necessary, and short longitudinals, resting on the center pivot, were framed between them. This construction allows deep cross girders without unduly increasing the distance from the floor to the masonry. The center pivot turns on three discs 30 in. in diameter, the middle one of hard bronze, and the top and bottom ones of hardened steel. There are four center wedges at the intersection of the trusses and cross girders. These move transversely, but the end wedges move longitudinally. There are four transverse balance wheels, two on each side, carried by cantilever brackets opposite the cross girders. Two pairs of longitudinal wheels are supported by the short transverse pinion girders. The two main pinions are on the center line of the bridge. The span is turned by two 52-h.p. motors, one adjacent to each pinion. The center and end wedges are driven simultaneously by a 20-h.p. motor near the middle of the span. One 20-h.p. gasoline engine in the operator's house is geared to operate the bridge slowly when the electrical equipment is out of service. A set of special transmission gears and clutches, adjacent to the engine, provides for forward and reverse movements of both the turning and wedging apparatus from the engine; which is of the ordinary non-reversible four-cycle, heavy, single-cylinder type. There is a band brake on the first counter shaft from each turning motor, also a similar brake on the gasoline engine transmission mechanism in the house.

12. Rim-bearing, highway swing span.—Figures 3M and 3N show the skeleton and sections of the Madison Avenue Bridge over the Harlem River, in New York City. The span is 300 ft., from center to center of the end supports. There are three trusses, spaced 30 ft. between centers. The two roadways are 27 ft. between curbs and each has an electric railway track near the middle truss. The two sidewalks are 9 ft. wide and are on cantilever brackets outside of the trusses. The entire floor is covered with buckle plates, filled with concrete, which supports a wood-block pavement on the roadways. The sidewalks have granitoid surfaces.

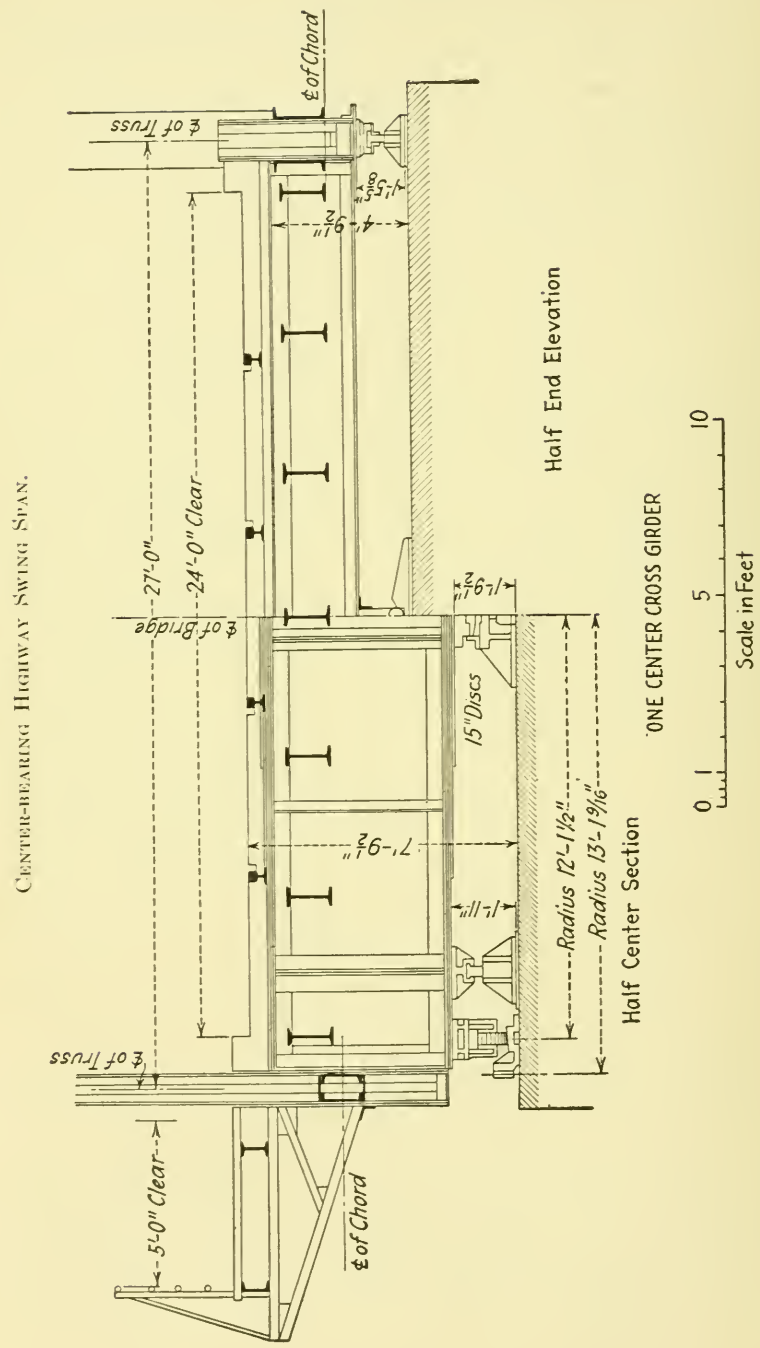
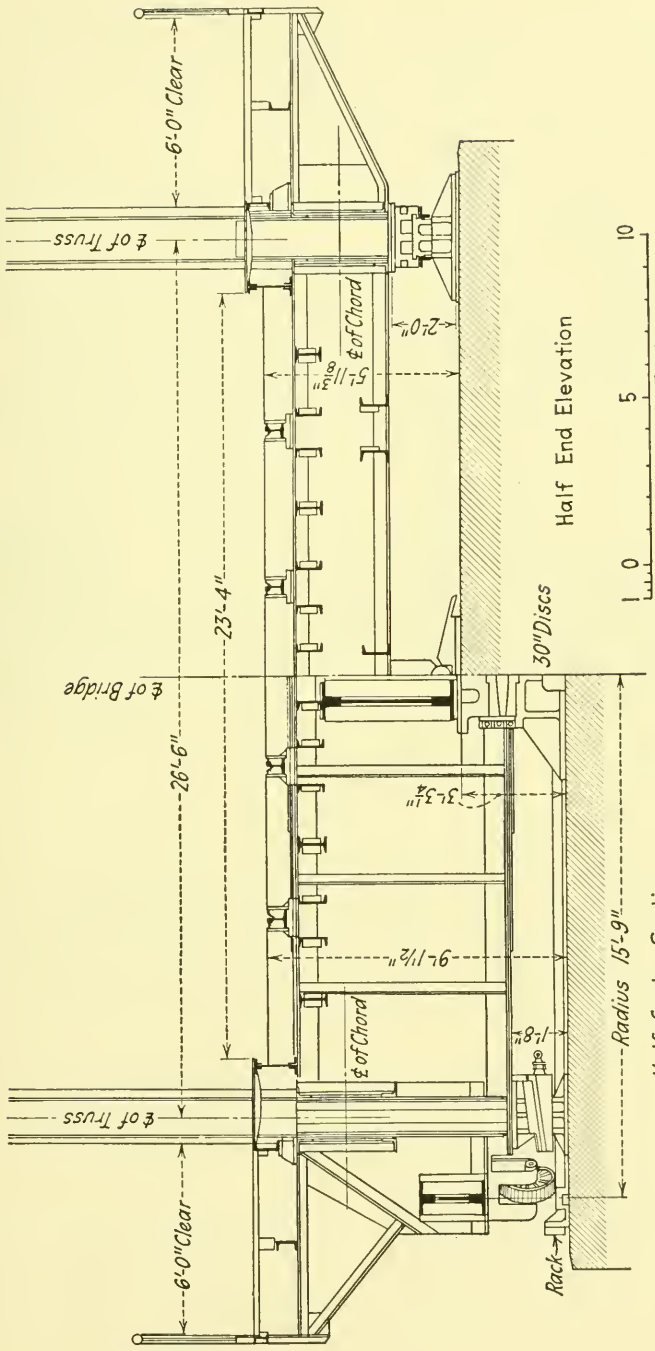

CENTER-BEARING HIGHWAY SWING SPAN.
℄ of Chord
℄ of Truss
4'-9½"
1'-5⅝"
27'-0"
24'-0" Clear
℄ of Bridge
1'-9½"
15" Discs
7'-9½"
1'-11"
Radius 12'-1½"
Radius 13'-1⅜"
℄ of Truss
5'-0" Clear
℄ of Chord
Half End Elevation
ONE CENTER CROSS GIRDER
Half Center Section
0 1 5 10
Scale in Feet

FIG. 3K.



Center-bearing Highway Swing Span.

Fig. 3L.

The system of distributing girders is shown on Fig. 3N, at (*c*). The drum is loaded at 12 points, and the girders are arranged so that the distribution is practically determinate. The drum turns on 72 rollers, 24 in. in diameter, with 12-in. faces. They run in an annular channel frame, with a third channel 3 ft. $6\frac{1}{2}$ in. inside of the center of the drum. The roller axles are short, only extending through the three channels of the circular frame. There are 16 radial struts connecting the roller frame with the central spider. The drum has 12 triangular radials, with three panels of bracing between each pair.

The end lifts are vertically moving rams, actuated by toggles. The mechanism is arranged so that in case of an over-drive the rams are lifted from their seats, and there is no danger of damage. The end latches are of the ordinary counterbalanced type, but the catches on the piers are pivoted to short vertical links and diagonal springs.

Rim-bearing Highway Swing Span.

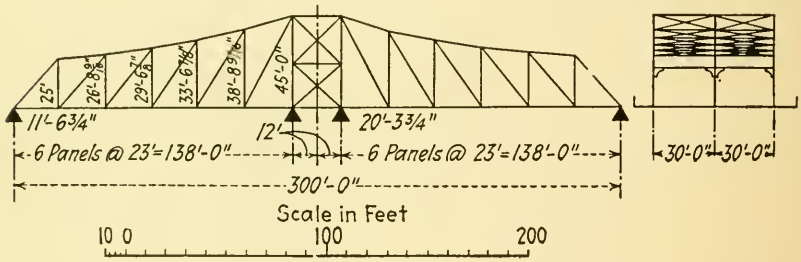


Fig. 3M.

When the bridge approaches with too great a velocity to allow the springs to stop it, the latch carries the catch away with it until it becomes disengaged. The bridge is then free to move on, and the catches are brought to central position by the springs. The springs are so designed as to be strong enough to turn the bridge slowly and bring it to true central position.

This bridge is turned by two 40-h.p. electric motors, geared to two main pinions. The mechanism has been satisfactory in operation. The steelwork and machinery weigh 3,130,000 lbs. The bridge was designed by the Engineers in the Department of Bridges, of the City of New York, and was built in 1910.

13. Distribution of loads to drums of rim-bearing draw bridges.— A study of the failures of the turntables of rim-bearing draw bridges indicates that they are nearly all due to unequal distribution of the loads to the drum and rollers upon which the span revolves. In a

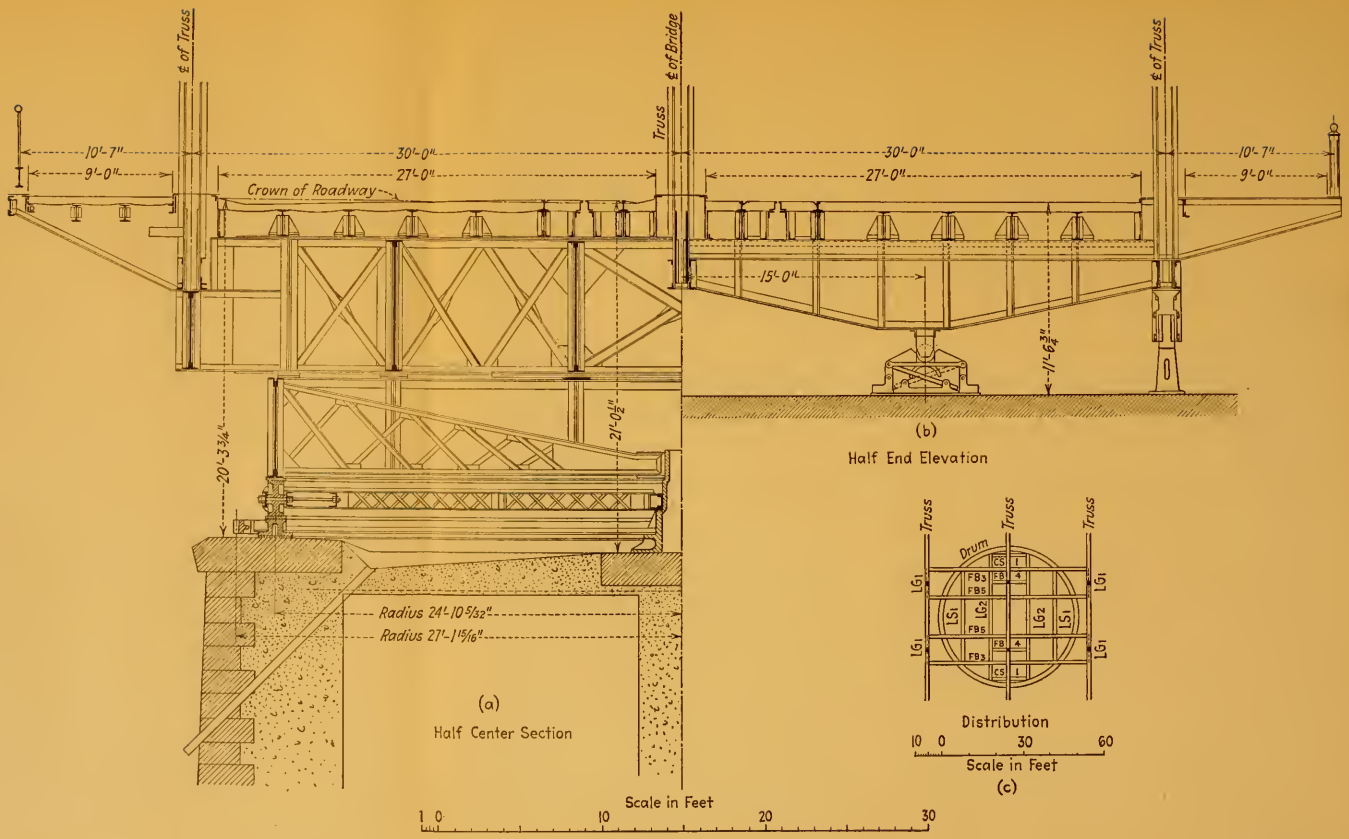


Fig. 3N.

(To face page 62.)

well-designed bridge the weight of the span is distributed to the drum by a system of girders which have definite and equal static reactions from the drum.

Figure 30 shows a few arrangements that have been used. In each sketch AB and CD are the lengths of the middle panels of the main

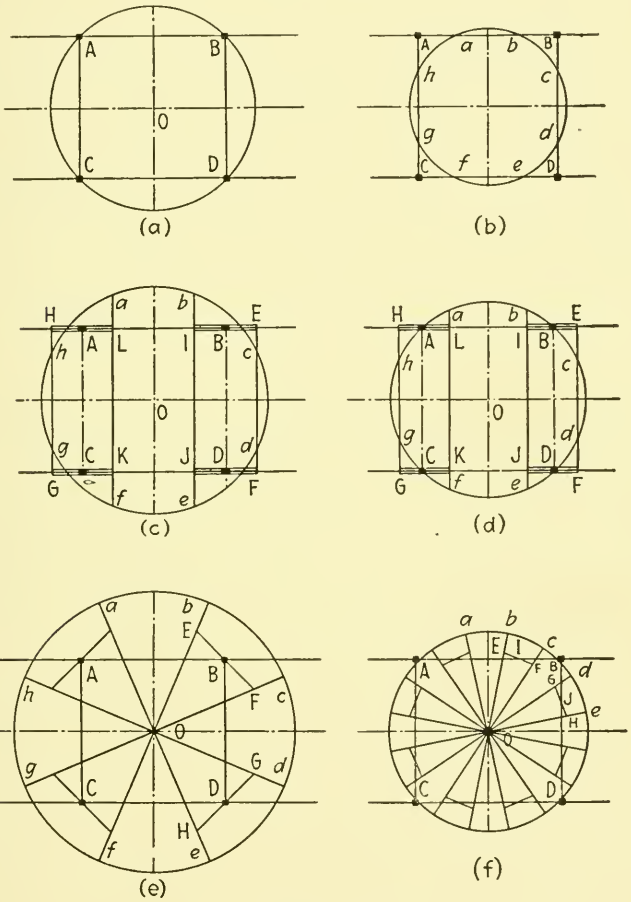


Fig. 30.

trusses, measured on their center lines. In order to compare the diameters of the drums required by the various designs, it is assumed that the main trusses, or girders, are 18.5 ft. apart, from center to center. This is about the average distance required for modern, heavy, single-track, railway swing spans.

In Fig. (*a*), the load is distributed equally to the four points A, B,

C, and D. The middle panel, AB, is the same as the truss spacing, or 18.5 ft. The diameter of the drum is $18.5 \times$ sec. 45°, or 26.16 ft. This design requires a large drum, which must be deep for stiffness; and the loads will be taken by a few of the wheels near the loaded points. It is suitable for light bridges only, when the depth from floor to masonry is ample.

Figure (*b*) shows a much better design. The load is distributed equally to the eight points *a*, *b*, *c*, *d*, etc. The diameter of the drum is $18.5 \times$ sec. $22\frac{1}{2}°$, or 20.03 ft. This design has been used for single-track and heavy double-track bridges with good results.

Figure (*c*) shows another method of distributing the load to eight points of the drum. Four transverse girders, HG, LK, IJ, and EF, rest upon the drum at points *a*, *b*, *c*, *d*, etc. The middle panel points of the main trusses are supported by the short girders HL, IE, etc., which are framed into the transverse girders. The diameter of the drum is made such that $HA = AL$, etc., or $D \times \cos. 22\frac{1}{2}° - 18.5 = 18.5 - D \times \sin 22\frac{1}{2}°$; in which D = the diameter of the drum. From this expression, $D = 28.32$ ft.

The designs in Figs. (*b*) and (*c*) are such that the drum diameter bears a fixed relation to the spacing of the trusses, and the middle panel must equal the truss spacing. This is sometimes inconvenient, as in a case where a new bridge must be built on an old pier. Fig. (*d*) is a variation of Fig. (*c*). The drum diameter is assumed and the length of the middle panel is made to fit the drum. Assume that the trusses are 18.5 ft. from center to center, and the drum is 25 ft. in diameter. Let x = the length of the middle panel; then $25 \times \cos 22\frac{1}{2}° - x = x - 25 \times \sin 22\frac{1}{2}°$, or $x = 16.33$ ft. It should be noted that while $HA = AL$, etc., the distance Ec is greater than bI, and the transverse girder, EF, must be made heavier than be.

The designs shown in Figs. (*a*), (*b*), (*c*), and (*d*), all distribute the entire load to the drum. The center carries no load, but must be anchored to the masonry to resist any horizontal forces developed during the rotation of the span. These forces become large unless the track, wheels, and live ring are very accurately made and carefully adjusted. The center pier is loaded only near its circumference. To assist in holding the center in place and to partly distribute the loads over the surface of the pier, a part of the weight can be delivered to the center pivot. This type is known as a combined rim- and center-bearing bridge.

Figure (*e*) shows a method of distributing the weight to the center and eight points of the drum. Eight radial girders *Oa*, *Ob*, etc., are framed into the drum, and attached to a revolving ring on the pivot.

Short girders at A, B, C, and D, are framed between the radials and support the middle panel points of the trusses. About one-fifth of the load should be delivered to the center. If less than this, the distances Eb, etc., become too short to provide for the shears at the outer ends of the radials. Using the same truss spacing and middle panel length as in the preceding cases, the diameter of the drum becomes, $D = \frac{5}{4} \times 18.5 \times \sec 45° \times \sec 22\frac{1}{2}° = 35.4$ ft. This design requires the largest drum and center pier of the six types shown.

In order to secure a better distribution to the drum and reduce its diameter, 16 radial girders may be used, as shown in Fig. (*f*). The diameter of the drum now becomes $D = \frac{5}{4} \times 18.5 \times \sec 22\frac{1}{2}° \times \sec 11\frac{1}{4}° = 25.52$ ft.

The diameters of the drums required by the several types of distribution, and the percentages of the smallest, are given in Table 3K.

TABLE 3K

Type	Diameter of Drum, Feet	Per Cent of (*b*)
Fig. (*a*)	23.16	130.6
Fig. (*b*)	20.03	100.0
Fig. (*c*)	28.32	141.4
Fig. (*e*)	35.40	176.7
Fig. (*f*)	25.52	127.4

Fig. (*d*) is omitted as the drum diameter may be varied.

For a span of a certain weight, Fig. (*a*) requires the deepest drum; but as the trusses rest directly upon it, the total depth from base of rail to masonry is not excessive. Figs. (*b*), (*c*), and (*d*) each require the same depth of drum and, as the girders rest upon the drums, the distance from base of rail to masonry is greater than for Fig. (*a*). Fig. (*e*) requires a drum as deep as for Figs. (*b*), (*c*), and (*d*), but, as the radial girders frame into the drum and the short girders into the radials, the distance from base of rail to masonry is less than for any of the preceding types. In Fig. (*f*), the drum is loaded at 16 points and can be the shallowest of all. The radials and short tangental girders frame flush with the drum, but the distributing girders AB, BD, DC, and CA, carry the entire load as cantilevers and must be deep and heavy. The distance from base of rail to masonry thus becomes greater than for Fig. (*e*) but less than for Figs. (*b*), (*c*), and (*d*).

Many other types of construction have been used, but these selected designs are all practical and efficient and have the advantage that the distributions are statically determinate. Any type in which careful adjustment for deflections must be made during erection should be avoided.

14. Single-track, railway, shear-pole swing span.—The general arrangement of a single-track, railway, shear-pole swing span is shown in Fig. 3P. This is the cheapest of all types of swing bridges. When the navigation traffic is small and openings infrequent, particularly in locations where navigation is closed during the winter, this design has been quite satisfactory. Most shear-pole bridges are on small railroads and branch lines, but there is one case where four double-track bridges have been in service under heavy main-line traffic for twenty years. The maintenance cost has been very small and no trouble has been experienced in operation.

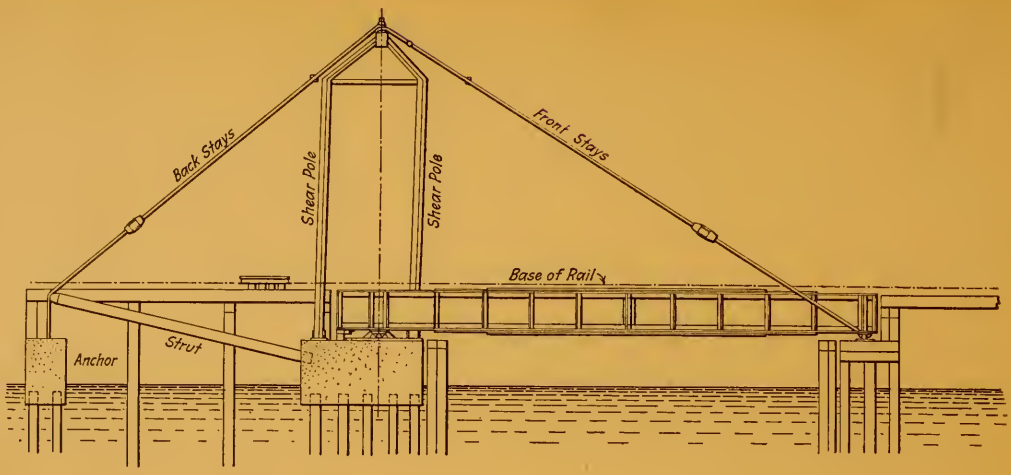
The shear poles are usually of steel. The centers are of the simple-pivot, disc type. Both the dead and live loads are delivered to the pivot by cross girders. Wedges are rarely used. Two transverse balance wheels steady the span, when swinging. The free ends of light bridges are supported by rollers, running up on an inclined track until they register against stops on the pedestals. For bridges under heavy traffic, wedges at both ends would be desirable. The center of the pivot on the shear pole is set slightly eccentric with respect to the pivot on the pier, in such a way that the free end of the bridge is lifted a little when moving to open position. This assists in closing. The most common method of operation is by means of hand winches on the draw protection. Two drums are provided, one for opening and the other for closing.

A single-track deck plate girder bridge which provided a 40-ft. clear opening, between fenders, was 56 ft. $2\frac{1}{2}$ in. long. The shear pole was of steel. The live load was Class E40 and the specifications were those of the American Bridge Co., of 1900, for medium steel, at 17,000 lbs. per square inch in tension.

The weight of the steelwork was	44,000 lbs.
The weight of the machinery was	13,000 lbs.
Total weight	57,000 lbs.

A similar double-track bridge, with two single-track spans opening in opposite directions, which gave a clear opening of 30 ft., was 46 ft. $5\frac{1}{2}$ in. long. One wide shear pole was used. When designed for E44 loading and for the same specifications, the total weight was 150,000 lbs.

Shear-pole Swing Span.



(a)

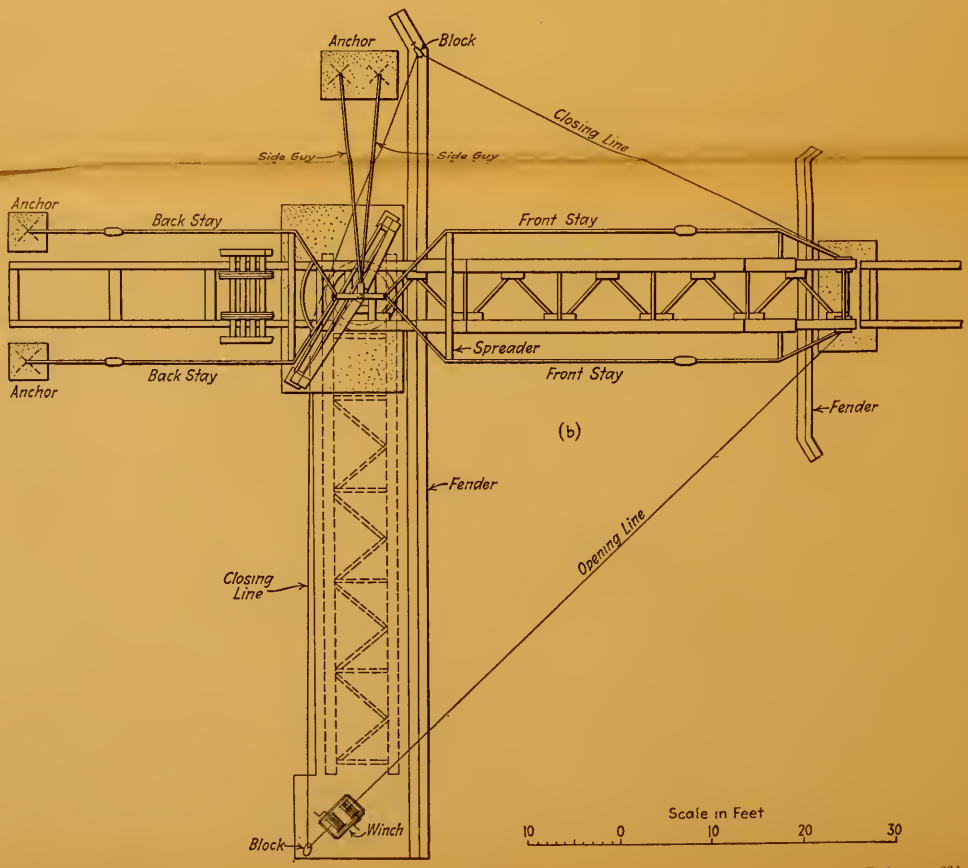


Fig. 3P.

(To face page 66.)

15. Approximate weights of railway swing spans.—Most first-class American railways now specify live loads of about Cooper's Class E60. A few still use E55, and some base their loads upon the Santa Fé type or Mallet locomotives, longer and heavier than E60. Full details of the Class E loads and a new series based upon the Mallet type and designated as Class M, may be found in the "Final Report on Specifications for Design and Construction of Steel Railway Bridge Superstructure," by the Special Committee of the American Society of Civil Engineers.[7] Many valuable tables of moments and shears for both systems are given.

The choice of live loads is made with reference to the rolling stock in use on the railway and with due allowance for the probable increase after the bridge is built. When a draw bridge is to be designed for a certain live load, it is necessary to determine the approximate weight of the steelwork before stresses can be calculated. The best method is to base the preliminary weight upon the known weights of existing structures of similar length, capacity, and design.

Treatises on bridge design seldom give adequate data concerning the weights of movable structures and such data are often omitted from descriptive articles in the engineering periodicals. In order to supply this lack, a few charts are given in Figs. 3Q to 3U, showing the actual weights by points plotted to scale. The span lengths are taken as abscissas and the weights, in hundreds of thousands of pounds, as ordinates. On each chart a curve is also shown. They are in the form represented by $W = aS^2 + bS$, in which W is the weight of the steel superstructure, complete, in pounds, and S is the span length, from center to center of end supports. The values of the coefficients, a and b, have been determined for the most probable curve of this type, the reductions having been made from the actual weights by the method of least squares.

When the weight of a swing span is known for a certain span length, live load, and method of design, the approximate weight for a heavier or lighter load is readily found by applying corrections based upon experience. For example, when the weight is known for a Class E50 load a similar bridge of the same length and for the same specification, but designed for an E60 load, should not weigh over 10 per cent, more than the E50 span. This assumes that both spans have been designed with economical outlines and floor depth. For approximate preliminary assumptions, known weights may be varied upward and downward by 10 per cent for each variation of ten units in the E Class designation of the live load.

[7] *Trans., Am. Soc. C. E.*, Vol. LXXXVI, p. 471, 1923. Also issued in pamphlet form.

Fig. 3Q. Southern Pacific Co.'s single-track, through, riveted, center-bearing, swing spans.—It is unusual to know the weights of a series of swing draw bridges of different lengths, all designed for the same

Single-track, Through, Center-bearing Swing Spans. Southern Pacific Co., Specifications 1003 of 1909.

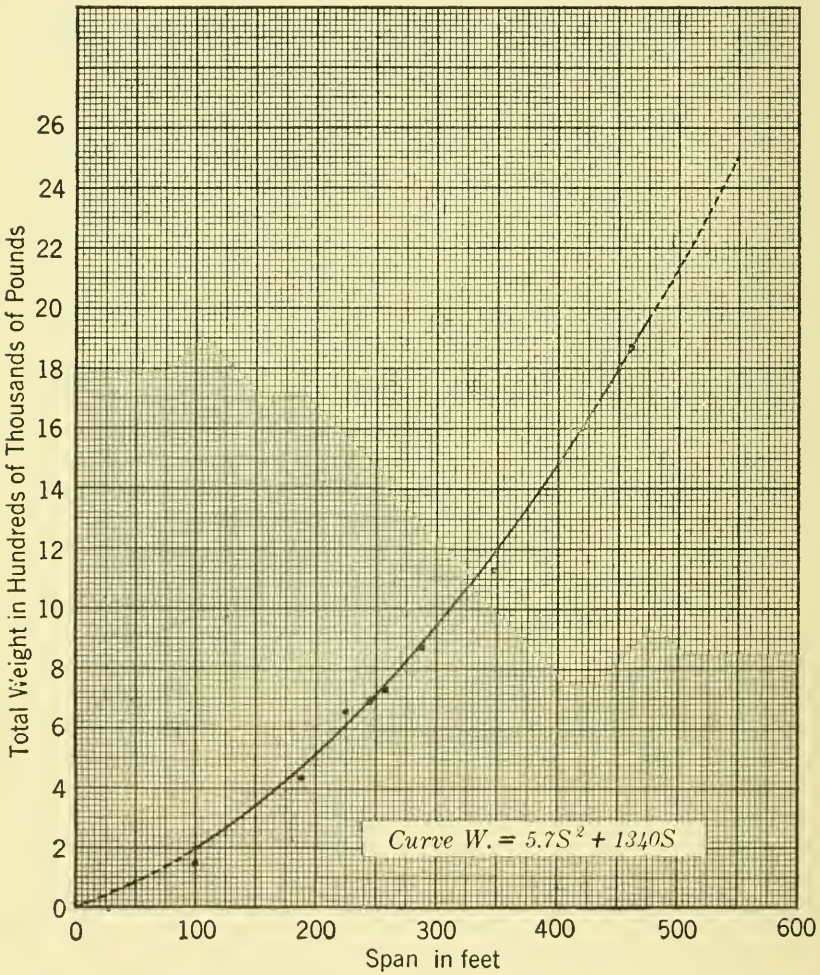


Fig. 3Q.

load and under the same specifications. The labor of making such a series of designs and estimates is almost prohibitive. Fortunately, it is possible to give the weights for one such series, which is valuable

as a basis of comparison. This includes eight single-track railway swing spans of various lengths from 101 ft. to 458 ft. Their weights are plotted on Fig. 3Q, by points. They were all designed in accordance with the Southern Pacific Co.'s Common Standard Specifications, 1006, of 1909. The live load was about Class E55, but the two engines were followed by 5000 lbs. per linear foot in place of 5500, as in E55. The method of design was based on the fatigue theory, using tension unit stresses of 8000 $\left(1 + \frac{\text{Min.}}{\text{Max.}}\right)$ lbs. per sq. in. reduced for compression by the formula,

$$C = 8000\left(1 + \frac{\text{Min.}}{\text{Max.}}\right) - 45\frac{l}{r}.$$

These bridges are heavier than if they had been designed for the same loads in accordance with the Specifications of the American Society of Civil Engineers, of 1923.

The equation of the most probable curve of weights is:

$$W = 5.7S^2 + 1430S. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The curve is extended at both ends by broken lines, to indicate the weights beyond the limits of the span lengths from which the equation was determined. The part for the shorter spans is of uncertain value.

Fig. 3R. Single-track, through, riveted, center-bearing swing spans.— The weights of twenty-eight single-track, through, riveted, center-bearing swing spans are plotted on Fig. 3R, as plain points. Those of five through plate-girder spans are shown by points in a circle. None of these bridges were designed for less than E50, others for about E55, and nearly one-third of them for E60 loads. Most of the specifications were similar to those of the American Railway Engineering Association, of 1910. Impact was calculated from the formula.

$$I = S\frac{300}{300 + L}.$$

The tension unit stress basis was in most cases 16,000 lbs. per sq. in. The bridges represented by points far from the curve were designed in accordance with special requirements. All spans were considered in calculating the curve, which may safely be taken as representing the weights of E50 spans up to about 225 ft., and for E55 for longer spans.

The equation of the curve is:

$$W = 8.92S^2 + 280S. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The weights become uncertain on the broken line extensions of the curve, particularly for the shorter spans.

Single-track, Through, Center-bearing Swing Spans.

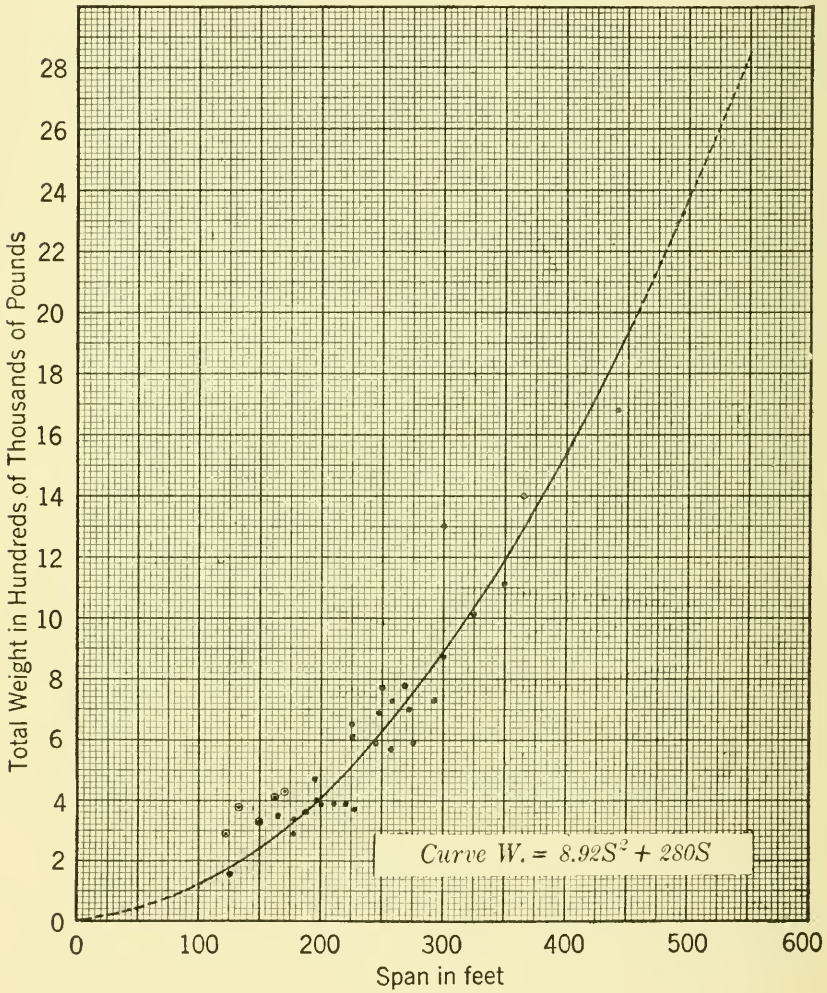


Fig. 3R.

Fig. 3S. Single-track, through, rim-bearing swing spans.—The weights of seventeen single-track, through-truss, rim-bearing swing spans are plotted by points in Fig. 3S. It will be noted that this curve closely corresponds to that in Fig. 3R. These rim-bearing bridges were

built earlier than most of the center-bearing bridges plotted in Fig. 3R, and for lighter loads. A few were designed for E40, some for E45, and most of the others for E50. The curve in Fig. 3S may be rather

Single-track, Through, Rim-bearing Swing Spans.

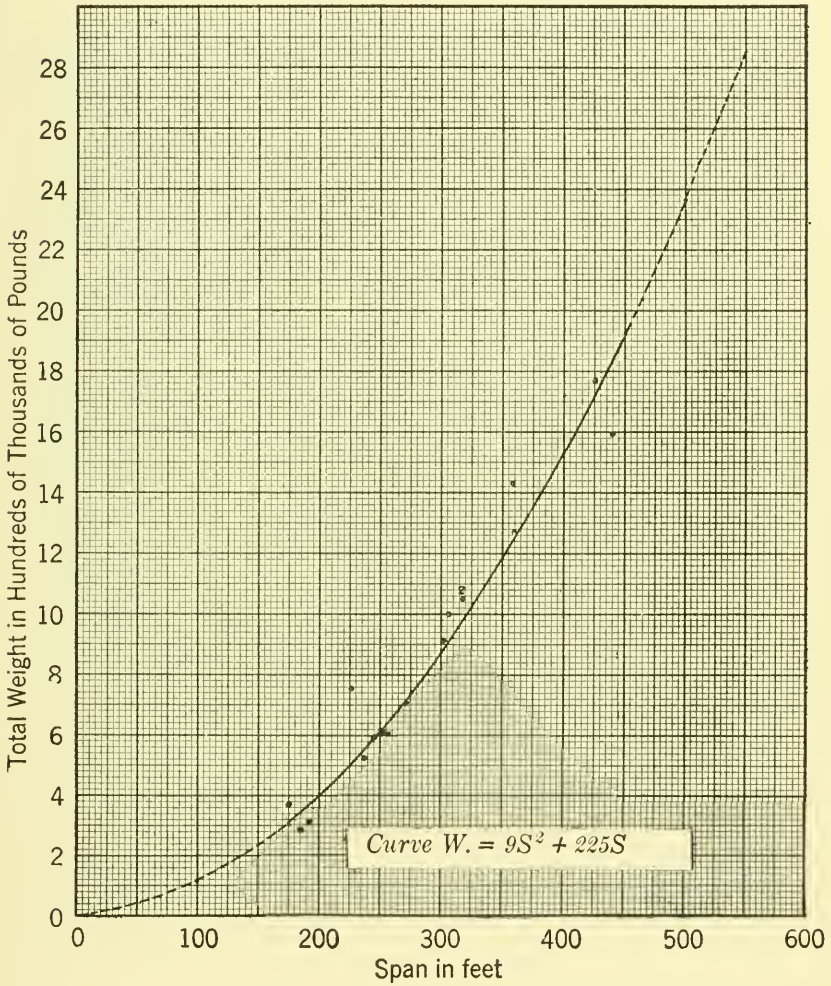


Fig. 3S.

light for E50, which explains the close agreement with that in Fig. 3R. Rim-bearing spans are heavier than center-bearing for the same conditions and span lengths. The equation of the curve is:

$$W = 9S^2 + 225S. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The broken line extensions are even more uncertain than those in the preceding article.

Fig. 3T. Double-track, through, riveted, center-bearing swing spans. —The weights of twelve double-track, through, riveted, center-bearing

Double-track, Center-bearing, Riveted Swing Spans.

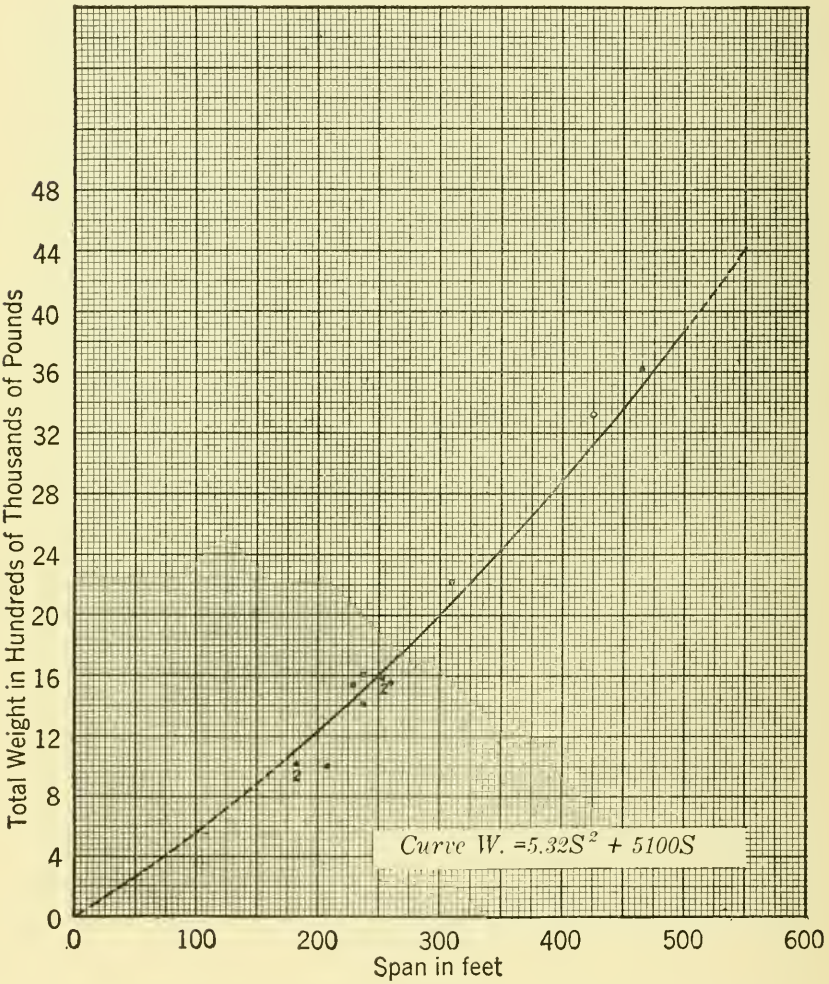


Fig. 3T.

swing spans are plotted by points in Fig. 3T. None of these bridges were designed for loads less than E50, and most of them for E55 and E60 loads. The curve of weights is safe for E55, up to about 150 ft.

span and probably for E60 for longer spans. The equation of the curve is:

$$W = 5.32S^2 + 5100S. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Double-track, Through, Rim-bearing Swing Spans.

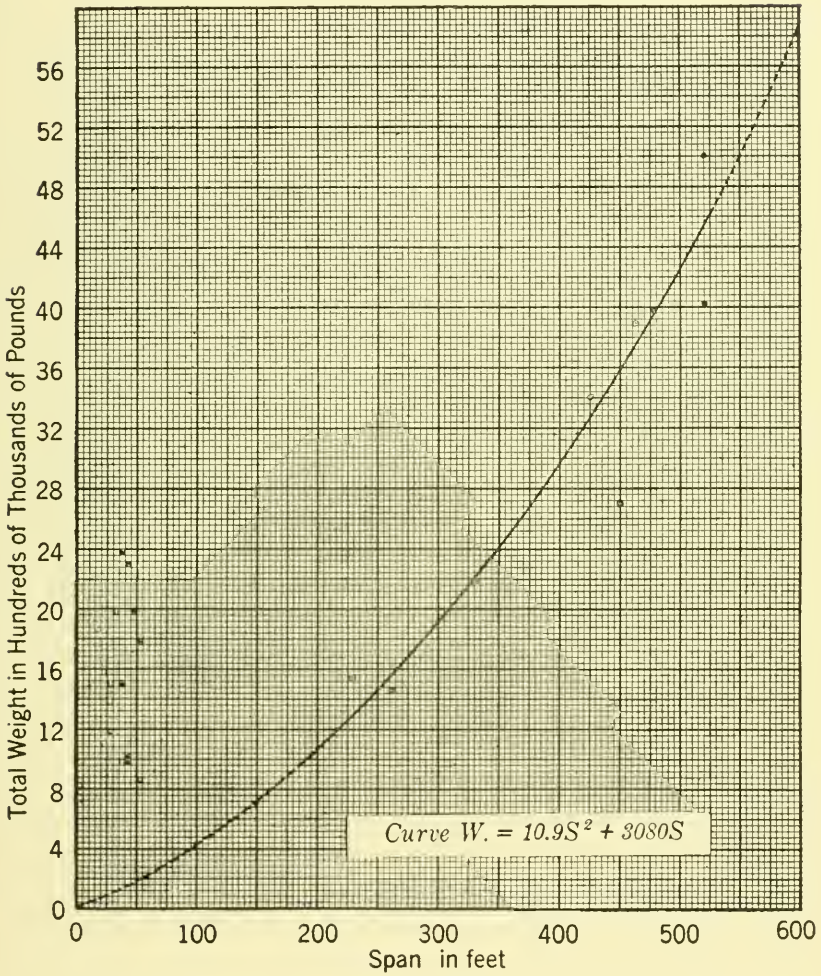


Fig. 3U.

The broken line extensions of the curve are quite uncertain, particularly for the shorter spans.

Fig. 3U. Double-track, through, rim-bearing swing spans.—The weights of nine double track, through, rim-bearing swing spans are

plotted by points in Fig. 3U. Nearly all are pin-connected truss spans. Most of the designs were for E50 to E60 loads, and were made in accordance with a variety of specifications. The curve represents about E50 loading up to 250 ft., and E55 for the longer spans.

The equation of the curve is:

$$W = 10.9S^2 + 3080S. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The weights in the broken-line extensions are very uncertain.

16. Approximate weights of through-truss, highway swing spans.—Railway bridges are usually single- or double-track structures, and their widths are nearly constant for each type. Thus it is easy to compare the weights of spans of different lengths and loadings. This is not true of highway bridges. The roadways may be of any width and many also have sidewalks outside of the trusses. Bridges of the same length and for the same live loads, when designed for the same specifications, may vary widely in total weight. Some basis of comparison is desirable.

The tendency of modern specifications for highway loads is to designate certain loads on parallel lanes of traffic. Such loads represent traffic conditions better than the former system of uniform live loads per square foot of roadway, combined with heavy truck or wagon loads on any part of the roadway. Two of the latest specifications designate loads to be applied on parallel lanes of traffic 9 ft. wide. They are the "Final Report on Specifications for Design and Construction of Steel Highway Bridge Superstructure," being the report of a Special Committee on Specifications for Bridge Design and Construction, of the American Society of Civil Engineers, dated Feb. 4, 1924.[8] Also the "General Specifications for Steel Highway Bridges," of the American Railway Engineering Association, Dec., 1923.[9]

A study of the weights of highway swing spans, reduced to the weights corresponding to a 9-ft. lane of traffic, demonstrated that they then formed a reasonable basis for comparison. While the available data are much less complete than those for railway structures, it is thought that the results of the study are of enough value to be given in convenient form for easy use. Figs. 3V, W, and X show the weights reduced to 9-ft. lane widths, plotted by points, and on each chart a curve is drawn. These curves were developed in the same manner as those shown in Art. 15. The lane-weights were taken from the expression:

$$\text{Lane-weight} = \text{total weight} \times \frac{9}{\text{width}}. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

[8] *Trans. Am. Soc., C. E.*, Vol. LXXXVII, p. 1273, 1924.

[9] *Bulletin 262* of the American Railway Engineering Association, Dec., 1923.

In this the term " width," is taken as the width of the roadway, when there are no sidewalks, or the width of the roadway plus the width of one sidewalk.

Highway, Through, Center-bearing Swing Spans.

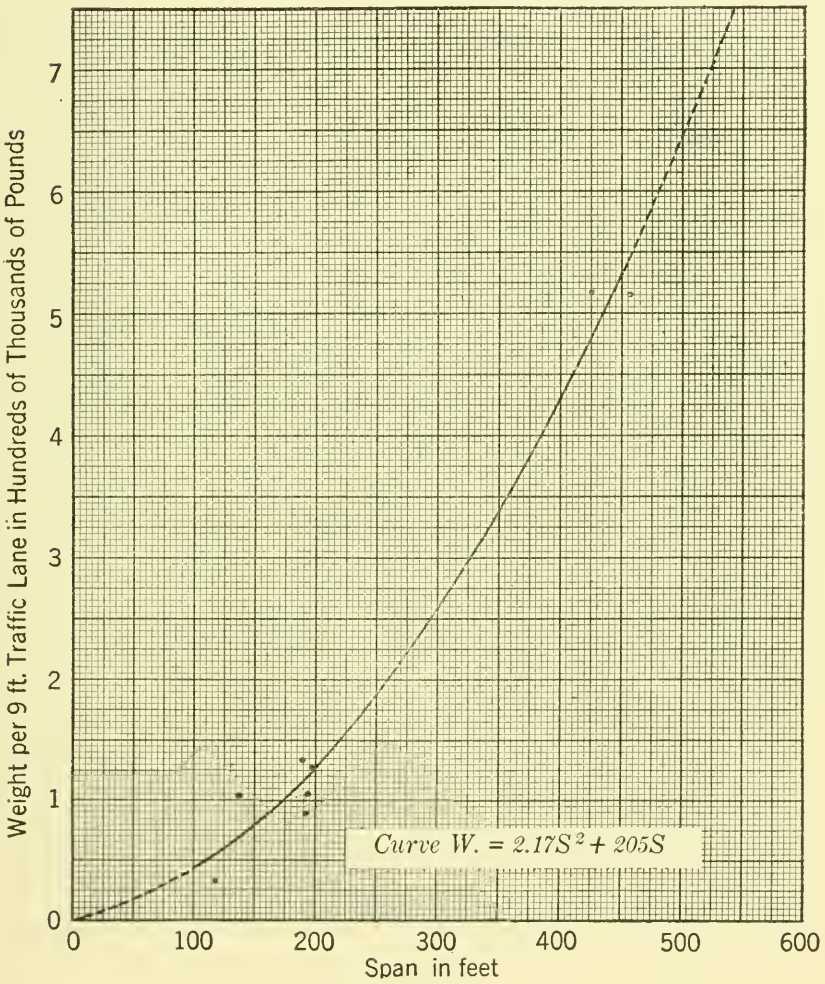


Fig. 3V

Fig. 3V. Ordinary highway, center-bearing swing spans.—The 9-ft. lane-weights of eight ordinary highway, center-bearing swing spans are plotted by points on Fig. 3V. The live loads were usually based upon

about 100 lbs. per square foot of roadway, combined with wagon, truck, or road-roller loads in considerable variety. The specifications were also quite varied. It is evident that the curve shown must be considered as only an approximate guide, and its accuracy is less than those given for the railway bridges.

The equation of the curve is:

$$W = 2.17S^2 + 205S. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The broken-line extensions for the shorter spans are of uncertain value, but such short spans are of little practical use.

It must be remembered that W in (7) is the weight corresponding to a single 9-ft. traffic lane. For example: if W is found for a certain span length and the roadway is 27 ft. wide, the weight of the span is $3 \times W$.

Fig. 3W. Light highway, rim-bearing swing spans.—The 9-ft. lane-weights of five light highway, rim-bearing swing spans are plotted by points on Fig. 3W. These bridges were all unusually light and were built several years ago. It will be noted that they weigh less than center-bearing bridges of the same lengths. If the loads had been as heavy as those for the center-bearing bridges, their weights would have been greater than for the corresponding center-bearing designs. Two of these spans had light, narrow sidewalks.

The equation of the curve is:

$$W = 1.58S^2 + 210S. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The broken-line extensions of the curve are of uncertain value, particularly for the shorter spans.

Fig. 3X. Heavy, city, through, rim-bearing swing spans. The 9-ft. lane-weights of ten heavy, city, through, rim-bearing swing spans are plotted by points on Fig. 3X. In calculating the lane-weights the term "width," in Eq. (6) was taken as the sum of the widths of the roadways and one sidewalk. Five of the bridges are in New York City, over the Harlem River, and are of quite modern design and for heavy loads.

The equation of the curve is:

$$W = 0.6S^2 + 1340S. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The broken-line extensions of the curve are of little value for spans less than 150 ft. long.

17. Approximate weight of machinery for swing spans.—The term machinery is assumed to include parts that must be made in a machine

shop, rather than in an ordinary structural shop. In center-bearing bridges the machinery includes:

All cast parts, like the center pivot, complete; the rack and track on the center pier; the center and end wedges, with their seats and pedes-

Highway, Through, Rim-bearing, Swing Spans.

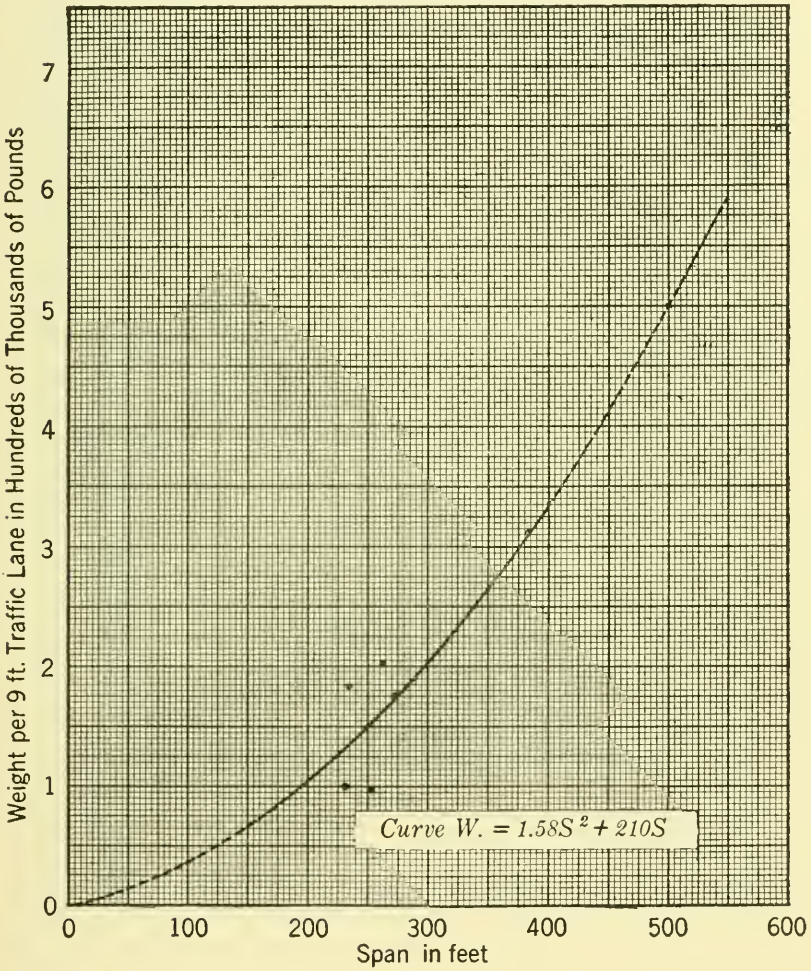


Fig. 3W.

tals; the end latches and catches; the end-rail operating mechanisms and rail locks; also all gears, shafts, bearings, and minor parts in the trains of operating mechanism for turning the span and driving the

wedges, beginning at the point of application of power. It does not include the motive-power units. Miscellaneous small structural machinery supports are usually included in the machinery, particularly when they must be fitted to the mechanism in the shop.

Heavy, City, Through, Rim-bearing Swing Spans.

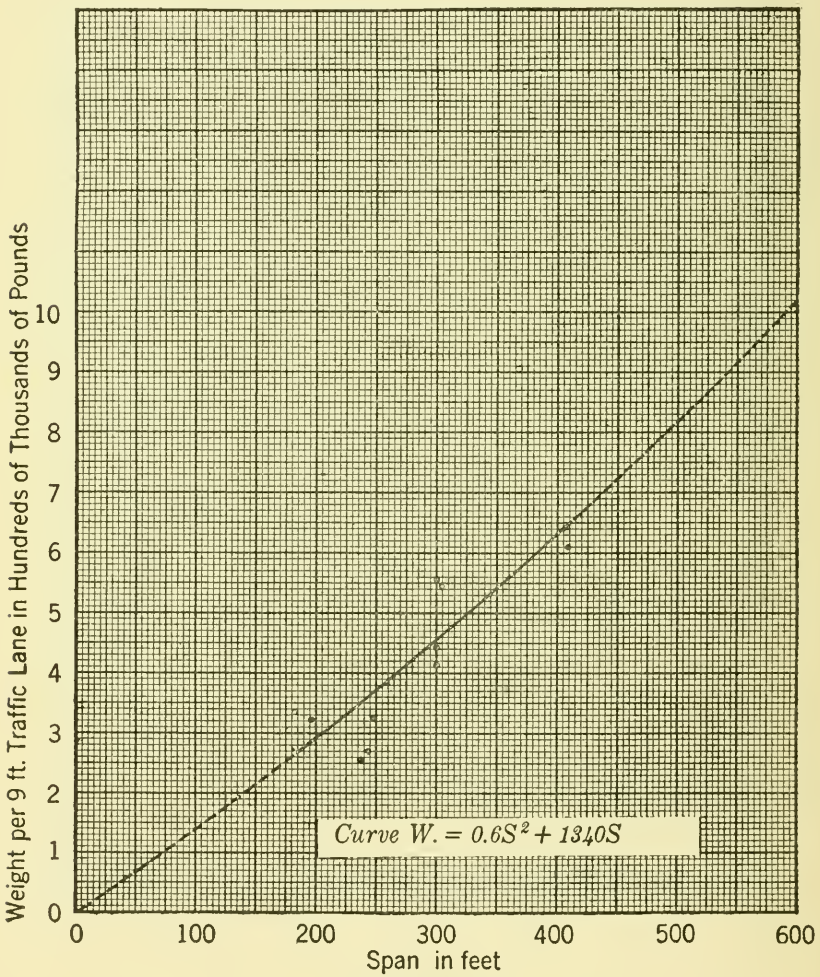


Fig. 3X.

In rim-bearing bridges, the same parts are included, so far as they apply, and in addition all parts of the live ring and the radials and bracing under the drum are included.

The weight of this class of machinery varies widely with the skill and experience of the engineer who devises the general arrangement and designs the details. As a result, all data given must be considered as approximate. They have been taken from average designs, and an experienced engineer should be able to design satisfactory machinery that will, on the average, weigh less than the amounts given.

Table 3K gives the weights of the machinery for various classes of bridges, all expressed in percentages of the structural steelwork. It must be noted that the weights shown on Figs. 3Q to 3X, and in Eq. (1) to (9), both inclusive, are all total weights, including the machinery.

TABLE 3K

MACHINERY WEIGHTS EXPRESSED IN PERCENTAGES OF STRUCTURAL STEEL WEIGHTS

Type of Superstructure	Span Lengths, in Feet				
	100–200	200–300	300–400	400–500	500–600
Railway Bridges					
Single-track, Center-bearing:					
Deck plate girders	17				
Through plate girders	18				
Truss spans	17	15	13	11	
Double-track, Center-bearing:					
Truss spans	18	15	12	10	
Single-track, Rim-bearing:					
Truss spans	23	22	19	17	
Double-track, Rim-bearing:					
Truss spans	22	20	18	16	20*
Highway Bridges					
Center-bearing:					
Truss spans	15†	14†	13‡	12‡	
Rim-bearing:					
Truss spans	22†	17†	17‡	17‡	

* Average of two spans with very heavy machinery.
† Ordinary highway spans.
‡ Heavy City bridges.

CHAPTER IV

BASCULE BRIDGES

1. General.—Many types of small bascule bridges were invented and built for military purposes long before the invention of the locomotive. A few had also been built for carrying highways across canals and small navigable streams, but the real development of the modern bascule began after the practical steam locomotive was invented, in 1829.

In 1831, a single-leaf, highway, simple-lever bascule was built over Bowcombe Creek, Kingsbridge, Devon, England. It was designed by Mr. J. M. Rendell.[1] It was 15 ft. 9 in. wide and 32 ft. long. The leaf turned on a " cast-iron shaft 7 ft. 6 in. from the inner end, working in the abutment pier, which is built hollow to receive it, and thus the part within the axle acts as a counterweight." The bridge was operated by an hydraulic pump which moved a rack, driving a pinion on a drum shaft on which a chain was wound, raising the end of the leaf.

In 1839, a small trunnion bascule was built at Selby, England, for the North Eastern Railway, over the River Ouse.[2] This was a double-leaf, simple-trunnion bascule, built of cast iron. When closed it acted as an arch for live loads. The shoreward ends carried the floor and counterweights, which revolved downward into a tail pit when the bridge was opened. The clear channel was 45 ft. when the leaves were in open position. Hand-power was used for operation.

The Knippel Bridge was built at Copenhagen in 1867. It was also of cast iron, and was a simple trunnion bascule, operated by hydraulic machinery. Before 1878, Mr. James Price remodeled an old retractile railway bridge into a bascule. It was across the River Shannon, at Drumsna. The approach girders carried the rails up to the center of the trunnions. The bascule girders were outside of the fixed approach girders. Each bascule girder had a separate trunnion turning in pedestals supported by longitudinal girders.

[1] A Paper by George Clarisse Dobson. *Min. Proc. Inst. C. E.*, Vol. II, p. 68, 1843.

[2] A Paper on Movable Bridges by Mr. James Price, M. Inst. C. E. *Min. Proc. Inst. C. E.*, Vol. LVII, Part III, p. 1, 1878–79.

In 1878, the Fijenoord trunnion bascule was built at Rotterdam, Holland. The clear span was 75 ft. Each leaf had four trusses. The two outer trusses acted as arches under live loads, and the inner ones were extended to the rear of the trunnions and were counterweighted. This bridge could be operated by hydraulic power, by gas, or by man-power. A highway bascule was built at Koenigsberg in 1880. It acts as a cantilever under live loads, rear anchors taking the uplift at the abutments.

During the decade from 1890 to 1900, the development of bascule bridges received much attention, and several of the modern types were designed. Some are based upon the older types, modified and improved to adapt them to modern traffic requirements, while others are new inventions.

2. Bascule bridge over the Tiber, near Rome.—A very interesting highway, double-leaf, simple-trunnion bascule bridge was built across the River Tiber, near Rome, before 1893. It was designed by Sig. Angelo Vescovali, Chief Engineer, Hydraulic Service of Rome.[3] The clear opening between the piers is 41 ft. 7 in., and the width of the roadway is 19 ft. 6 in. There are two sidewalks, 5 ft. $5\frac{1}{2}$ in. wide. Fig. 4A is a longitudinal and Fig. 4B a transverse section at one of the two main piers. Each bascule leaf has five longitudinal girders, carrying the floor and counterweights. The trunnions extend through these girders and turn in ten pedestal bearings, supported by a transverse box girder, also passing through the five girders, and supported by the extended ends of the two fixed, longitudinal, approach girders which are outside of the bascule leaf. The bascule girders have curved openings arranged to give clearance for the transverse box girder when the leaf is opened. The break in the floor is slightly to the rear of the trunnions. The leaves are operated by hydraulic cylinders, within the piers, by means of flat steel wire ropes passing over deflecting sheaves on the piers and a quadrant on the ends of the two outer bascule girders. The leaves are nearly balanced, and close by gravity, but are under control by the hydraulic mechanism. There are two fixed spans of 114 ft. 8 in. each on each side of the draw span, the whole bridge being 563 ft. long.

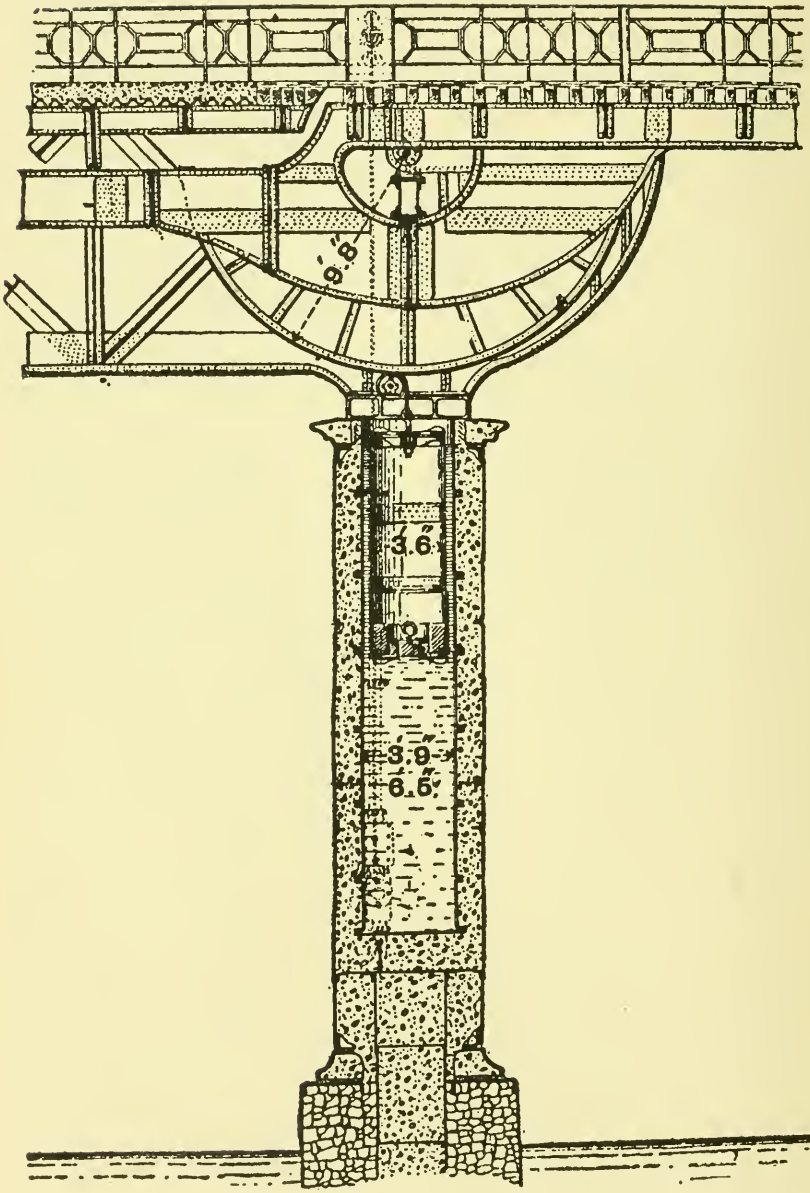
3. The Tower Bridge, London.—The bridge over the River Thames, near the Tower of London, is among the earliest of modern, heavy bascule bridges.[4] Many of the principles involved in its design have

[3] *Industries*, London, Vol. 14, p. 316, 1893.

[4] "The Foundations of the River-Piers of the Tower Bridge," by George Edward Wilson Cruttwell, M. Inst. C. E. *Min. Proc. Inst. C. E.*, London. Vol. CXIII, Part III, p. 117, 1892–93.

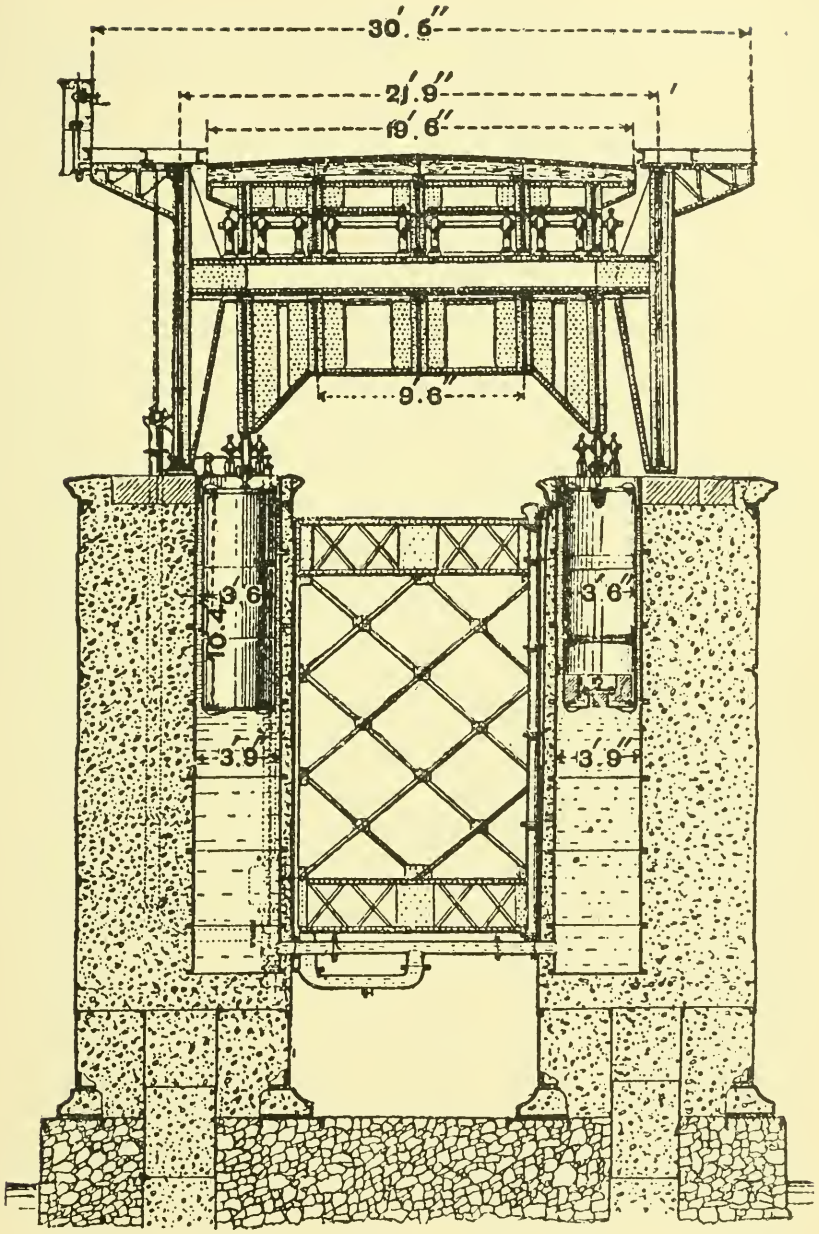
"The Tower Bridge: Superstructure," by G. E. W. Cruttwell, M. Inst. C. E., *Min. Proc. Inst. C. E.*, Vol. CXXVII, Part I, p. 35, 1895–97.

"The Machinery of the Tower Bridge," by Samuel George Homfray, M. Inst. C. E. *Min. Proc. Inst. C. E.*, London, Vol. CXXVII, Part 1, p. 54, 1896–97.



Industries.

Fig. 4A.



Industries.

FIG. 4B.

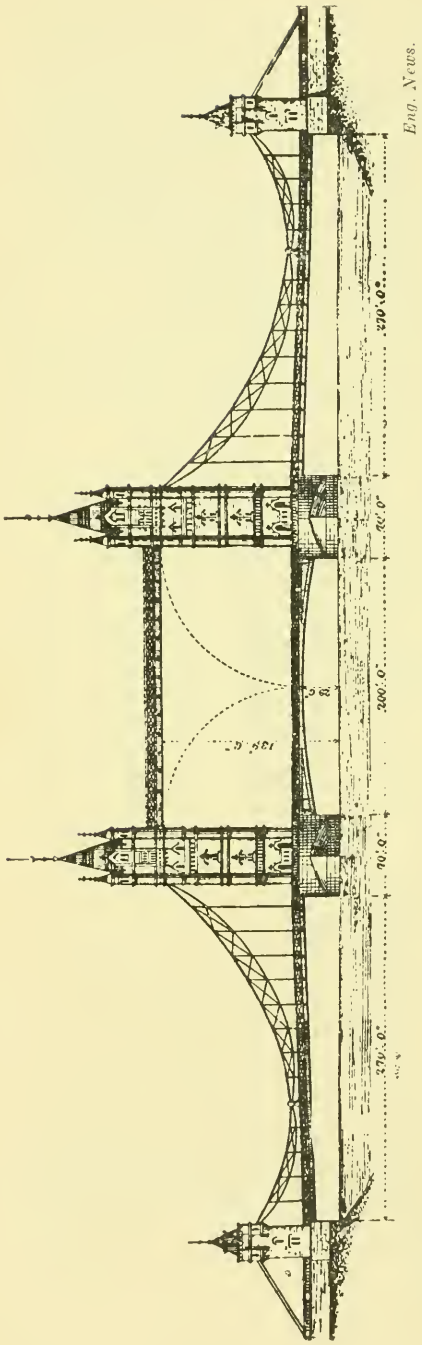
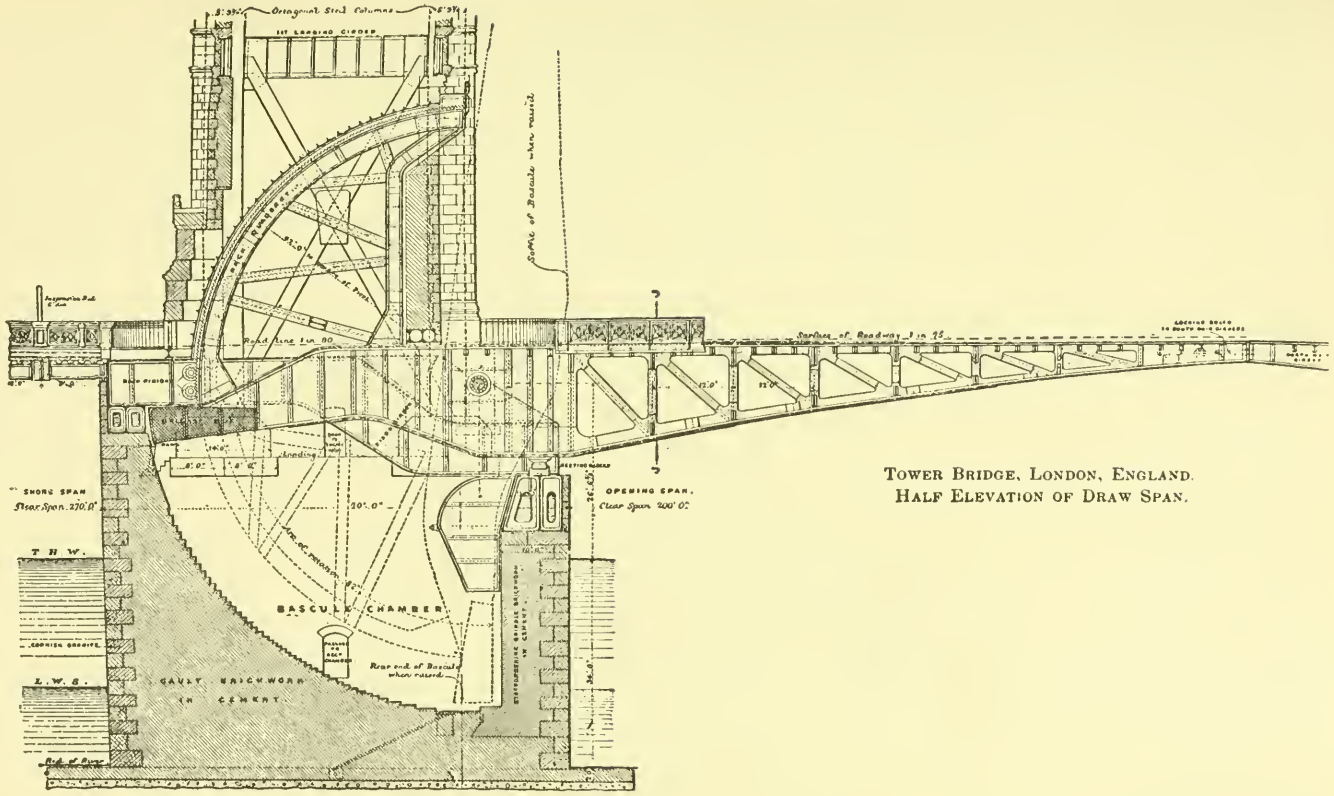


Fig. 4C.

been followed in later structures. The bridge consists of a double-leaf, simple-trunnion bascule span, which provides a clear waterway of 200 ft. The central bascule span is flanked at each end by a suspension span with 270 ft. clear waterway. The total length of the superstructure is 940 ft. out to out of the abutments. The north, or Middlesex, approach is 1260 ft. long, and the south or Surrey, approach is 780 ft., a total of 2980 ft. The width between parapets is 60 ft., except for the bascule span which is 49 ft. in clear width. The roadway on the bascule is 32 ft. wide, for four lines of vehicles. The two sidewalks are each 8.5 ft. wide. Fig. 4C is a general elevation of the bridge, taken from a drawing. Fig. 4D is a half sectional elevation of one of the bascule leaves.

Towers.—The main piers are surmounted by towers 119 ft. 6 in. high. They carry the loads from the suspension chains of the 270-ft. spans, the weight of the two high-level footwalks spanning the channel, and a part of the weight of the masonry screens around them. The screens were designed to give a monumental appearance to the bridge. Each tower contains two hydraulic lifts for passengers, and two flights of



TOWER BRIDGE, LONDON, ENGLAND.
HALF ELEVATION OF DRAW SPAN.

FIG. 4D.

stairs to give access to the high-level footwalks. The footwalk trusses carry the weights of two tension members, each consisting of 8 plates, 24 in by 1 in., riveted together and connected to the suspension links of the side spans. The reactions from the suspension system are delivered to the towers by 4 nests of segmental expansion rockers. The rockers are 20 in. in diameter and each nest consists of 8 segments, 3 ft. long, in 2 groups of 4 each, placed end to end. They are secured in proper position by means of teeth on their ends which engage the upper and lower bearing plates. The load on each rocker nest is 2,464,000 lbs. and the rocker-bearing pressure is expressed by $P = 428 \times D \times L$. In this formula D is the diameter and L the length of the rockers, both in inches. The live load was taken without impact.

Where the tower posts are surrounded by masonry, the steel is wrapped with oiled canvas, so as to permit free expansion. Layers of canvas and red lead were placed under the large tower column bases on the masonry, to secure a uniform distribution of the great loads. The column bases are of octagonal shape and the load at each column is 2867 tons.

Bascule span.—Each bascule leaf consists of four lattice girders extending 100 ft. beyond the face of the piers. The rear ends are of plate construction and their ends are 62 ft. 6 in. to the rear of the pier faces. They are spaced 13 ft. 6 in. from center to center. The panel length is 12 ft. and the floor is carried by floorbeams and stringers, with $\frac{3}{8}$-in. buckled plates on them. Outside cantilever brackets carry the parapets and sidewalks. The main trunnions are 13 ft. 3 in. to the rear of the pier faces and the span is 225 ft. 6 in. from center to center of trunnions. The rear ends of the bascule girders are floored over and carry the counterweights, which weigh 410 tons, of 2000 lbs., per leaf. Each consists of 84 tons of cast-iron blocks, the remainder being of lead, on account of limited clearances. Each leaf weighs 1198 tons.

The trunnions are 21 in. in diameter and 48 ft. long, of forged steel. They are keyed to cast-iron flanges bolted to the girders. The trunnions pass through the 4 main girders and turn in 8 pedestal bearings, supported by longitudinal girders parallel with the bascule girders, and spanning the tail pits. The pedestal girders also carry the floor over the tail pits. Each trunnion pedestal contains a roller bearing with 15 rollers, $4\frac{7}{16}$ in. in diameter and $22\frac{1}{2}$ in. long. The maximum pressure per roller unit, when turning, is expressed by $P = 727 \times D \times L$, in which D is the diameter and L the length of the rollers, both in inches.

When the leaves are closed their rear ends engage anchor girders riveted between the fixed trunnion girders. Tail locks are driven under the rear ends of the bascule girders, when closed. The free ends

are provided with shear locks, 5 in. in diameter, at each of the 4 joints. Pier-rest castings are built into the masonry about 9 ft. in front of the trunnions, and wedge-shaped bearing blocks are driven to firm bearing with the main girders, when the bridge is closed. A rack quadrant is attached to the upper side of the rear end of the two outer bascule girders. The pitch radius is 42 ft. and the circular pitch is 5.9 in. Two pinions engage each rack.

A steam boiler and engine plant is installed in arches in the Surrey abutment. Two double-tandem, compound, surface-condensing engines, of 360-h.p., drive hydraulic pumps. One engine can operate both leaves and the auxiliary machinery, and one is held in reserve. The pumps discharge into an accumulator in each pier. The plunger is 22 in. in diameter and its stroke is 18 ft. The pressure is 700 lb. per square inch. Two hydraulic engines are installed in each end of each main pier. The smaller of each pair is intended for ordinary operation, and the larger for emergencies due to heavy winds. The large engines have proved to be unnecessary and are used as reserves. Reduction gearing connects the engines with the main pinion shafts. The front and rear locks and the pier bearing blocks are all driven by hydraulic cylinders, controlled from a central operating station in one of the piers.

The pavement is unusual. The buckle plates are filled with concrete. A transverse layer of creosoted pine plank rests on the concrete and supports a longitudinal layer of greenheart planking. Creosoted pine blocks 4 by 9 by $4\frac{1}{2}$ in. are laid diagonally on the greenheart planks, to which they are secured by 5-in. galvanized coach screws, 2 ft. apart. The blocks are doweled together on all vertical surfaces by oak pins $\frac{3}{4}$ in. in diameter, $1\frac{3}{4}$ in. long, with a middle collar $\frac{1}{4}$ in. wide to space the blocks. The joints are filled with asphalt.

This bridge was designed by Sir. John Wolfe Barry, M. Inst. C. E., Chief Engineer. Mr. G. E. W. Cruttwell, M. Inst. C. E., was the Resident Engineer, and Sir William Arrol & Co., were the contractors for the superstructure. The foundations were begun in March, 1886, and the bridge was formally opened for traffic on June 30, 1894. The costs were:

Iron and steel superstructure	$1,685,500
Operating machinery	426,100
Piers and abutments	653,700
Masonry superstructure	745,600
Paving and lighting	151,600
Total for the bridge proper	$3,662,500
Approaches	484,300
Total	$4,146,800

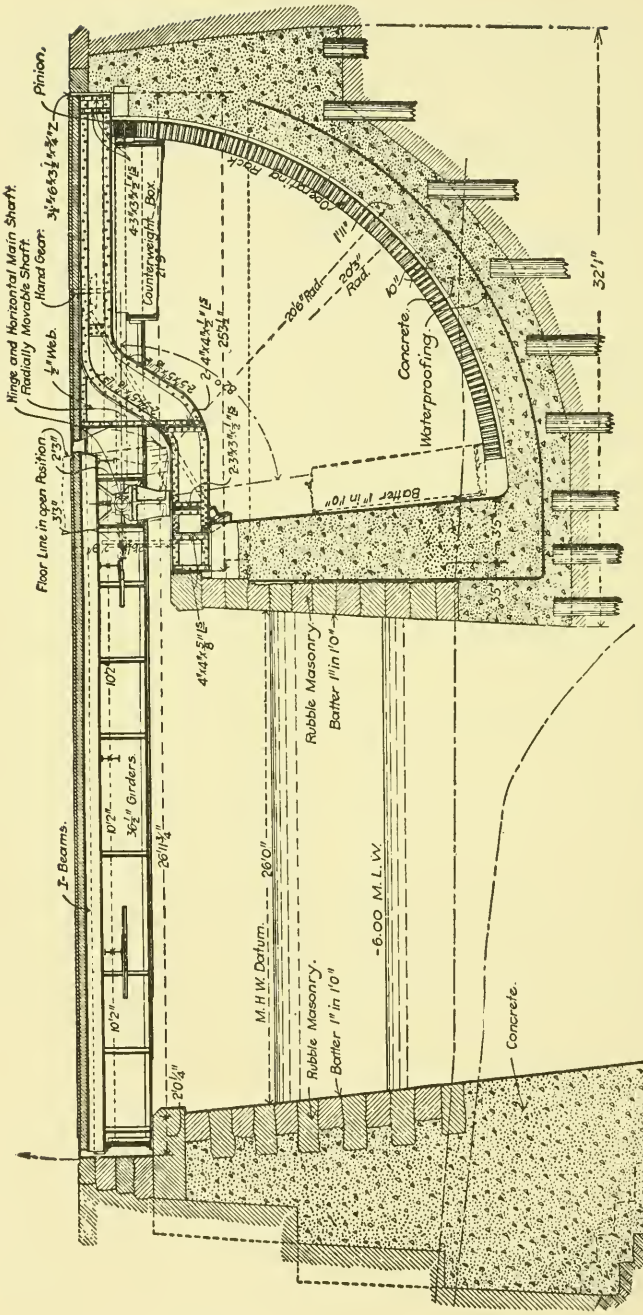


Fig. 4E.

Proc. Bkln. Engr's. Club.

4. 135th Street Bridge, New York City.—A small, simple-lever bascule bridge was built to carry 135th Street over the Mott Haven Canal, in New York City, in 1902.[5] The clear span is 26 ft., and the width 66 ft. over all. The bridge carries a roadway 42 ft. wide and two sidewalks of 11.5 ft. each. The general arrangement is quite similar to that of the Tower Bridge, except for the method of operation. There are 7 symmetrically arranged bascule girders, with a floor system of transverse I-beams, supporting a double plank floor. The four outer girders end at the trunnion, and the three interior girders extend rearward and carry the counterweight. The trunnions are 7 in. in diameter and each turns in 2 pedestals supported by longitudinal girders, which are curved upward to carry the floor across the counterweight pit. Fig. 4E gives a longitudinal sectional elevation of the bridge. The break in the floor is 3.25 ft. to the rear of the trunnions. A quadrant rack is mounted on the inner curved face of the abutment, and a pinion at the rear of the bascule girders engages it. Clearance is allowed between the rack and the masonry to permit dirt to fall out of the rack.

An electric operating motor is mounted on the main pier, connected by a train of gears with the main driving shaft, which extends to the main pinion, being supported by bearings on the girders. Provision is also made for hand operation by a capstan on the fixed span, with a diagonal driving shaft connected to the main gear train. The bascule can be opened in 20 seconds by the 18-h.p. motor. The bridge was designed for the heaviest road roller and city traffic. The weights follow:

Structural steelwork	66.5 tons
Machinery	29.0 tons
Cast iron sectional counterweight	65.5 tons

The cost was $30,000.

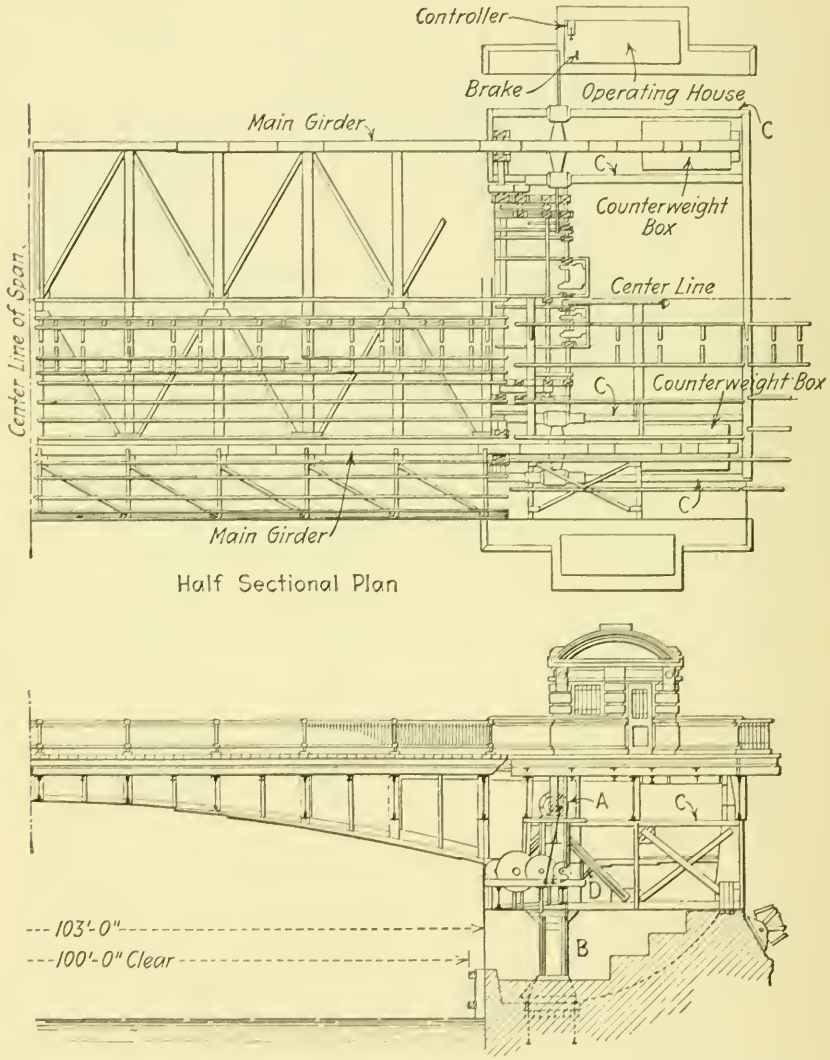
The bridge was designed by Mr. George R. Ferguson, for the Department of Bridges of the City of New York.

5. Anacostia Bridge, Washington, D.C.—The highway, double-leaf, simple-trunnion bascule draw span of the bridge across Anacostia Creek, in Washington, D.C., is arranged quite differently from those already mentioned.[6] Fig. 4F shows an elevation and plan of one-half of the draw span. The clear waterway is 100 ft., and the bridge carries a roadway 35 ft. wide and two 6.5-ft. sidewalks. The clear distance between the main piers is 103 ft., and the trunnions are 7 ft. $8\frac{1}{2}$ in. back

[5] "Types of Movable Bridges," by J. S. Langthorn, M. Bklyn. Engrs. Club. *Proc. Bklyn. Engrs. Club*, 1904, p.,122.

[6] *Eng. News*, 1905, II, p. 655.

of the pier faces, making the span length 118 ft. 5 in. between trunnion centers. There are two main girders 33 ft. between centers. They



Eng. News.

Fig. 4F.

extend 21 ft. back of the trunnions. Each girder has a trunnion attached to flanged castings bolted on each side of the girder web. Each trun-

nion turns in 2 pedestal bearings. They are supported by parallel longitudinal girders, 6.5 ft., center to center, which rest on short vertical columns under the trunnions at one end and on the masonry at the other.

Steel counterweight boxes are framed around the rear ends of the bascule girders, occupying the space between the ends of the bearing girders. Transverse girders are framed across the rear ends of the bearing girders. The rear ends of the bascule girders have heavy lugs, and when the bridge is closed they engage spring buffers, with oak stops, which are attached to the bottoms of the transverse girders. The main girders thus act as cantilevers for live loads on the channel arms.

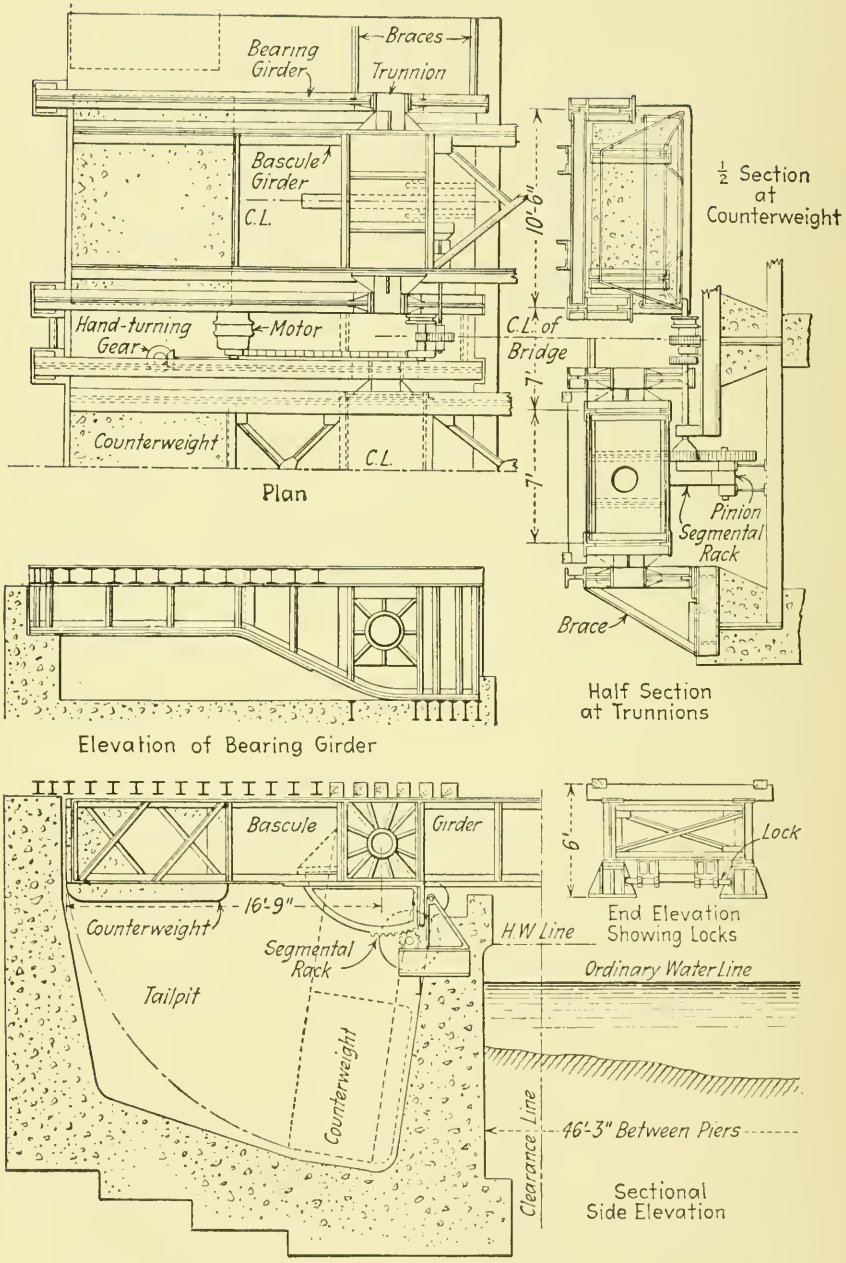
The bearing girders are extended in front of the trunnions, and a transverse girder, 7 ft. 5 in. from the trunnions, is fitted with adjustable wedge blocks, so that when the leaves are closed the trunnions are relieved of a part of the live load. The free ends of the bascule girders are fitted with shear locks, $4\frac{1}{2}$ in. in diameter. When the bridge is closed the locks are driven transversely through holes in overlapping plates on the ends of the girders. The floor over the counterweight pit is carried by the 4 trunnion-bearing girders. The break in the floor is a few feet in front of the trunnions.

The leaves are operated by electric motors, mounted on framing between the fixed girders, and a train of reduction gearing connects them with the two main operating pinions. These engage cast quadrant racks, bolted to the bottoms of the bascule girders. The counterweights are composed of scrap iron and concrete, and weigh about 350 lbs. per cubic foot.

The bridge was designed by Mr. W. J. Douglas, M. Am. Soc. C. E., under the direction of Col. John Biddle, Engineer Commissioner, District of Columbia, and was built in 1905 to 1907.

6. Père Marquette railroad bridge.—In 1915, the Père Marquette Railroad built a small double-track, railway, single-leaf, simple-lever bascule bridge, across the Paw Paw River, at Benton Harbor, Michigan.[7] The bascule span consists of two parallel single-track deck leaves which can be operated singly or together. Fig. 4G shows some of the more important details. The main girders are 70 ft. 9 in. long, the channel arm being 54 ft. and the counterweight arm 16 ft. 9 in. long. The trunnions are steel castings, bolted to the main girder webs, which are connected by short transverse, square, box girders. They turn in bearings inserted in the webs of fixed girders, parallel with the bascule girders, and supported by the masonry. The tail ends of the bascule girders are connected by cross frames and carry a counterweight block, of concrete and steel punchings, which weighs about 300 lbs. per cubic foot.

[7] *Eng. News*, 1915, I, p. 929.



Eng. News.

FIG. 4G.

The track is carried over the counterweight pit by transverse I-beams, supported by the fixed girders. When the bridge is closed the tail ends clear these I-beams by a small distance. The break in the floor is to the rear of the trunnions. The rails across the counterweight span rest in channels, built of an 18-in. base plate and two 3-by-3-in. guard angles. On the channel span they are supported by ordinary wood cross ties, fitted with timber guard rails.

When the bridge is closed, the free end is locked down to cast pedestals, by bolts moving transversely and engaging slots in the pedestals, which are anchored to the masonry.

The bridge is operated by a 25-h.p. electric motor, connected by a chain drive to two gear trains, each ending in a pinion which engages a segmental quadrant rack, attached to the transverse, box, trunnion girders, midway between the main bascule girders. Hand gear is provided also. The bridge can be opened as a single unit in about 30 seconds, by power. Hand operation requires 12 minutes for each of the two leaves.

The bridge is proportioned for Cooper's E60 loading.

7. Chicago Type, single-lever, trunnion, bascule bridges.—In 1899, many of the old swing bridges over the various branches of the Chicago River, in Chicago, were found to be in bad condition, some of them being beyond economical repair, and a few in dangerous condition. At the same time a movement was proceeding with the object of gradually abolishing the swing type in the city, on account of the obstruction to navigation by the piers and draw protection fenders. In May, 1900, the Commissioner of Public Works invited competitive designs and proposals for the construction of a bascule bridge to replace an old swing span that had failed by breaking apart at the center when the bridge was open.

The engineers of the city prepared three designs, and outside bidders submitted five others. A board of three engineers recommended the adoption of one of the city designs, with a few suggested modifications. This design was made under the direction of Mr. John Ericson, City Engineer, and Mr. Edward Wilmann, City Bridge Engineer. With later improved and simplified details, it has become well known as the Chicago Type Bascule.

The design follows that of the Tower Bridge, in London, in a general way, but with changes to adapt it to the local conditions at Chicago. In most cases the bridges are half-through with riveted connections, but deep enough at the trunnion ends to allow the use of substantial overhead bracing. Fig. 4H shows one end of the Northwestern Avenue Bridge, which is typical of several of the earlier bridges. Here, three

trusses were used. The counterweighted ends are relatively deep, ending in a quadrant-shaped member that carries the operating rack, which is on the convex side of the chord. The counterweights are on the bottom of the rear frames and below the floor. The counter-

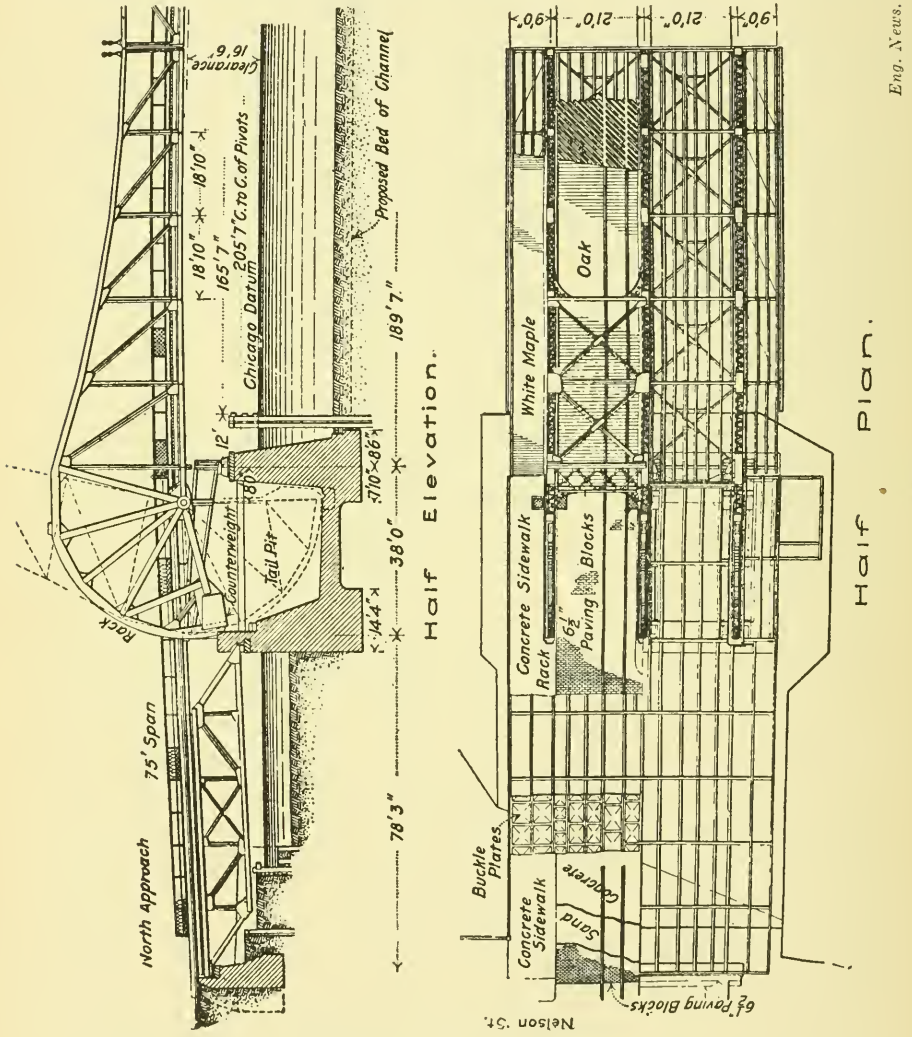


Fig. 411.

weighted curved ends move in slots in the floor. Both of the roadways and the sidewalks are supported by the fixed parts and the piers. The break in the floor is in front of the trunnion. The trunnions are supported by fixed girders, resting on the masonry, and they are usually in pairs, parallel with the bascule trusses.

When the bridge is closed, the two leaves are connected at the center by shear locks. The rear of the bascule engages anchorages, so that the leaves act as cantilevers for live loads. The first panel point in front of the trunnion rests on a pier pedestal, which relieves the trunnion and increases the stiffness of the structure under live loads. Usually, the rear end of the counterweight truss has a tail lock, which must be disengaged to open the leaf. Most of the bridges have only about 16 ft. of clearance above the water at the center of the span. When in open position, the counterweights are below the water level and water-tight pits are provided.

The Northwestern Avenue Bridge.[8]—Refer to Fig. 4H. This bridge has a span of 205 ft. 7 in. between trunnion centers. The clear waterway is 165 ft. 7 in., and the pier rests are 8 ft. in front of the trunnions. The trunnions are $17\frac{1}{4}$ in. in diameter, each turning in 2 bearings $26\frac{1}{2}$ in. long, supported by longitudinal girders resting on the masonry. Each leaf weighs 1,370,000 lbs., which includes 700,000 lbs. of counterweight, composed of concrete with embedded pig iron. The pressure between the trunnions and bearings is about 500 lbs. per square inch of projected area, when in motion. The maximum pressure is, then, about $500 \times \frac{4}{\pi} = 637$ lbs. per square inch. The live load on the roadways and walks increases these pressures to about 855 and 1087 lbs. per square inch, respectively, when the pier rests are not in action. These are very conservative values.

The operating rack has a pitch radius of 30 ft. $1\frac{5}{8}$ in., and consists of $4\frac{1}{2}$-in. round pins, 12 in. long, inserted in the structural work, 9 in. from center to center. This forms an open rack, not likely to be damaged by accumulations of dirt or by snow and ice. Each leaf is operated by two 40-h.p. electric motors, which can hold the leaf against a 70 to 80-mile wind, when fully open. The reduction gearing contains worm-and-wheel units that serve to lock the bridge and prevent any undue acceleration when it is being closed. Band brakes are provided also.

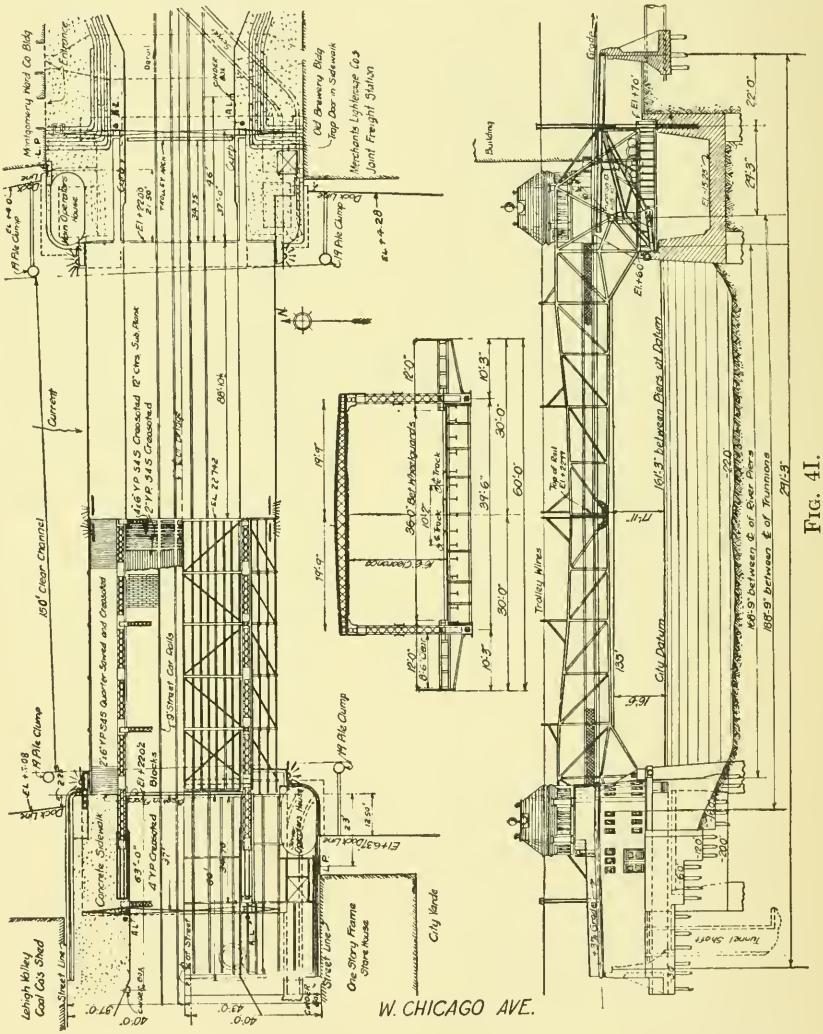
The superstructure was designed for 100 lbs. per square foot, extending over the roadways and sidewalks, while the floor system and details provide for the heaviest road rollers and street cars, the latter weighing 22 tons when empty and 35 tons when fully loaded.

The bridge was completed in December, 1903.

West Chicago Avenue Bridge.—Figure 4I was taken from a general drawing of the West Chicago Avenue bridge. It is typical of several of the Chicago Type bascules. The span is 188 ft. 9 in. between trunnion centers, and the clear channel is 150 ft. The bridge has two

[8] *Eng. News*, 1905, I, p. 65.

trusses instead of three, as in some of the earlier structures. The roadway is 36 ft. wide, the two sidewalks are each 8.5 ft. wide, and the over all width is 60 ft. The main features of rear anchorages, tail locks, shear locks, and a front pier rest are all continued. The floor-breaks


W. CHICAGO AVE.
150' Clear Channel
Current
Trolley Wires
City Datum
Tunnel Shaft
Concrete Sidewalk
Lehigh Valley Coal Co's Shed
One Story Frame Store Houses
City Yards
Old Brewery Bldg
Merchants Lighterage Co's Joint Freight Station
Montgomery Ward Co. Bldg
Main Operator's House
Counter Weight
Machinery Pier
Fender Pier
Fender Pier
Fender

improved by buildings, even closer to the channel than the ends of the bascules and approach spans. It is evident that a swing bridge would have been impossible in such a location.

This bridge was finished in 1914.

The West Lake Street Bridge.—The West Lake Street Bridge is a simple-lever, trunnion, double-leaf, double-deck bascule.[9] The clear channel is 209.25 ft. The span is 245 ft. between trunnion centers and 217 ft. between centers of front pier rests. The lower deck carries a 38-ft. roadway and two 16-ft. sidewalks. The Chicago and Oak Park Elevated Railroad has two tracks on the upper deck. This bridge differs from those previously mentioned in that the trunnion pedestals are supported by heavy transverse girders passing through the trusses, and the rack is on the concave side of the rear framing. Both of these features are the subjects of United States patents. The first was granted to Mr. Joseph B. Strauss and the second to Mr. A. von Babo.

The floor-break in the lower deck is in front of the trunnion, and that in the upper deck is behind it. The remaining features of the design are much the same as for the earlier bridges. Fig. 4J shows the general outlines and arrangement of the structure.

This bridge was finished March 4, 1916.

The Michigan Boulevard Bridge.[10]—Figure 4K, from a photograph, shows the general appearance of this bridge. The clear channel is 220 ft., and the span is 256 ft. between trunnion centers. The bridge is a double-deck structure, the upper deck carrying two boulevard roadways, each 27 ft. wide, and two sidewalks of 15 ft. each. The lower deck has two 18-ft. roadways, for heavy trucking, and two 6-ft. sidewalks. There are two outside and two inside trusses. One outer and one inner truss, on each side of the center, are connected by the floor systems and bracing. The two inner trusses are only 6 ft. between centers. The structure is really two parallel two-truss bridges. Diaphragms connect the two inner trusses, so that in normal operation the two parallel leaves are moved together. In case of emergency, however, they can be disconnected and operated as separate units. It is estimated that 20 men can disconnect the parallel units in about 8 hours.

The trunnion pedestals are supported as described for the West Lake Street Bridge. The rack is concave also. The Michigan Boulevard Bridge is the heaviest known bridge of this type. Each bascule lead, of two units, weighs about 6,700,000 lbs. The load on the outside trunnions is 1,800,000 lbs., and on the inside, 1,540,000 lbs. The outer

[9] *Eng. News*, 1915, II, pp. 876, 934, 1267; 1916, I, p. 756.

[10] *Eng. News-Rec.*, 1920, II, p. 508.

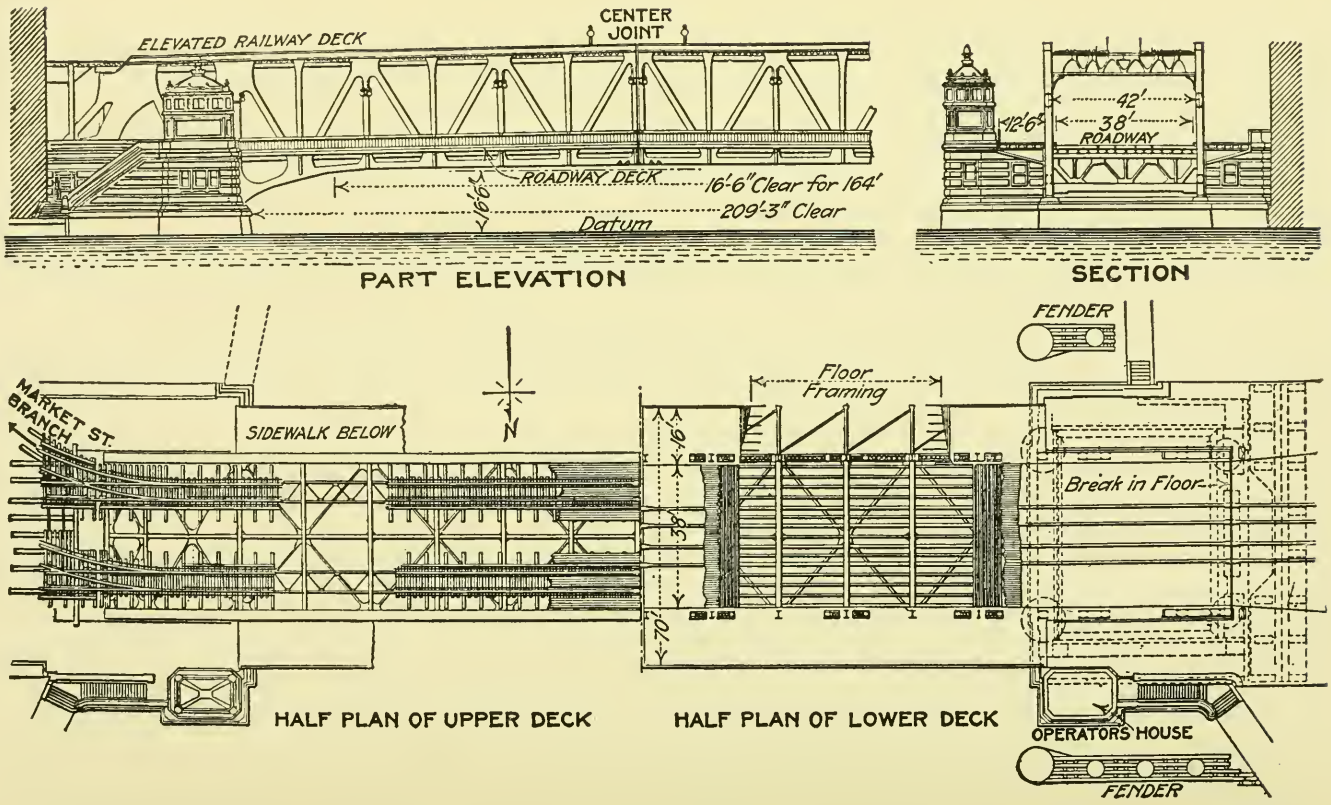

ELEVATED RAILWAY DECK
CENTER JOINT
ROADWAY DECK
16'-6" Clear for 164'
209'-3" Clear
Datum
PART ELEVATION
42'
38'
ROADWAY
12'-6"
SECTION
FENDER
Floor Framing
MARKET ST. BRANCH
SIDEWALK BELOW
N
16'
38'
70'
Break in Floor
HALF PLAN OF UPPER DECK
HALF PLAN OF LOWER DECK
OPERATORS HOUSE
FENDER
Eng. News.

FIG. 4J.

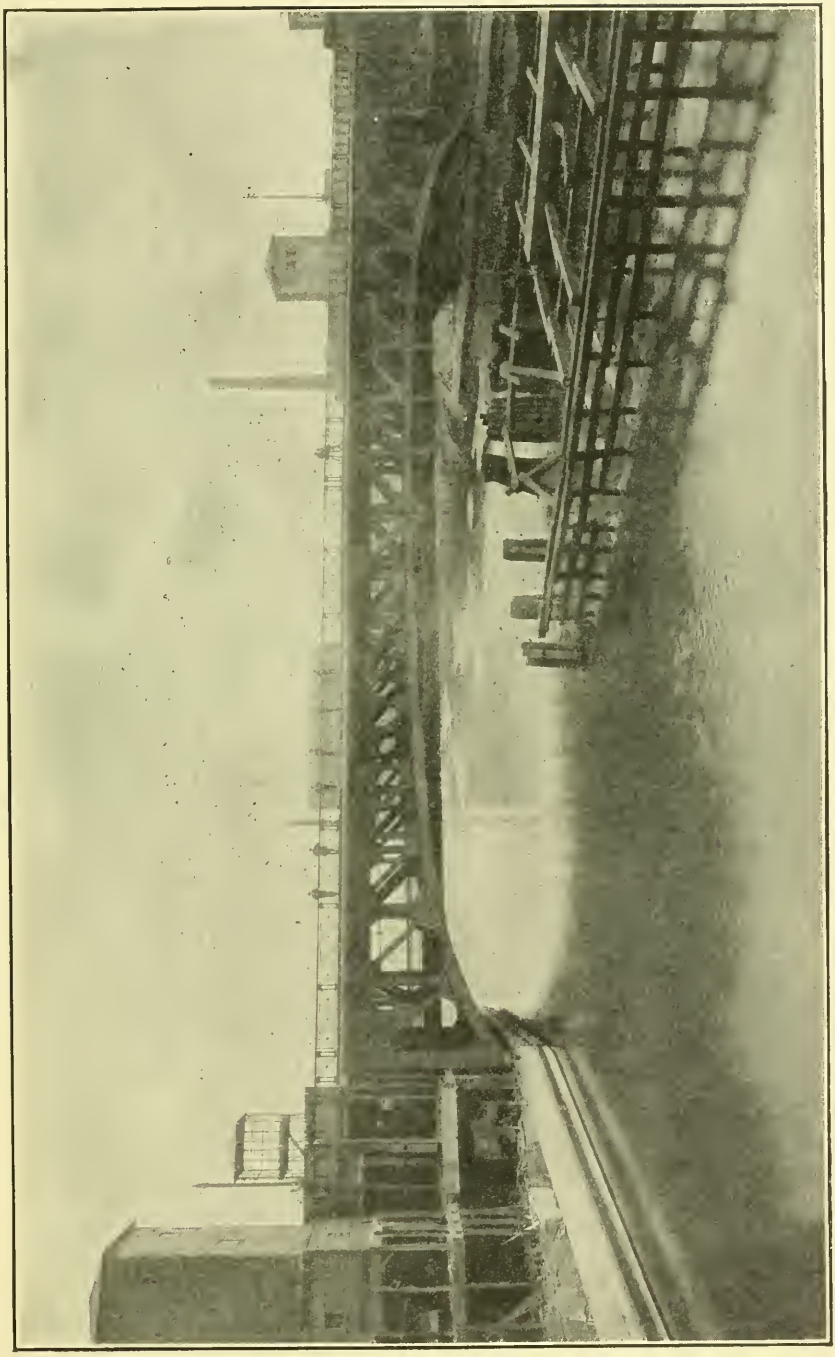


FIG. 41.

trunnions are 26 in. in diameter and the inner $24\frac{1}{2}$ in. They were designed for bearing pressures of 1800 lbs. per square inch, and extreme fiber stresses of 16,000 lbs. per square inch. Rear anchorages and tail locks, as well as center shear locks, were used. Special, rear safety-brakes were provided. These brakes are described in Volume II, Chapter III, Art. 9.

The bridge was designed for 100 lbs. per square foot on the entire upper deck. The load on the lower deck was taken as 2000 lbs. per foot on each car track, assumed to occupy 10 ft. in width; with 100 lbs. per square foot on the remainder. The floor systems of both decks, except for the car tracks, were proportioned for trucks weighing 24,000 lbs. on two axles, 10-ft. centers. The street cars were assumed to be 50 ft. long and to weigh 100,000 lbs. Impact was added to the live-load stresses according to the formula:

$$I = S\frac{100}{NL + 300},$$

in which S is the live-load stress, L the loaded length, and N is $\frac{1}{10}$ of the loaded width. Members having reversal of stress were proportioned after adding one-half of the smaller to the stresses of opposite signs. The maximum chord stresses just to the rear of the trunnions are 2,692,000 lbs. in the outer trusses. This gives some indication of the magnitude of the structure.

The counterweighted arms extend 42 ft. to the rear of the trunnions. The counterweights, for each leaf, consist of:

Concrete, weighing 145 lbs. per cubic foot	930 tons
Concrete and steel punchings, weighing 300 lbs. per cubic foot	665 tons
Total per leaf	1,595 tons
Total for the bridge	3,090 tons

The bridge was designed in the Bridge Designing Section of the Bureau of Engineering, Dept. of Public Works, of the City of Chicago. Mr. Thomas G. Pihlfeldt was the Engineer of Bridges, and Mr. Hugh E. Young was then in charge of Bridge Design.

The bridge was opened for traffic in May, 1920.

Three bridges in Seattle.—During the period from 1915 to 1919, the city of Seattle built three double-leaf bascule bridges which were of practically the same design as the later Chicago Type bascules. The floor-breaks are in front of the trunnions. Rear anchors, rear locks, shear locks, and pier rests are all embodied in the designs. The trunnion pedestals are supported on transverse girders through the trusses,

and the racks are concave. They were proportioned for the following loads:

Each car track, a 60-ton car and a 50-ton trailer, each 47.5 ft. long. The floors, 24-ton trucks, with wheel base 7 ft. wide and 16 ft. long, with 15 tons on the rear axle. Alternate, 100 lbs. per square foot. Impact on the stringers, 60 per cent; on the floor-beams, 50 per cent; on trusses,

$$I = S\frac{100}{1 + \frac{L^2}{20{,}000}} \times \frac{1}{2}.$$

The machinery was designed to operate the bridge against a wind pressure of 10 lbs. and to hold it against 15 lbs. per square ft. Each leaf has two electric motors of 100 h.p., for 550-volt direct current. Both solenoid and hand brakes are provided.

These bridges were built across the Puget Sound and Lake Washington Canal, at Freemont and Eastlake Avenues and 15th Avenue, N. W., Further data as to dimensions and weights are given in Table 4A.

Statistics.—The principal dimensions, facilities afforded, and the weights of the structural steelwork and operating machinery of fifteen Chicago Type bascule bridges are given in detail in Table 4A. In this series of bridges, the weight of the machinery is shown to vary from 9.2 to 17.9 per cent of the weight of the structural steelwork. For well-designed machinery, the weight may be taken as 15 per cent of that of the steelwork, for preliminary estimates.

It must be remembered that the machinery weights given do not include the motors or power plants, but only the machine parts that form a part of the bridge proper and the gear trains, beginning at the point where the power from motors, or engines, is delivered to the gear train.

8. Scherzer rolling-lift bridges.—This type is commonly known as a bascule, but the term "rolling-lift" is more correct and descriptive. Like several other types of bridges moving about a horizontal axis, it was used long ago in small structures of simple design. See Chapter I, Art. 4, Fig. 1F. In 1893, the late William Scherzer was retained by the Metropolitan West Side Elevated Railroad Company, of Chicago, to design a four-track draw bridge across the Chicago River, near Van Buren Street. He developed the first of the structures since known as the Scherzer Rolling-lift Bridge. The bridge was patented, and has been vigorously advocated and widely used in the United States and, in several cases, in other countries. The type is attractive on account of its simplicity and the small power required for operation.

TABLE 4A

CHICAGO TYPE, SIMPLE LEVER, TRUNNION, BASCULE BRIDGES

Location and Name	Number of Leaves	Clear Waterway, Feet	Bascule Span c. to c. of Trunnions, Feet	Counterweight Arms of Approaches, Feet	Roadway, Feet	Sidewalks, Feet	Weight of Structural Steel, Pounds	Weight of Machinery, Pounds	Mchy. to Structural, Per Cent	Total Weight, Pounds	Date
Chicago Bridges											
Division Street	2	.....	149	.....	2 @ 20	2 @ 9	.....	.....	.....	1,055,000	1901
95th Street	2	100	128	60	2 @ 20	2 @ 9	.....	.....	.....	1,163,000	1901
Clybourn Place	2	100	128	90	2 @ 18	2 @ 9	.....	.....	.....	1,830,000	1901
West Division Street	2	.....	172.67	.....	2 @ 20	2 @ 9	.....	.....	.....	1,498,000	1903
Northwestern Avenue	2	165.58	205.58	105 and 47	2 @ 18	2 @ 9	2,180,000	200,000	9.2	2,380,000	1903
Archer Avenue	1	.....	136c. to end	2 @ 36.67	2 @ 18	2 @ 7	1,027,000	129,000	12.6	1,156,000	1906
North Avenue	2	.....	172.67	87.33	2 @ 18	2 @ 9	1,309,000	235,000	17.9	1,544,000	1907
West Lake St. & El. R.R.	2 leaves 2 decks	195	245	2 cwt. arms @ 39	1 @ 38	2 @ 16	3,970,000	584,000	14.7	4,554,000	1916
Belmont Avenue	2	.....	188.75	2 cwt. arms @ 29.875	1 @ 36	2 @ 12	2,057,000	189,000	9.2	2,246,000	1916
Franklin-Orleans	2	210	251.83	North 58.17 South 47.33	1 @ 38	2 @ 8.33	3,739,000	386,000	10.3	4,125,000	1920
Michigan Boulevard	2 leaves 2 decks	220	256	2 cwt. arms @ 42	Upper 2 @ 27 Lower 2 @ 18	Upper 2 @ 15 Lower 2 @ 6	6,122,000	950,000	15.5	7,072,000	1920
Wells Street & El. R.R.	2 leaves 2 decks	220	268	2 cwt. arms @ 42	1 @ 38	2 @ 13.5	5,170,000	598,000	11.6	5,768,000	1921
West Madison Street	2	188.75	221.23	East 35.59 West 22.14	1 @ 38	2 @ 14	2,676,000	391,000	14.6	3,067,000	1922
Seattle Bridges											
Freemont Avenue	2	200	242	2 cwt. arms @ 35	1 @ 40	2 @ 7.5	.....	.....	.....	2,631,000	1916
15th Avenue, N. W.	2	180 (about)	218	2 cwt. arms @ 35	1 @ 40.25	2	2,208,000	236,000	11.7	2,444,000	1917
Eastlake Avenue	2	180 (about)	218	2 cwt. arms @ 35	1 @ 40.25	2 @ 6	1,993,000	229,000	11.5	2,222,000	1919

The leaf extending over the channel is continued shoreward, in the form of a circular quadrant, which carries the counterweight; and, in operation, the entire structure moves shoreward on the quadrant, which rolls on a horizontal track, usually in the form of a heavy girder. The track is provided with teeth meshing in openings in the bottom of the quadrant girder, to insure certainty of operation and accuracy in registration when closing. When opened, the bridge recedes from the channel, the span is shortened, and the necessary angular movement is reduced. Rolling friction is substituted for that of trunnions turning in journal bearings, with consequent reduction in resistance to motion. This is the only type of draw bridge, moving about a horizontal axis, that approaches a swing bridge in the small power required for operation. When the leaf is open, it exposes a large vertical area to wind pressure, as in a bascule, and power must be added to provide for opening and holding it against wind pressure.

When the bridge is opening, the entire mass moves horizontally along a path of the length,

$$\frac{2\pi r\alpha}{360},$$

in which r is the radius of the quadrant on which it rolls, and α is the angle of opening of the leaf, in degrees. This results in a shifting of the maximum load on the foundations, which is undesirable in case of heavy bridges and compressible foundations. On account of the concentration of large loads on the line contacts between the rolling quadrant and the horizontal track, much care and skill must be exercised in the design and construction of these parts. This matter will receive further consideration in Appendix B. When the details are adequate and the foundations good, this type is suitable for many locations.

The principal features of a single-leaf bridge are shown in Fig. 4L. This is a double-track railway bridge on the Newburg and South Shore Railway, across the Cuyahoga River, in Cleveland, Ohio. The span is 160 ft. between end bearings and provides a clear channel of 120 ft., normal to its direction, which is at an angle of 60° 30′ 30″ with the bridge. The superstructure was proportioned for Cooper's Specifications for Class E50 loads. The counterweights are of cast-iron blocks. In later designs, concrete has been used in most cases.

The operating machinery has been placed on the movable span. The main pinions are at the center of the quadrant, on which the bridge rolls, and a straight rack, which engages the pinion, is attached to a rigid frame on each side of the bridge, and outside of the outer clearance of the trusses. The mechanism and rack thus become simple,

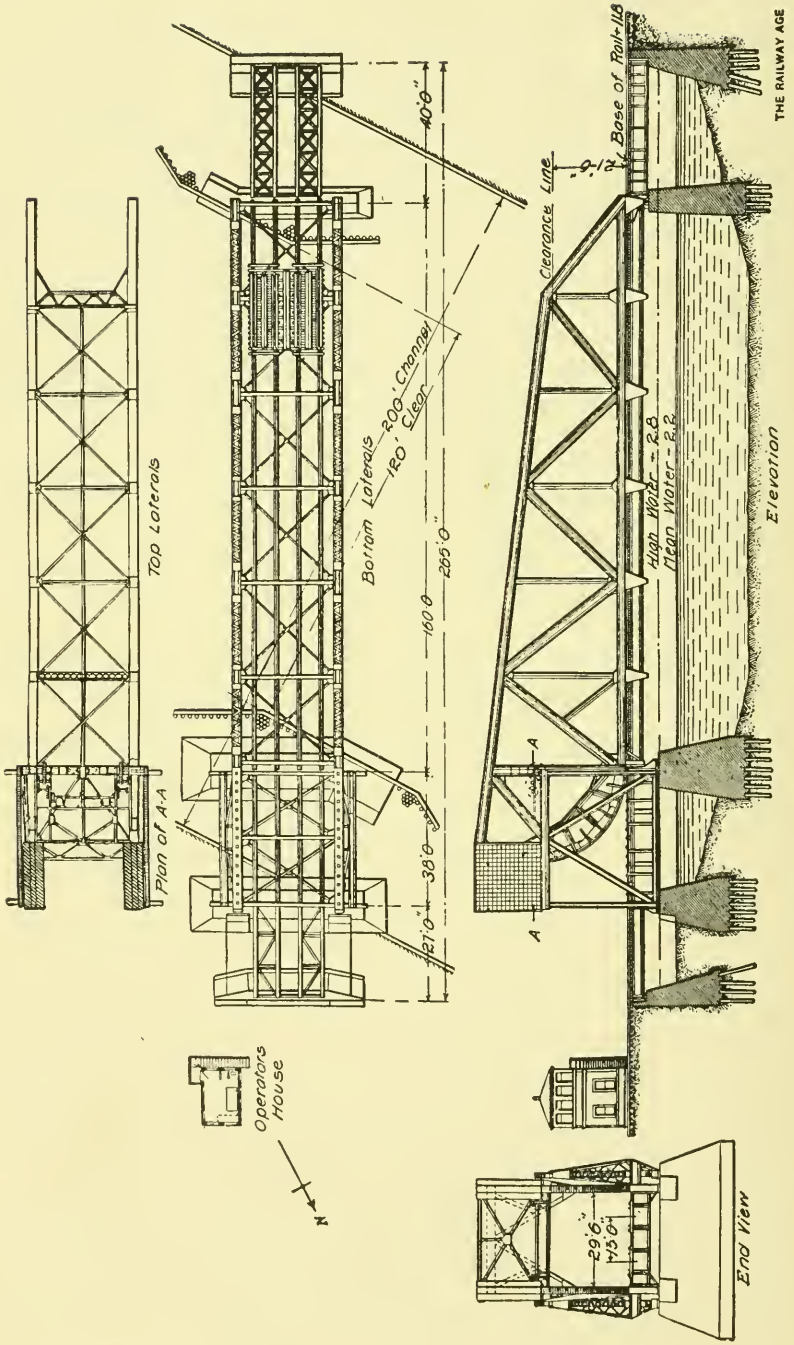

THE RAILWAY AGE
Top Laterals
Plan of A-A
Bottom Laterals
200' Channel
120' Clear
40'0"
160'0"
265'0"
38'0"
127'0"
Clearance Line
9'-12"
Base of Rail-118
High Water - 2.8
Mean Water - 2.2
Elevation
A
A
Operators House
N
29'6"
25'0"
End View

FIG. 4L.

and the action positive and certain. As it is difficult to fabricate and erect the two rack frames exactly alike and truly opposite, an equalizer should always be introduced in the gear train, preferably in the first counter shaft from the motor.

This bridge was built in 1904.

Figure 4M shows the general elevation of a small, double-track, through plate girder bridge, on the Central Railroad of New Jersey, across the Elizabeth River, at Elizabeth, N. J. The span is 73 ft. between end bearings and the clear channel is 60 ft. The method of operation is the same as for the larger bridge shown in Fig. 4L. The counterweight is of concrete and extends across the entire width of the structure.

This bridge was built in 1912.

Figure 4N represents a typical, small, double-leaf highway bridge. Center shear locks and rear anchorages and locks are required, to enable the leaves to act as cantilevers for live loads. The breaks in the floor of such a deck structure must be well to the rear of the center of the main pinions, on account of the recession of the leaf in opening. The same movement makes a simple center lock feasible. This lock is a tongue-and-groove device on the ends of the main girders of the leaves, as shown in Fig. 4O. No mechanical parts are needed, for the tongue can be entered by controlling the relative positions of the closing leaves, as indicated at (*a*). As the leaves advance, in closing, the tongues are driven into positive locked position, as shown at (*c*). The rear end of the counterweight frame engages overhanging anchorage girders, carried by columns extending into the masonry to secure ample resistance to the uplift caused by cantilever action of the channel leaves. When live load is also carried by the extensions of the main girders beyond the counterweights, it will also be found that tail locks are needed to take the reactions from the loads on the girder extensions.

Statistics.—The principal dimensions, facilities afforded, and the weights of the structural steelwork and operating machinery of several Scherzer Rolling-lift bridges are given in Table 4B. It will be noted that the percentage of machinery to structural parts is less than in Table 4A, for the simple-lever trunnion type. This is due to the simplicity of the designs and the small rolling friction developed. The proportion varies widely for the different designs and is larger for those with two leaves. The weight of the motors and power plant proper is not included, but only the gear trains from the motors to the point of application to the bridge, including the racks.

9. Rall bascule bridges.—This type was developed and patented by Mr. Theodore Rall, and is controlled by the Strobel Steel Construc-

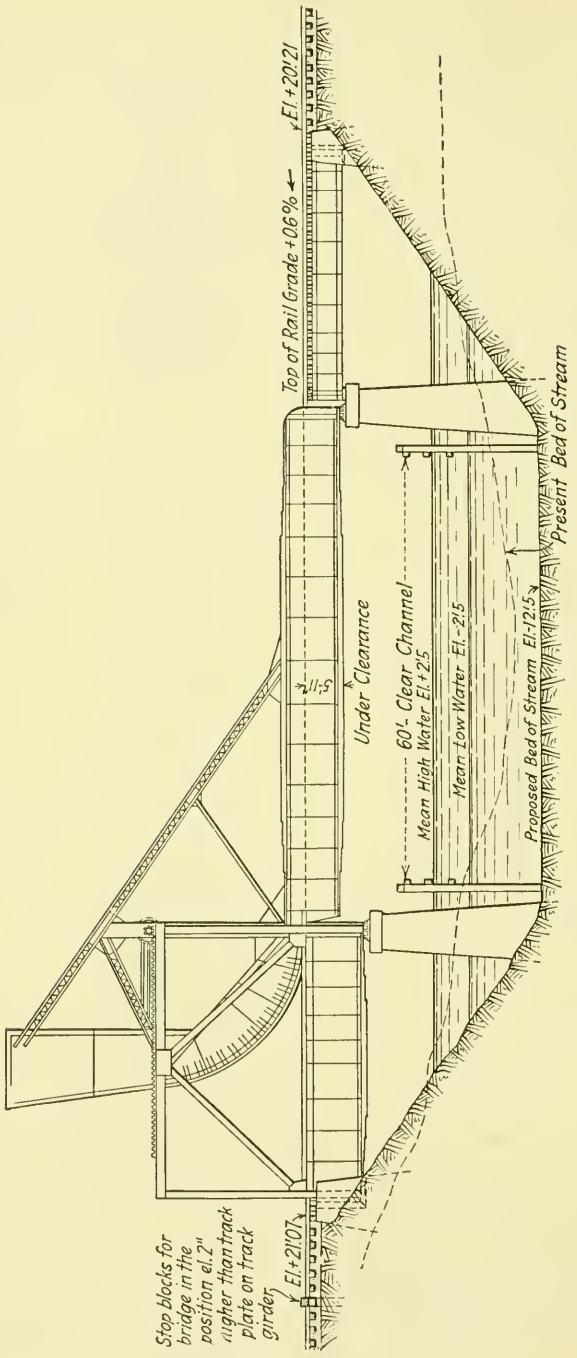

El.+20'2"
Top of Rail Grade +0.6% ←
Under Clearance
60'-0" Clear Channel
Mean High Water El.+2'5
Mean Low Water El.-2'5
Proposed Bed of Stream El.-12'5
Present Bed of Stream
Stop blocks for bridge in the position el.2" higher than track plate on track girder
El.+21'0"

FIG. 4M.

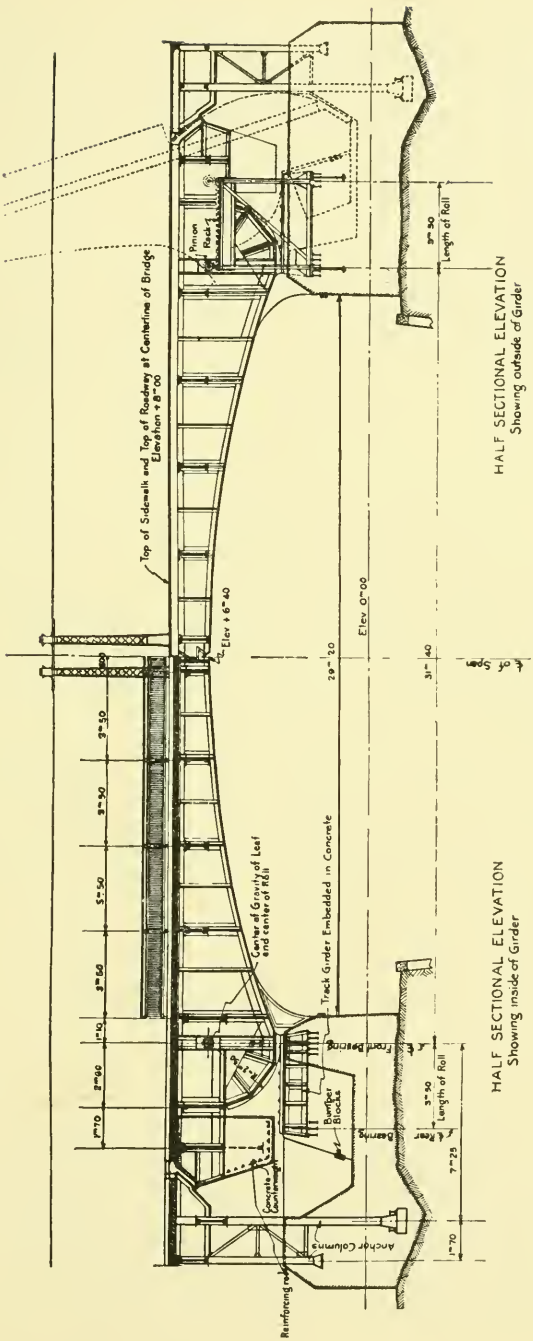

Top of Sidewalk and Top of Roadway at Centerline of Bridge
Elevation + 8'-00
Pinion
Rack
Elev + 6'-40
Elev 0'-00
29'-20
31'-40
3'-30
Length of Roll
HALF SECTIONAL ELEVATION
Showing outside of Girder
Center of Gravity of Leaf
and center of Roll
Track Girder Embedded in Concrete
Floor Beams
Bumper
Blocks
Concrete
Counterweight
Anchor Columns
Reinforcing rods
HALF SECTIONAL ELEVATION
Showing inside of Girder

Fig. 4N.

tion Co., of Chicago. The bascule leaf opens by a combined rolling and swinging movement that can best be explained by reference to Fig. 41P, which is a diagram of one end of a double-leaf, electric railway bridge over the Illinois River, at Peoria, Illinois.[11] The span is 142 ft. between centers of end supports. The leaf is carried at its center of gravity

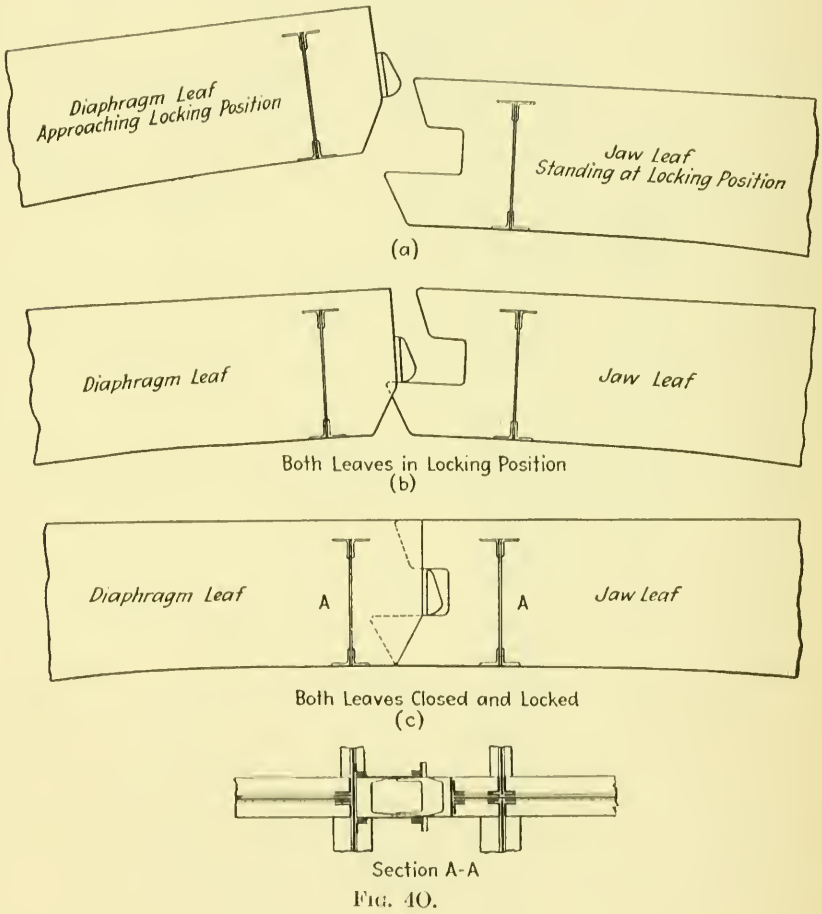


FIG. 40.

by a shaft A, having at each end a roller B, which rolls on a horizontal track girder C. The counterweighted end of the main girder is connected to the pier by the swinging-strut EF, which turns on pins. When the bridge is closed, pedestal G, on the main girder, seats on pin F, on the pier, and roller B is slightly lifted from its track C. Operating

[11] *Eng. News*, 1907, II, p. 56.

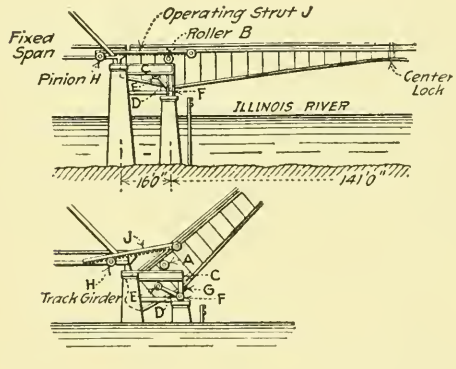
TABLE 4B

Scherzer Rolling-lift Bridges

Name or Location	Design	Number of Leaves	Clear Channel-Feet	Span c. to c., Feet	Roller Track Span, Feet	Tracks, Number or Roadway, Feet	Sidewalks, Number, Width	Weight of Structural Steel, Pounds	Weight of Machinery, Pounds	Machinery to Structural, Per Cent	Total Weight, Pounds	Date Built
Railway Bridges												
Newburg & South Shore Ry., Cuyahoga River, Cleveland, Ohio	Thro. Riv.	1	120	160 skew	38	2	.....	1,327,000	106,000	7.99	1,433,000	1904
New York, New Haven & Hartford R.R., Cos Cob, Conn.	D. P. G.	2 parallel	67.5	81	20.17	2 @ 2	.....	1,741,000	99,000	5.69	1,840,000	1906
New York, New Haven & Hartford R.R., Naugatuck Jct., Conn.	Thro. Riv.	2 parallel	81	108	34.67	2 @ 2	.....	1,839,000	89,000	4.84	1,928,000	1907
New York, New Haven & Hartford R.R., Connecticut River, Lyme, Conn.	Thro. Riv.	1	149.25	158.25	38	2	.....	2,060,000	91,000	4.42	2,151,000	1907
New York, New Haven & Hartford R.R., Pelham Bay, N. Y.	Thro. Riv.	3 parallel	66.75	81.58	29.5	3 @ 2	.....	1,993,000	61,000	4.57	2,054,000	1908
New York, New Haven & Hartford R.R., Bronx River, N. Y.	Thro. Riv.	3 parallel	66.75	103.25 skew	37.17	3 @ 2	.....	3,222,000	108,000	3.55	3,330,000	1908
Chicago Lake Shore & Eastern R.R., East Chicago Canal, Indiana Harbor, Ind.	Thro. Riv.	1	86.5	96	.....	2	.....	637,000	41,500	6.52	678,500	1908
Buffalo Creek R.R. Co., City Ship Canal, Buffalo, N. Y.	Thro. Riv.	1	.....	151	28.5	2	.....	1,350,000	42,000	3.12	1,392,000	1909
New York Central & Hudson River R.R., Peekskill, N. Y.	T. P. G.	1	50	67	16	2	.....	488,500	16,000	3.28	504,500	1913
River Terminal Ry., Cuyahoga River, Cleveland, Ohio	Thro. Riv.	1	.....	148	47.25	1	.....	622,500	20,500	3.29	643,000	1914
Maine Central R.R., Wiscasset, Me.	T. P. G.	1	.....	83	48.5	1	.....	332,500	16,500	4.96	349,000	1916
Maine Central R.R., Thomaston, Me.	D. P. G.	1	.....	30.04	.....	1	.....	76,000	6,500	8.55	82,500	1921
Maine Central R.R., Georges River	T. P. G.	1	39	44.92	30	1	.....	120,000	7,500	6.25	127,500	1921
Highway Bridges												
Newtown Creek, Brooklyn, N. Y.	Thro. Riv.	2	150	172 skew	2 @ 80	40	2 @ 8	2,210,000	200,500	9.07	2,410,500	1902
Malden River, Boston, Mass.	D. P. G.	2	50	56	.....	28	1 @ 8	226,000	35,000	15.49	261,000	1905
Saugus River, Boston, Mass.	D. P. G.	2	50	56	.....	26	1 @ 6	210,000	35,000	16.67	245,000	1905
Kimberley Ave., New Haven, Conn.	D. P. G.	2	.....	51	23	44	2 @ 10.58	343,000	51,000	14.87	394,000	1906
Buffalo Creek R.R., Union Canal, Buffalo, N. Y.	Thro. Riv.	1	.....	90	.....	27	2 @ 6	428,000	41,500	9.70	469,500	1906
Charles River Basin Lock, Boston, Mass.	D. P. G.	2 parallel	.....	52	21	2 @ 31.17	2 @ 9.83	457,000	45,000	9.85	502,000	1907
Pelham Bay, New York, N. Y.	Deck Riv.	2	60	80	.....	34	2 @ 7.5	401,000	32,000	7.98	433,000	1908
Taunton Great River, Fall River, Mass.	D. P. G.	2	.....	113.5	.....	44	2 @ 8	878,500	56,500	6.43	935,000	1908
Mill Creek, Bayville, L. I.	D. P. G.	2	79	85	8.5	21	1 @ 5	139,000	30,000	21.58	169,000	1922

strut J is hinged to the main girder and has a bottom rack, engaging pinion H, on the fixed span or the abutment. Rotation of the pinion draws the rack rearward, first causing the roller to contact with its track. The span then turns about the trunnion in the roller and the anchor pin F, causing a combined rotary and retractile movement of the leaf. Point E rotates in the arc of a circle about the center F, and other parts of the span move in various curves.

As in case of a simple rolling-lift bridge, the span length may be a minimum and the angular motion is less than for a simple trunnion bascule. The weight of the entire leaf is carried by the rollers and travels backward across girder C, when opening. This shifts the load upon the piers and foundations in a manner similer to that of a simple rolling-lift span. The weight must also be taken by line contacts between the rollers and their tracks, so that these parts require the best of design, material, and workmanship. The friction of motion is greater than that of a rolling-lift, for the trunnions must turn in the rollers under maximum load and the hinged links will also offer some resistance.



Eng. News.

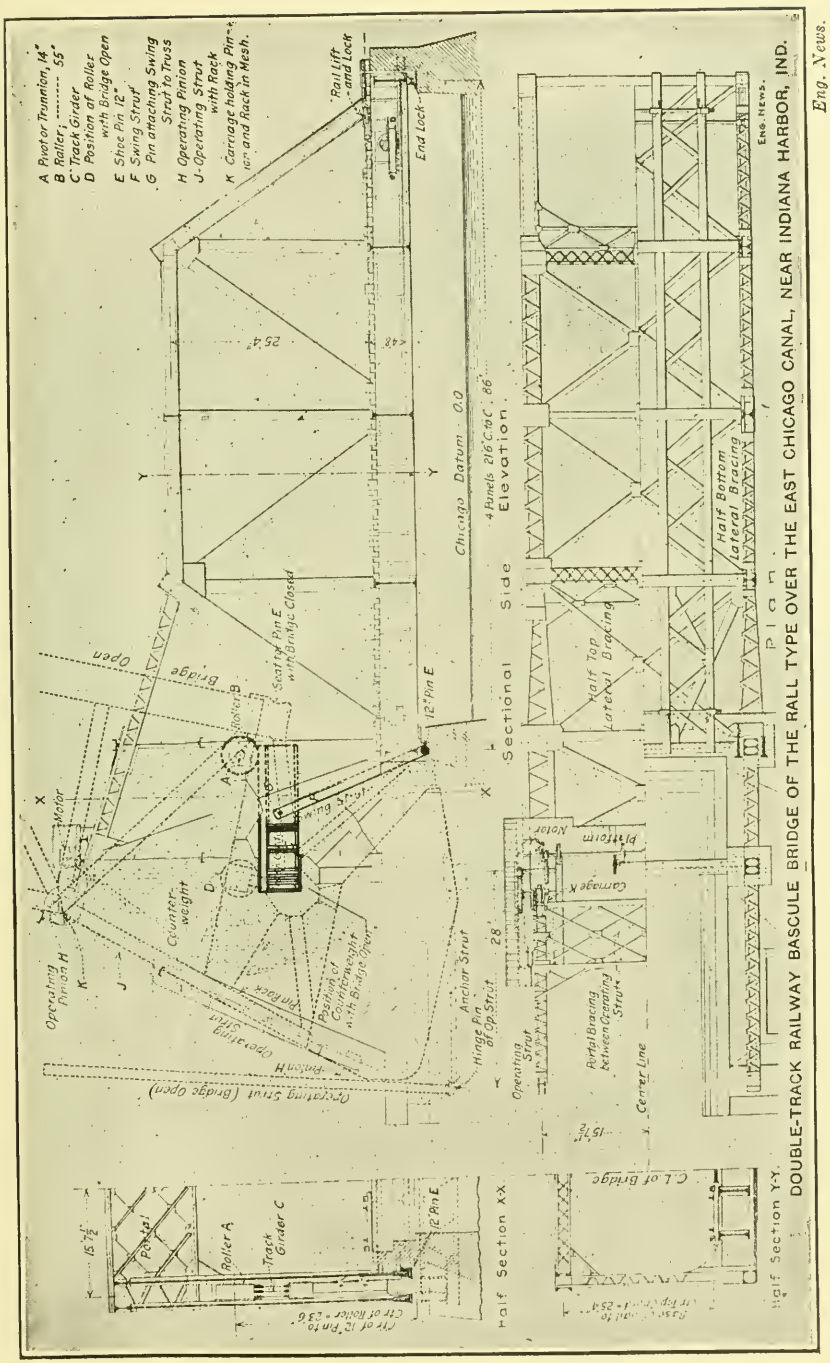
Fig. 4P.

One excellent feature of the design is that the rollers can be removed, for repairs or replacement, when the bridge is in its closed position, for they are then relieved of all load. Further, the rollers are of such size that they can be cast solid, and take load directly, while the rolling-lift quadrants are usually so large that they must be built up from structural plates and shapes, with a joint between the girders and the curved tracks attached to their circumference.

This bridge was finished in April, 1907

East Chicago Canal Bridges.—Figure 4Q shows the general design of four double-track, railway, Rall bascule bridges over the East Chicago Canal, near Indiana Harbor, Ind.[12] Two of these were built for the Lake Shore and Michigan Southern Railway, and one each for the Pittsburgh, Fort Wayne and Chicago Railway and the Baltimore and Ohio Railroad. The span length is 86 ft. between centers of end bearings. When the bridge is closed, all live load is delivered

[12] *Eng. News*, 1909, I, p. 295.



Eng. News.

FIG. 40.

directly to the piers. Ordinary pier pedestals are used at the free end of the span. At the trunnion end a pedestal is attached to the bottom chord, under the end panel point, which seats on pin E, in a pedestal built and anchored into the masonry. The small deck bridge, in Fig. 4P, had longitudinal operating racks. In the through design in Fig. 4Q, the racks are articulated to the abutment masonry and extend in a diagonally upward direction, in the plane of the trusses, to the top of the counterweight frames, where the main pinions engage the racks. The motor and reduction gearing are mounted on the rear end of the counterweight frames. The main pinion is held in mesh with the rack by a frame, with rollers which travel along the operating strut. When the bridge is open, the strut is nearly vertical. When the span is closed, the free end is secured by a thrust lock, interlocked with the signals. Rail locks are provided at each end of the leaf.

Each bridge is operated by two 35-h.p., 220-volt, direct-current electric motors. The actual power required to move the bridge, under ordinary wind conditions, is about 25 h.p. Thus, one of the motors may be held in reserve. The rail locks at each end of the bridge and the end locks are operated by two $\frac{1}{2}$-h.p., direct-current motors. Each main motor has a solenoid brake and there is an hydraulic emergency brake on the driving shaft, actuated by a hand pump in the operator's house. The several motors are electrically interlocked with each other and with the signals, so that all operations must be performed in proper sequence.

These bridges were proportioned in accordance with the specifications of the Pennsylvania Lines, for a live load of about E60 engines, followed by 5000 lbs. per linear foot of track. The weight of structural material and machinery is about 326 tons for each bridge and the total rolling load is 555 tons. The trunnions within the rollers were proportioned for 1600 lbs. per square inch of bearing, and the rollers for $400 \times D \times L$, at the surface of contact with the track. The rollers are 55 in. in diameter, with 25-in. faces, and are of cast iron, with rims and hubs 3 in. thick and with 2-in. webs. Steel tires, $1\frac{1}{2}$ in. thick, were shrunk around the cast-iron cores. Later practice has been to use cast steel for the rollers, and in some cases special alloy steel has been provided.

Broadway Bridge, Portland, Oregon.[13]—This is the largest Rall bascule yet built. It is a double-leaf, city-street structure, 278 ft. between end bearings and 70 ft. wide. There is one 46.5-ft. roadway and two 9-ft. sidewalks. The clear distance between the tops of the open leaves is 250 ft. The draw is the middle of three river spans,

[13] *Eng. News*, 1913, II, p. 704.

each of the two flanking fixed spans being 297.213 ft. long. The roller-track girders, their supports, the swinging-struts, and the machinery are supported in the first long panel of the adjoining fixed spans. The closed leaves act as cantilevers. The horizontal end bottom reaction is taken by the bottom chord of the fixed span. The tension at the top is delivered to the top chord by a long link, which is slotted and moves on a pin at the upper hip panel point. The counterweight frames are inside of the bascule trusses and the counterweights move between the end posts of the anchor spans.

Each leaf has two operating struts, each placed 12 ft. from the longitudinal center of the span. They are attached by pins to the moving leaf and extend back over the end of the fixed span. Racks on their bottom faces mesh with the main driving pinions. The operating struts are 91 ft. long and weigh 21 tons each. They are stiffened by top vertical diaphragm-girders, and are held in mesh by a frame at the pinion, with guide rollers running on the flanges of the struts. The slotted tension anchors are 52 ft. long with a travel of 35 ft. They are anchored to the bascule leaves by 13-in. pins and move on $10\frac{1}{2}$-in. pins in extensions of the top chords of the anchor spans. Adjustment is provided, so that when the leaves are closed the slots come to exact bearing on the pins and all of the live load is taken. Hydraulic buffers, within the anchor slots, assist in bringing the closing bridge to bearing without shock. The swing-struts are articulated to the counterweight frame at one end and to the track girder at the other. They are 23 ft. long and swing through an arc of 105°.

The rollers on which the leaves move are the most unusual features in the bridge. Figure 4R shows one of them, with its track and the adjacent steelwork. They are of nickel-chrome steel, 100 in. in diameter with 40-in. faces. The track is of the same material and is 20 in. deep to distribute properly the great concentrated load to the webs of the the supporting box girders. The nickel-chrome steel was specified to have an elastic limit of 52,000 lbs. per square inch, an ultimate tensile strength of 102,000 lbs. per square inch, an elongation of not less than 15 per cent, and a reduction of area at the fracture of not less than 17 per cent.

The load on each roller is about 2,000,000 lbs. The load per linear inch of roller is 50,000 lbs. Thus the load may be expressed by the formula,

$$P = 500 \times D \times L,$$

in which D is the diameter of the roller and L its length, both in inches.

The bridge was designed for a load of 2000 lbs. per linear foot on each

street-car track, and for 100 lbs. per square foot on the balance of the roadways and the sidewalks, making a total load of 7600 lbs. per linear

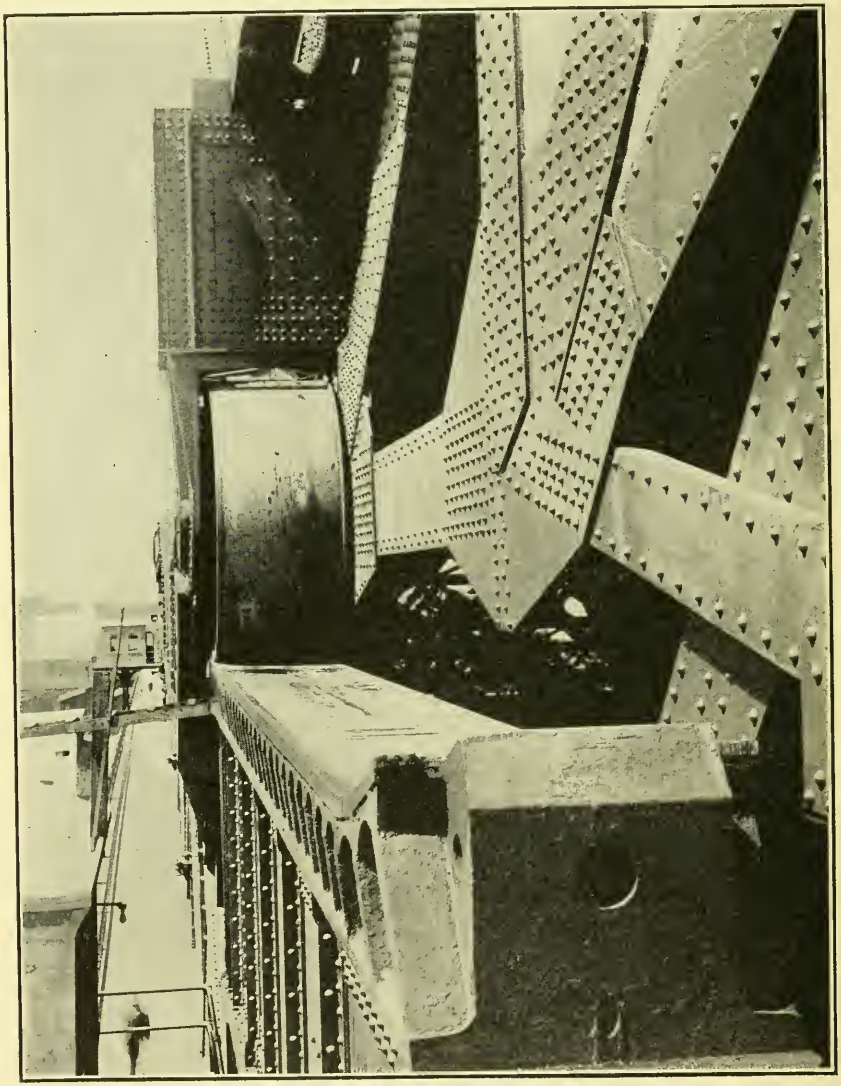


Fig. 4R.

foot of bridge. A special feature of this bridge is that the specifications required the trusses of the bascule span to be in line with the trusses of the approach spans. This necessitated special recesses in the coun-

terweight boxes and their supports to enable them to pass between the trusses of the approach spans, and rendered the design of the bracing rather difficult.

The bascule span was designed by the Strobel Steel Construction Co., of Chicago, subject to the approval of Mr. Ralph Modjeski, of Chicago, who was chief engineer in charge of the design and construction of the entire bridge, for the city of Portland, Oregon.

The bridge was opened for traffic in April, 1913.

The estimated weight of the steelwork is given below.

APPROXIMATE ESTIMATED WEIGHT OF STEELWORK

One Bascule Span, 250 Ft. Clear Opening

Structural Steel	Pounds	Pounds	Per Cent of Steelwork
Movable part:			
Floor system	590,000		
Overhead bracing	160,000		
Main trusses, river arms	700,000		
Main trusses, tail arms	460,000		
Cross girders over wheels	120,000		
Counterweight boxes	160,000		
Live load anchor struts, Structural part of operating struts, Controlling struts	190,000		
		2,380,000	
Stationary part:			
Track girders, Supporting columns, Bracing		400,000	
Stairs, machinery supports, floor, bolts, spikes, and other miscellaneous parts		70,000	
		2,850,000	
Mechanical parts:			
Cast steel wheels	110,000		
Cast steel tracks	106,000		
Forged hollow trunnions	20,000		
Forged solid trunnions	10,000		
Bronze bushings	4,000		
		250,000	8.77
Operating machinery		150,000	5.26
Total steelwork		3,250,000	14.03

10. Strauss bascule bridges.—The first Strauss bascule bridge was completed in 1905. It was patented by Mr. Joseph B. Strauss, and this and his later patents are controlled and advocated by the Strauss

Bascule Bridge Co., of Chicago. There have been more bascule bridges built from the various Strauss designs than from those of any other single type of bascule.

The first bridge was built for the Wheeling and Lake Erie Railroad over the Cuyahoga River, in Cleveland. It is a single-track, single-leaf, through, riveted structure. The bascule span is of the overhead counterweight type and is 150 ft. long. The counterweight is carried by a pair of struts articulated to rearward extensions of the main trusses. The tower that supports the main trunnions, on which the span revolves, is extended up to the top of the counterweight. A link is pinned to the top of the tower and to the counterweight, and lies in a plane parallel to the plane through the main trunnion and the pin in the end of the main truss which supports the counterweight struts. These four joints are at the corners of a parallelogram. When the leaf opens, the counterweight is held parallel to its original position and a balance is maintained as the bridge revolves. This parallelogram, in some form, is used in practically all of the Strauss bascule designs.

This series comprises designs of three general types:
(1) The vertical overhead counterweight type.
(2) The underneath counterweight type.
(3) The heel trunnion type.

(1) *Vertical overhead counterweight type.*—Figure 4S is a diagram of this type. The center of gravity of the moving leaf is at g and that of the counterweight at g'. Trunnion A and joint B are in the same plane as g, and link CD is parallel to gAB. $ABDC$ is a parallelogram. Hence, in any position, closed or in motion, $Px = Wy$, or $Px' = Wy'$; and a balance is maintained throughout the movement of the leaf. In this type the dead load on the trunnion is $P + W$, and the reaction is always positive and constant. The introduction of the parallelogram permits the center of gravity of the counterweight to be at any convenient height above B, while in a simple trunnion bridge it must be at B, in the plane of gAB. The break in the floor is in front of the trunnion.

Chambly Canal bridge.—Figure 4T shows the Chambly Canal bridge at St. Johns, Quebec. The clear channel is 70 ft. and the moving leaf is 85.75 ft. between bearings. It is a typical small highway bridge of this design. The roadway is 17 ft. wide and there are two sidewalks 4 ft. wide. The bridge was designed for a 12-ton road roller and 100 lbs. per square foot, and in accordance with the specifications of the Dominion Government. It is operated by two electric motors of 7.5 h.p. each. A quadrant rack is attached to the heel of each truss, under the roadway, and meshes with the main pinion, driven by the motor

through a train of reduction gearing. When closed, the end of the leaf is locked down. As the floor breaks in front of the trunnion, rear locks

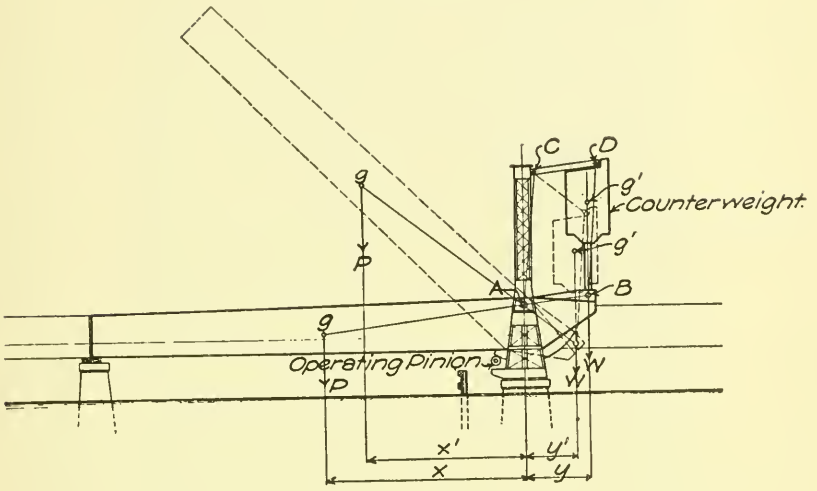


Fig. 4S.

are not needed. The live-load reaction at the trunnion pin is taken by the main trunnion. The estimated quantities follow:

Trusses, floor system, and laterals	80,900 lbs.	
Tower and machinery supports	28,200 lbs.	
Counterweight frame, links, and anchorages	10,700 lbs.	
Total structural	119,800 lbs.	
Trunnions, pins, and bearings	4,600	3.84 per cent of structural
Operating and locking machinery	12,200	10.18
Total mechanism	16,800 lbs.	14.02 per cent of structural
Counterweight, concrete at 148 lbs. per cu. ft		80 cu. yds.
Reinforcing steel		3,000 lbs.

This bridge was completed in 1916.

(2) *Underneath counterweight type.*—Figure 4U is a diagram of this type. The method of obtaining a balance is the same in principle as that shown in Fig. 4S. The parallelogram is *ABDC*. The counterweight may be extended above the attachment *B* and cored out to clear the floor framing, when a compact arrangement is necessary. When two leaves are used, front shear locks and rear anchorages

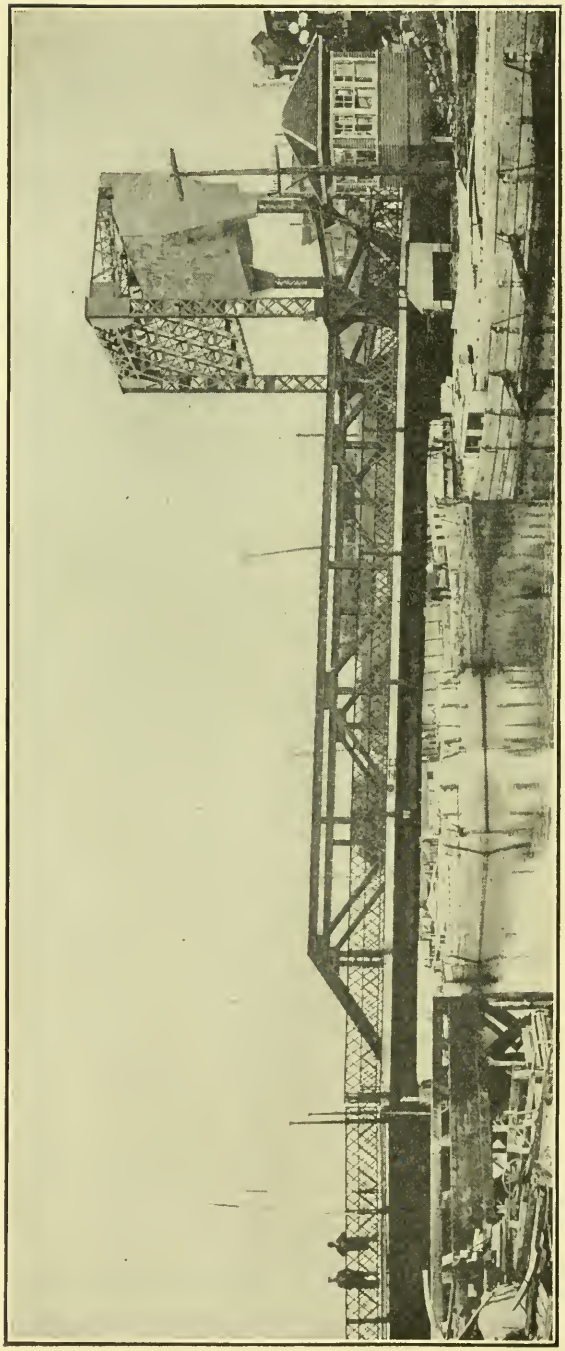


Fig. 4T.

must be provided. The dead load on the trunnions is $P + W$, and is constant, and the reaction is positive. The break in the floor is in front of the trunnions.

Genesee River bridge.—Figure 4V shows the Genesee River bridge at Charlotte, N. Y. It is a double-leaf deck structure, 180 ft. between trunnions. It carries a roadway 25 ft. wide, with two car tracks, and two sidewalks 6 ft. wide. It was designed for two 40-ton cars and one 15-ton truck, under Mr. J. E. Greiner's specifications. The operating machinery is driven by four electric motors of 35 h.p. each, and the shear lock by one motor of 5 h.p. The main pinions, at the end of the gear train, mesh with quadrant racks on the bottom of the rear extensions of the main trusses. The counterweights are pivoted to the rear

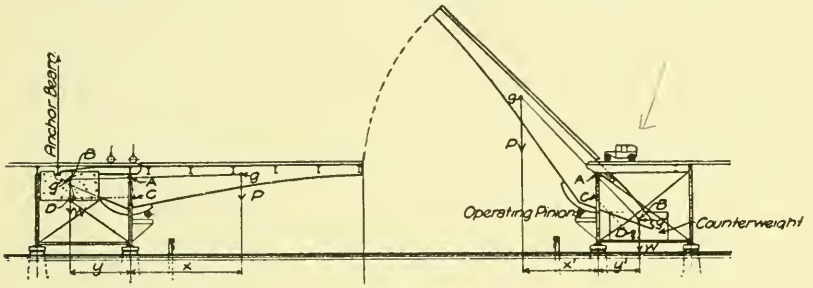


Fig. 4U.

ends of the trusses, as indicated by Fig. 4V. The trunnion posts are vertically beneath the trunnions and allow clearance for the moving leaf between them. They are braced by effective outriggers, as shown by the cut. The estimated quantities follow:

Trusses, floor system, laterals, and miscellaneous	435,000 lbs.	
Towers and machinery supports	100,000	
Counterweight frame, links, and anchors	26,200	
Total structural	561,200 lbs.	
Trunnions, pins, and bearings	17,000	3.03 per cent of structural
Operating and locking machinery	58,000	10.34
Total mechanism	75,000 lbs.	13.37 per cent of structural
Ladders and railings	3,000 lbs.	
Counterweights, concrete at 148 lbs. per cu. ft.	252 cu. yds.	

When this bridge is closed it presents a very good architectural appearance. In locations where the roadway can be high above the water and there is adequate height so that the counterweights can swing

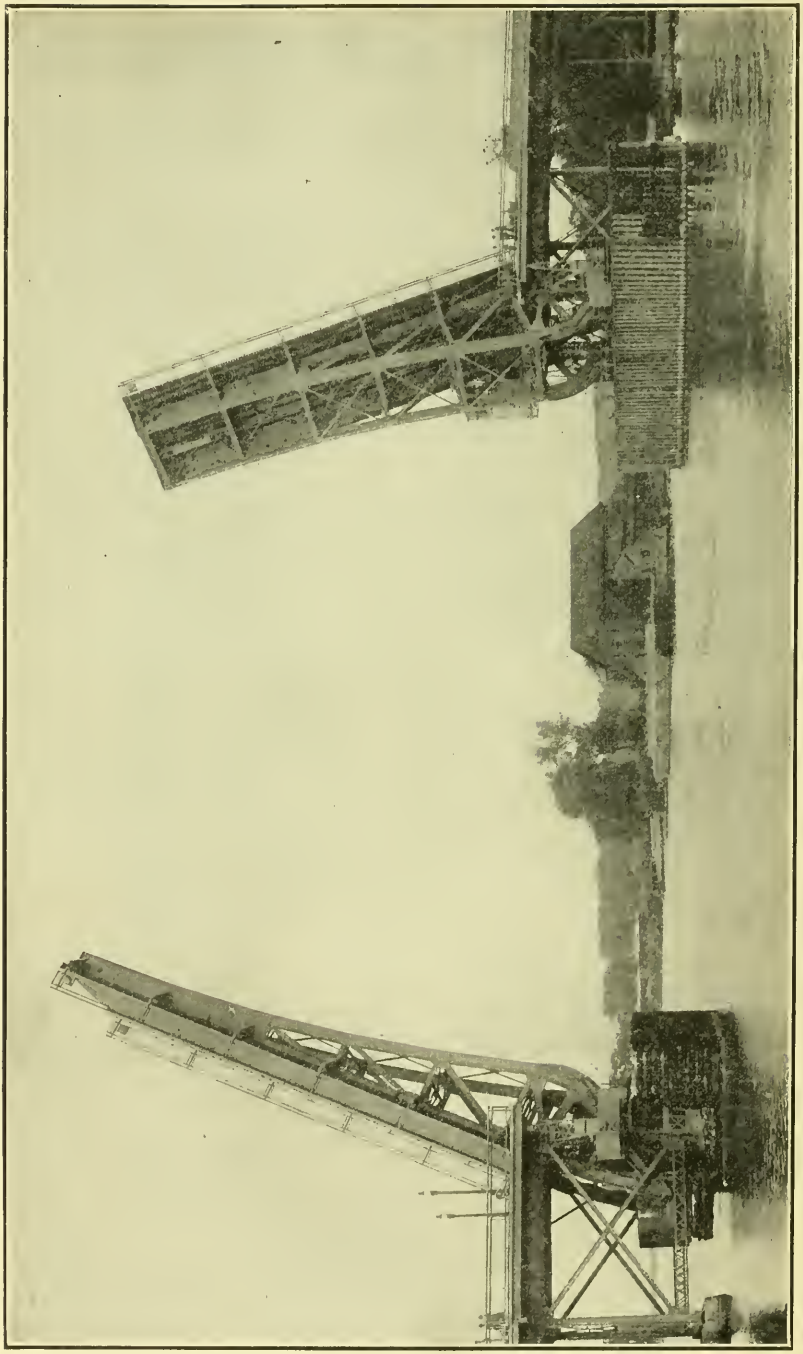


FIG. 4V.

clear when the bridge is open, this type will be found preferable to Type I.

(3) *Heel trunnion type.*—Figure 4W is a diagram of this type. The leaf trunnion is at A and the counterweight-truss trunnion at C. A tension link joins B and D. $ABDC$ is the parallelogram. The center of gravity of the counterweight is placed at g', in the plane Cg' parallel to the plane Ag passing through the main trunnion A and the center of gravity of the moving leaf g. The weight of the counterweight and span are made inversely proportionate to their lever arms, y and x, and maintain this relation throughout the movement of the span, thus preserving a balanced condition.

This design divides the great weight of long spans and their counterweights between two piers, while Types 1 and 2 concentrate it upon one. The reaction at C is always positive, but the vertical component of the dead-load reaction at A is negative in most cases. The leaf, being balanced, has no reaction at Pier I. The vertical reaction at A is the load P, acting as a shear at A, less the vertical component of a reaction at A, parallel to BD and equal to $\frac{Px}{z}$. Whenever this vertical component is greater than P, the reaction at A acts downward, or is negative. There is also a reaction at A, acting toward the right, equal to the horizontal component of the reaction parallel to BD. As the proportions of most bridges of this type are such that the vertical component of the dead-load reaction at A is negative, and often of considerable magnitude, the trunnion caps must be proportioned and anchored for this force. Whenever the live-load positive reaction at A exceeds the vertical dead-load negative reaction, the stresses at A are reversed.

Analysis of the reactions on Piers II and III, of Fig. 4W, demonstrates that both are always positive and constant for dead load, provided that the effect of the unbalanced condition due to the operating struts is neglected. The reaction at Pier II is equal to the weight of the moving leaf P, and that at Pier III is equal to the weight of the counterweight and its frame W.

The leaf is moved by operating struts, articulated to the trusses at B and extending to the tower, where their racks mesh with the main operating pinions. These struts are long and heavy. In closed position their weight at B tends to hold the span down, but when the lead approaches the open position their rear ends act as cantilevers, with reactions at the pinion and at B, and tend to assist the counterweight and hold the bridge open. It is usual to balance the bridge for some intermediate angle and to provide power to overcome the action of

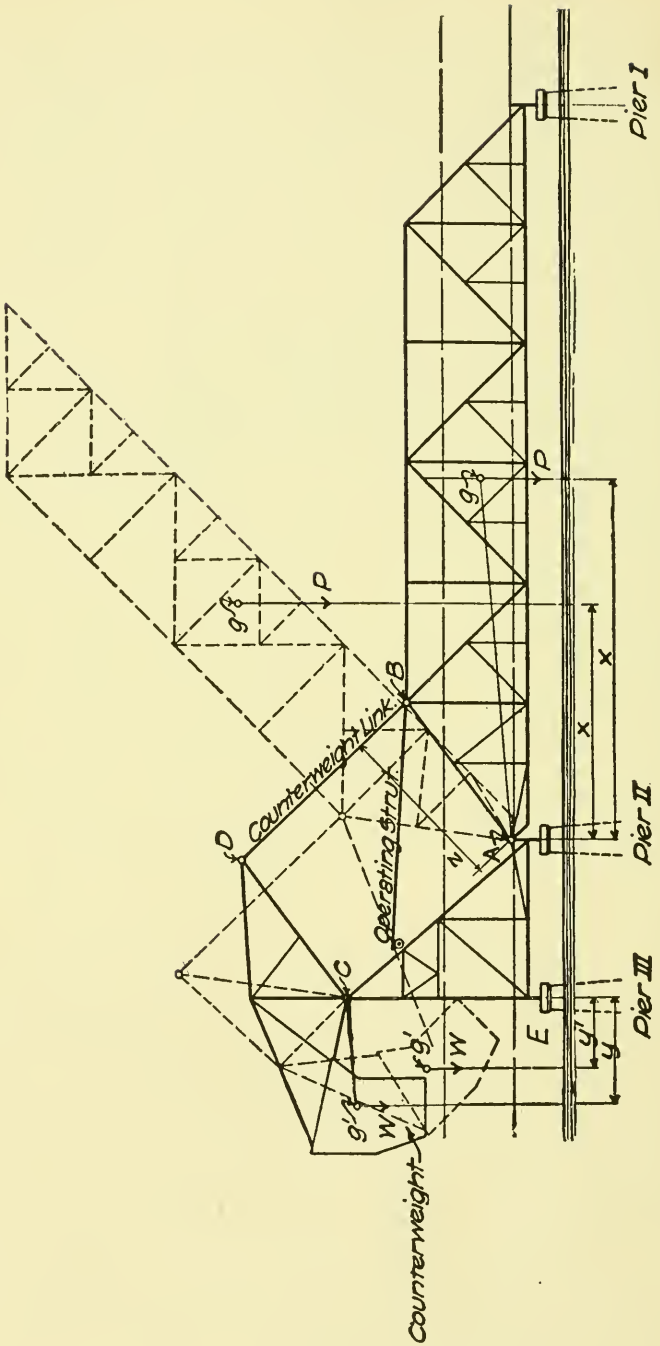

Pier I
Pier II
Pier III
Counterweight Link
Operating Strut
Counterweight
P
W
W'
D
B
C
E
A
x
y
y'

FIG. 4W.

the struts. It will be noted that the trunnions at points *A*, *B*, *D*, and particularly at *C*, are under heavy stress during the entire movement of the span.

St. Charles Air Line bridge.—Figure 4X shows the general design of the St. Charles Air Line bridge over the Chicago River, at 16th Street, Chicago. This is the longest single-leaf bascule bridge that has been built. It is a double-track structure, crossing the channel at an angle of 63°, and affords a 200-ft. clear waterway. The span is 260 ft. between centers of bearings. In addition to the great size and weight of the bridge, a feature is the use of two " wing " counterweights, one on each side of the bridge, which move outside of the planes of the trusses and the railway clearance. Their leverage and efficiency is thus increased, for, when in open position, they can swing considerably below the elevation of the tracks. Each counterweight is about 37 by 60 ft. in size and over 5 ft. thick. They are of concrete, weighing 148 lbs. per cubic foot, and the two weigh about 1770 tons.

The main trunnions are 25 in. in diameter and 27 in. long. Those of the counterweight trusses are 46 in. in diameter and 44 in. long. Phosphor-bronze bushings, free to turn in the cast-steel bearings, were used around these unusually large trunnions. Double lubrication is thus provided. There are lubricating grooves between the bushings and the bearings, as well as on the inner surface, which carries the trunnion sleeve. On account of the great size and weight of the bridge, air buffers and oak bumping blocks were used in the operating struts.

The operating machinery and control apparatus is in an operator's house on the counterweight tower, above the railway clearance line. Power is applied by two 150-h.p. alternating-current, 3-phase, 60-cycle, 440-volt electric motors. They drive the main pinions through equalizer and reduction gear trains. A 55-h.p. gasoline engine emergency drive was also provided. The bridge is fully opened or closed in $1\frac{1}{2}$ minutes by the main motors.

In addition to the customary solenoid motor brakes, special pneumatic emergency brakes, acting directly on the operating struts, were provided. The ends of the closed bridge are latched to the pier pedestals by motor-operated latch bars, under the bottom chords. They move longitudinally. The break in the floor is in front of the trunnions. The end rails have square ends and are held in position by fixed yokes. Further details of the design are given in *Eng. News-Rec.*, 1919, II, p. 1056.

The bridge was proportioned for Cooper's Class E60 loading. The substructure was begun in October, 1917, and the bridge was finished in August, 1919.

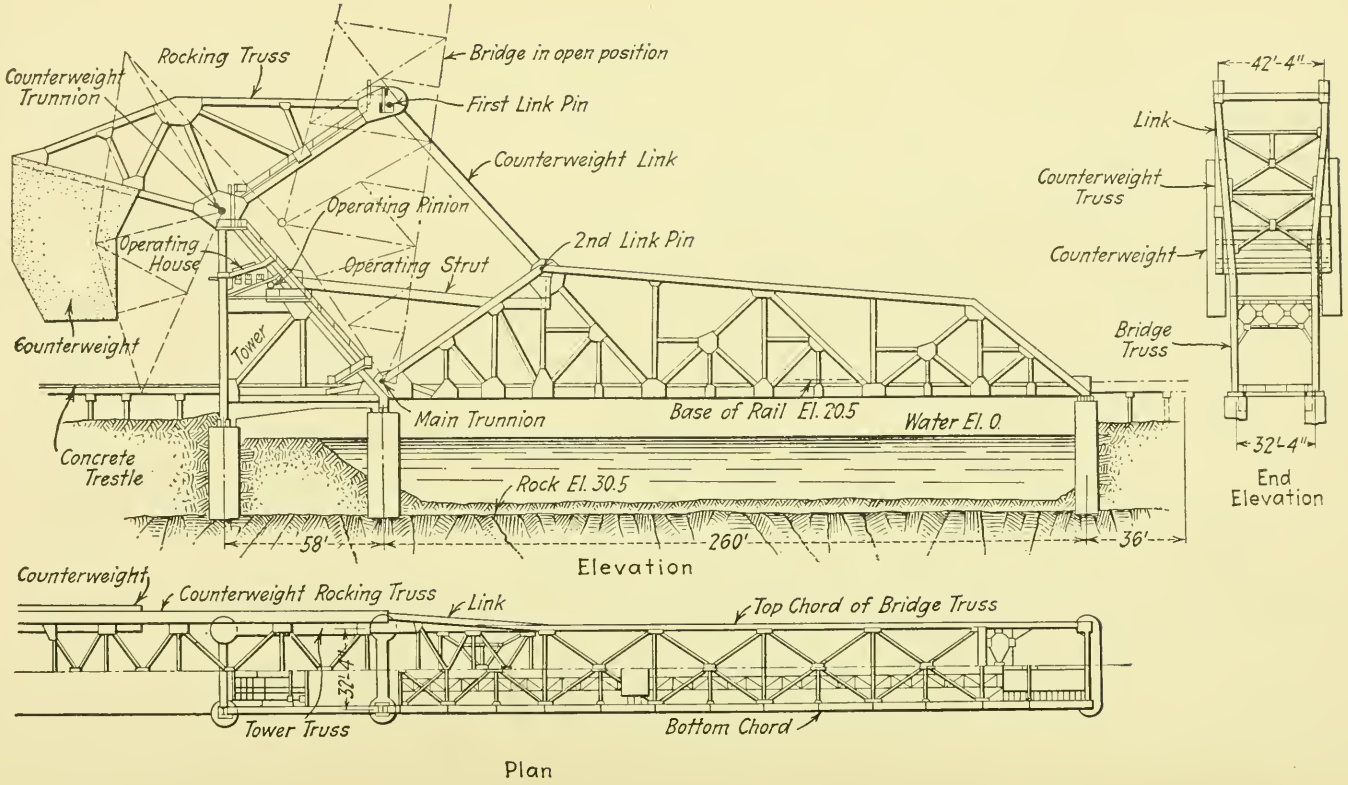

Counterweight Trunnion
Rocking Truss
Bridge in open position
First Link Pin
Counterweight Link
Operating Pinion
Operating House
Operating Strut
2nd Link Pin
Counterweight
Tower
Main Trunnion
Base of Rail El. 20.5
Water El. 0
Concrete Trestle
Rock El. 30.5
58'
260'
36'
Elevation
42'-4"
Link
Counterweight Truss
Counterweight
Bridge Truss
32'-4"
End Elevation
Counterweight
Counterweight Rocking Truss
Link
Top Chord of Bridge Truss
32'-4"
Tower Truss
Bottom Chord
Plan

FIG. 4X.

Thames River Bridge, New London, Conn.—Figure 4Y shows the double-track bridge across the Thames River, on the New York, New Haven, and Hartford Railroad, at New London, Conn. The bascule span is a double-track, riveted structure. The draw span is between two fixed spans. The main trunnions and draw rests are on 12-ft.

FIG. 4Y.

cantilever extensions of the adjoining fixed spans. The counter-trunnions are at the first main top chord panel point of a 327-ft. span, which carries the counterweight and machinery. The end panel of this span is made of special length and height to provide space for the swinging counterweight and to permit effective bracing. The counterweight moves between the fixed trusses, and swings down nearly to the tracks when the bridge is open.

The foundations at this site were deep and difficult, and a material economy was effected by the use of a single pier to carry the ends of the fixed and movable span.

The bascule span is 188 ft. between end bearings, and its piers 212 ft. between centers. The clear channel is 150 ft. between fenders.

On account of limited space for the counterweight, the bottom and rear portions were made of concrete loaded with steel punchings to a weight of 318 lbs. per cubic foot. The remainder was of ordinary concrete weighing about 150 lbs. per cubic foot. Pockets for adjustment weights, also of concrete, were cored in the mass. The heavy concrete was so heavily loaded with punchings that it had little strength. On this account and to facilitate construction, the whole counterweight was built in a steel box. The back and bottom had heavy girders to carry the weight of the concrete. Most of the loaded concrete was placed at the back of the box, to increase the lever arm of the mass. The weight of the counterweight is:

Steel box and girders	183,000 lbs.
Heavy concrete	920,000
Ordinary concrete	1,921,000
Adjustment blocks	149,000
Total	3,173,000 lbs.

The trunnion sizes are:

Moving leaf heel trunnion	19½ ins. diam.,	25 ins. long
Second link trunnion	29	21½
First link trunnion	18	34¼
Counterweight trunnion	45	39¾

They are all sleeve trunnions on pins except the first link trunnions, which are plain pins.

The machinery is on the fixed span and is operated by four alternating-current motors of 82 h.p. The current is 3 phase, 60 cycles, 440 volts. The motors are wired and controlled so that any pair can be used together, or all four when required. An auxiliary motor of 37 h.p. can be substituted for the regular motors for emergency operation. In case of failure of line current, current for the small motor can be supplied by a 45-k.v.a. gasoline engine motor-generator set. Each motor is furnished with a solenoid brake for regular use, and a pneumatic emergency brake is also fitted to each operating strut. The rest girders on the fixed span carry a pneumatic buffer to assist in gently seating the closing span. Thrust end locks were used. The

TABLE 4C

Strauss Bascule Bridges

Name or Location	Design	Number of Leaves	Clear Channel	Span c. to c., Feet	Counterweight Span, Feet	Tracks, Number or Roadway, Feet	Sidewalks, Number, Width	Weight of Structural Steel, Pounds	Weight of Machinery, Pounds	Machinery to Structural, Per Cent	Total Weight, Pounds	Date Built
Railway Bridges												
Wheeling & Lake Erie R.R., Cleveland, Ohio	Thro. Riv.	1	120	150 skew	.....	1	.....	899,000	50,000	5.56	949,000	1905
Chicago & Western Indiana R.R.	Thro. Riv.	1	.....	186	50	2	.....	2,044,000	168,000	8.22	2,212,000	1910
Buffalo Creek R.R., Buffalo, N. Y.	Thro. Riv.	1	.....	165 skew	45	2	.....	1,753,000	153,000	8.73	1,906,000	1913
Illinois Central R.R., New Orleans, La.	Thro. Riv.	1	.....	99.25	62	1	.....	633,000	26,000	4.11	659,000	1914
Baltimore & Ohio R.R., Whiting, Ind.	T. P. G.	1	.....	88.5	.....	2	.....	572,000	118,000	20.19	690,000	1914
Chicago & Northwestern Ry., Deering, Chicago, Ill.	Thro. Riv.	1	145	182.5 skew	55.5	3	.....	2,662,000	273,500	10.25	2,935,000	1916
Yazoo & Mississippi Valley R.R., Allen, Miss.	T. P. G.	1	64	75	75	1	.....	390,000	23,500	6.03	413,500	1917
Cleveland, Cincinnati, Chicago & St. Louis Ry., Cleveland, Ohio	Thro. Riv.	1	140	175 skew	45	1	.....	1,039,000	115,500	11.12	1,154,500	1919
New York, New Haven & Hartford R.R., Thames River, New London, Conn.	Thro. Riv.	1	.....	188	on fixed span	2	.....	1,633,000	205,000	12.55	1,838,000	1919
Illinois Central R.R., et al., 16th Street, Chicago, Ill.	Thro. Riv.	1	200	260 skew	58	2	.....	3,105,000	356,000	11.47	3,461,000	1919
U. S. Govt., Mare Island Navy Yard	D. P. G.	1	74	94	14	1 and highway	.....	304,000	53,000	17.43	357,000	1919
Highway Bridges												
Elizabeth, N. J., Elizabeth River	Pony Riv.	1	60	78	.....	1 @ 25	none	233,000	33,000	14.17	266,000	1908
Petaluma, Cal., Petaluma Creek	D. P. G.	1	50	60	2 @ 25	1 @ 24	2 @ 6	141,500	33,500	23.67	175,000	1914
Buffalo, N. Y., Black Rock Harbor	Thro. Riv.	1	.....	165	55.5	1 @ 30	2 @ 8	1,028,500	132,500	12.88	1,061,000	1914
Walnut Grove, Cal., Sacramento River	Thro. Riv.	2	220.5	226	2 @ 57	1 @ 18	.....	521,000	54,000	10.37	575,000	1915
Rio Vista, Cal., Sacramento River	Thro. Riv.	2	220.5	226	2 @ 57	1 @ 21	.....	542,000	56,000	10.33	598,000	1918
New Brunswick, N. J., Albany St.	D. P. G.	1	.....	58	.....	1 @ 24.83	2 @ 10	166,000	27,000	16.27	193,000	1919
Elizabeth, N. J., Elizabeth River	Thro. Riv.	1	.....	131.63	.....	1 @ 18	1 @ 4	462,000	60,000	12.99	522,000	1921
Troy–Cohoes, N. Y.	Thro. Riv.	2	.....	224	.....	1 @ 26	2 @ 6	1,429,000	165,000	11.55	1,594,000	1922
Isleton, Cal., Sacramento River	Thro. Riv.	2	220.5	226	2 @ 57	1 @ 18	.....	542,000	56,000	10.33	598,000	1922
Cohasset, Minn., Mississippi River	Thro. Riv.	1	.....	68	.....	1 @ 19	.....	152,000	22,500	14.81	174,500	1923
Los Angeles, Cal., Badger Avenue	Thro. Riv.	2	187.92	220	2 @ 50	2 @ 15.5	1 @ 6	4,761,000	342,000	7.18	5,103,000	1923

break in the floor is in front of the trunnions. The rail ends are scarfed, by turning the webs aside, and seat in suitable shoes, about 2 ft. from the floor-breaks.

The entire structure was designed for E60 loading, and the piers were built large enough to provide for a duplicate structure alongside, when needed. Silicon steel with a minimum yield point of 45,000 lbs. per square inch, was used for the trusses, floorbeams, stringers, and the counterweight links, trusses, and box. The bracing, shoes, operating struts, and rivets are of ordinary carbon steel.

The bridge was completed in 1919.

Statistics.—The principal dimensions, facilities afforded, and the weights of the structural steelwork and machinery of a few railway and highway Strauss bascule bridges are given in Table 4C. In most cases the weights of the trunnions, pins, and bearings are included in the machinery item, and properly so, for their unit cost is nearly as much as that of the operating gear trains. The weight of the motors and power plants proper are not included in the machinery.

The proportion of the weight of the machinery to that of the structural steelwork varies widely with the size of the bridge and the type of the design, and the data given are only suitable for approximate preliminary estimates.

11. Brown bascule bridges.—In 1896 a published reference was made to a method of balancing a bascule leaf by means of a counterweight moving vertically, which has since been developed into the present Brown bascule type. This and subsequent designs were patented and are controlled by Mr. Thomas Ellis Brown, Jr., of New York City.

Competitive designs for a proposed bridge across Newtown Creek, at Vernon Avenue, Brooklyn, N. Y., were requested in the summer of 1896, and on July 23 the plans of eight engineers were examined.[14] The design submitted by the late Thomas Ellis Brown was adopted and purchased. Various causes delayed the construction of the bridge, and it was finally built in 1902 from the plans of the Scherzer Bascule Bridge Co., as a rolling-lift, double-leaf structure. The original Brown design was also a double-leaf bridge and was 176 ft. 8 in. between centers of end trunnions. Cast-iron counterweights moved vertically in towers on the shores. Steel-wire cables extended upward from the counterweights and over 10-ft. sheaves on the tower-tops, thence diagonally downward toward the middle of the moving leaf. Near the trusses they connected to an eyebar linkage arranged so as gradually to reduce the effective leverage of the counterweight ropes as the leaf was opened. Bumpers on the span engaged the ropes in case of an over-

[14] *Eng. News*, 1896, II, p. 292.

drive at the open position. The power was hydraulic and was designed to control the movement against a wind pressure of 30 lbs. per square foot, and at velocities under control of the operator.

Ohio Street bridge, Buffalo, N. Y.—The first Brown bridge actually built was that at Ohio Street, Buffalo, across the Buffalo River.[15] In this bridge Mr. Brown substituted a cam girder, attached to the trusses, for the articulated eyebar linkages of the earlier design. The counterweight ropes are wrapped around the face of this girder and anchored to the main trusses at the first lower chord panel point from the trunnion. The face of the girder carries grooves for the ropes and is curved so that as the leaf is opened and the ropes are unwrapped from the cam girders, their effective lever arms are reduced and a balance of the moving leaf is obtained in all positions, the counterweight moving vertically.

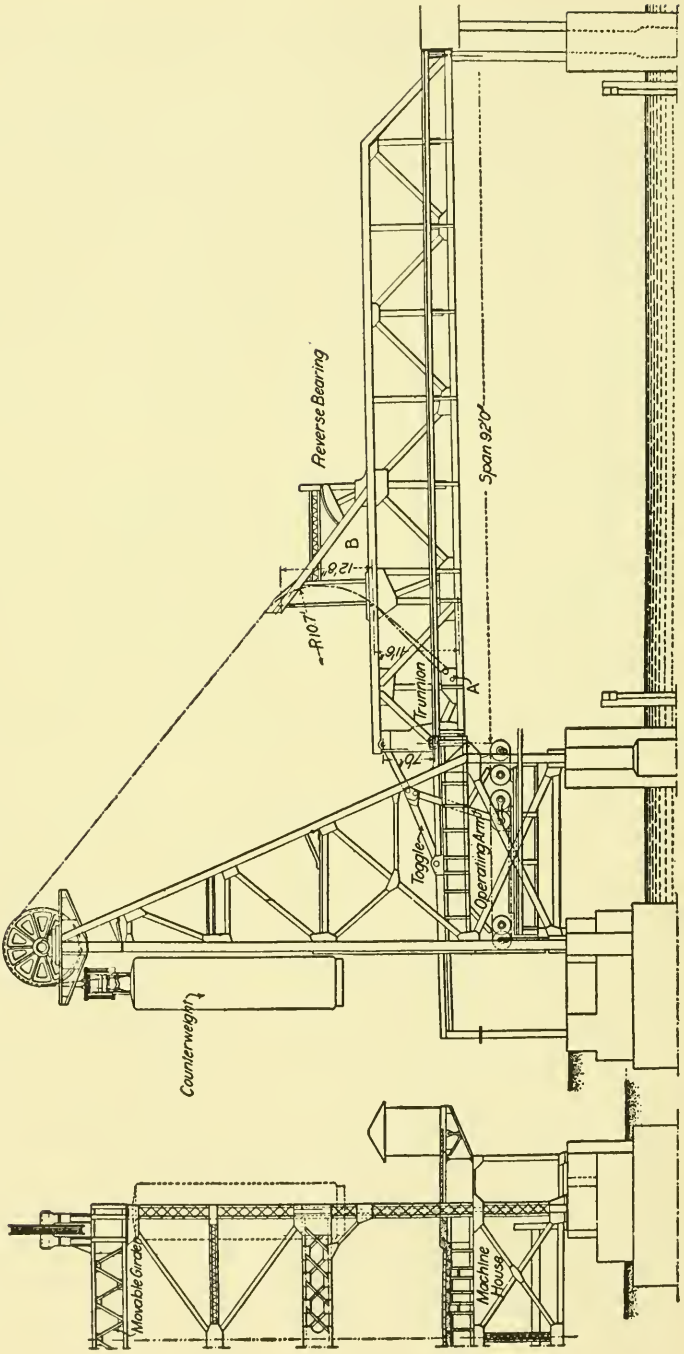
Power was applied by hydraulic accumulators and rams. The bridge is a single-leaf city-street structure, 166 ft. between end bearings, and provides a clear channel of 140 ft. between fenders. It has one 30-ft. roadway and two 7.5-ft. sidewalks. It was completed in 1906.

Hamburg Turnpike bridge, Buffalo, N. Y. This bridge was built in 1913.[16] It is a single-leaf bascule span, 92 ft. between centers of end bearings, and carries a city street across the Buffalo Ship Canal. Figure 4Z shows some of the principal features of the design. The method of balancing is the same as that used at Ohio Street, but the operating machinery is different.

The arms of a toggle joint connect the hip of the bascule girder with an anchorage on the fixed tower. The operating arm connects the middle joint of the toggle with a carriage, free to move longitudinally on a track on the top of the side girder of the tower. The carriage carries two sets of sheaves. Fixed sheaves are attached to the front and rear of the tower in the plane of the bascule trusses. The bights of two wire ropes are rove around the pairs of movable and fixed sheaves, and their ends are passed over guide sheaves and attached to a drum, 5 ft. in diameter, on the longitudinal center line of the tower. This drum is driven by two 30-h.p. electric motors, through two trains of reduction gearing. The arrangement is symmetrical on the two sides of the tower. All sets of ropes are operated simultaneously by the drum. When the drum turns in one direction the foot of the operating link moves away from the span, depressing the toggle and opening the leaf. Reversing the drum closes the bridge. When one set of ropes is wound on the drum, its complementary set is unwound. Thus the leaf

[15] *Eng. News*, 1908, 1, p. 51.

[16] *Eng. Record*, August 1, 1914.



Eng. Record.

FIG. 47.

is under full control at all angles and cannot be moved except by rotating the drum. Adjustment is provided so that when the free end of the leaf bears on the rest-pier pedestals, the toggle joint is straight. This automatically slows down the bridge, and the span is seated gently and firmly, even when the motors run at full speed. An overdrive would slightly open the bridge without damage. No free-end locks are required. The movement of the bridge is smooth, and the whole mechanism operates satisfactorily.

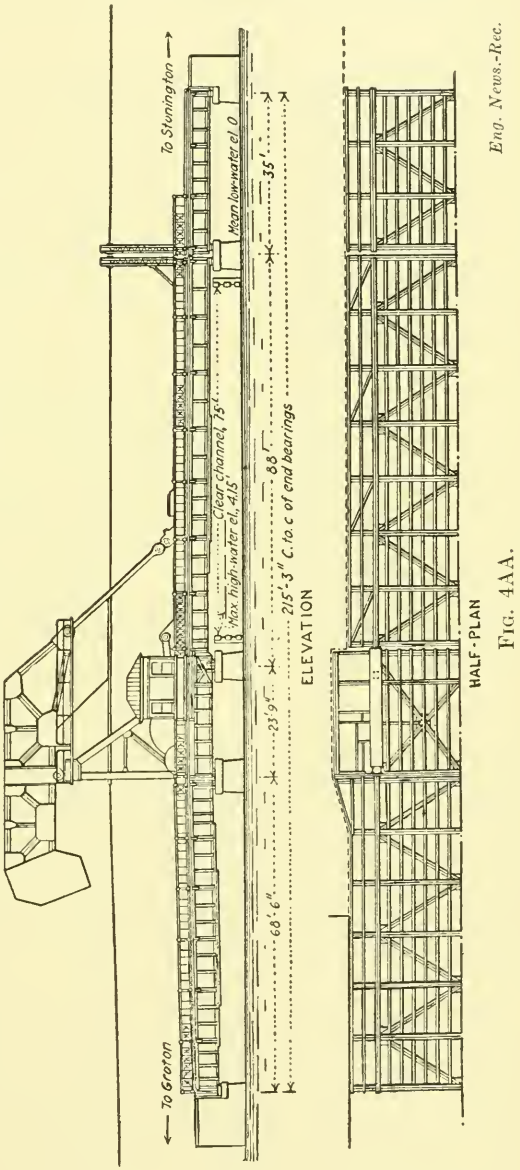


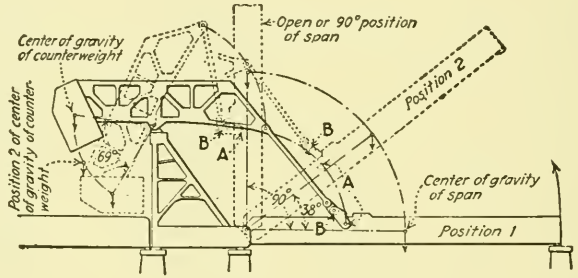
FIG. 4AA.

Bridge at Mystic, Connecticut. — Another Brown bascule bridge was completed during 1922.[17] Figure 4AA shows its general elevation and plan; Fig. 4BB is a diagram illustrating the method of balancing the bascule leaf. The bascule span is 84 ft. 8 in. between centers of end bearings and provides a clear channel of 75 ft. between fenders. The roadway is 33 ft. wide, and the width of each sidewalk is 5 ft. Vertically moving counterweights connected to

[17] *Eng. News-Rec.*, 1923, II, p. 374. An article by Thomas Ellis Brown, Jr., M. Am. Soc. C. E., and William G. Grove, M. Am. Soc. C. E.

the span by wire ropes were not used. The method of varying the effective leverage by means of articulated links is a simplification of that originally proposed in the 1896 design for the Newtown Creek bridge.

The movable span is counterbalanced by two concrete weights placed on the center lines of the main girders and carried by balance beams in the form of trusses which rest on trunnions at the tops of the tower bents. These trusses are about 60 ft. long over all, and their forward ends are connected with the plate girders of the moving span by hanger links or suspenders consisting of eyebars and short links having double sets of pivots at the span end. Butt blocks or stops are suitably placed on the upper flange of the movable-span girders to make contact with the short links at the proper time during motion, and thus change the point of rotation of the eyebar hangers from one set of pivots to the other.



Eng. News-Rec.

FIG. 4BB.

Figure 4BB is a diagram showing the action of the pivots and butt blocks as the span rotates through its 90° of motion. Between positions 1 and 2 of the span, or for about the first 38° of motion, the hangers rotate about the primary pivots *A*. The butt blocks then make contact with butt castings on the short links, preventing their further rotation about pivot *A*, as shown in position 2. From this position throughout the remainder of the opening, the eyebars rotate about the pivots *B*, on the ends of the short links.

On account of the relative position of the centers of gravity of the span and counterweight and the location of the link joints, while the span moves through 90°, the angular movement of the counterweight is only about 69°. Thus the counterweight may be behind the tower post when the bridge is fully open and the main trusses, links, tower frames, balance beams, and counterweights are all in the same vertical plane. This is an obvious advantage, for it allows full transverse bracing in the tower.

The span is operated by a strut articulated to the main girders above and in front of the trunnions and extending rearward, where it is attached to a crank pin on a segmental bull wheel, the outline of which may be seen in Fig. 4AA, just to the rear of the operator's house. When the bridge is closed the truss pin, the crank pin, and the center of the bull wheel are in a straight line, thus forming a toggle joint. When the span is fully open a similar condition obtains. End locks are not required. Exact control of the motors when bringing the bridge to rest is relatively unimportant, for the reasons given above in connection with the Hamburg Turnpike bridge. When in open position, the bull wheel and operating strut are at a point of maximum leverage and there is no danger of the wind overturning the span against the tower.

With only two butt-link joints, an exact balance is not attained, but in the Mystic bridge the maximum error was only about one-half of one per cent. The location of joint *B* was purposely selected so that the unbalancing would be in the fully open position and would tend to prevent opening the bridge too far. It thus serves as an automatic buffer, which builds up rapidly during the last few degrees of motion and without shock.

Power is applied by two 36-h.p., 3-phase, 60-cycle, 220-volt alternating-current motors, placed on frames with worm gear units of the ratio 20 : 1. Two gear reductions were used between the worm gear units and the main pinions. The machinery is so compact that it was placed under the sidewalks of the tower span.

The weight of structural steel in the bascule and tower spans is about 493,000 lbs., with 81,500 lbs. of machinery, a total of 574,500 lbs. The weight of the machinery is 16.5 per cent of that of the structural steelwork. The concrete counterweights weigh 400 tons and the total weight of all moving parts is about 650 tons.

New Brown type.—The Brown type has recently been modified and improved by locating the articulated butt-links on the channel end of the balance beams.[18] Figure 4CC shows the general features of a bridge of this design in closed and open positions. Better details result from moving the butt-links to the end of the balance beams, and control of the balancing is equally simple. The bull-wheel toggle action affords an effective and simple method of operating a small bridge, like the Mystic, but for large and heavy structures an operating strut and rack would be chosen. The unbalancing of the leaf by cantilever action of these struts, near the open position, can be corrected by

[18] *Journal of the Western Society of Engineers*, 1924, Vol. XXIX, p. 195. An article by Thomas Ellis Brown, Jr.

the proper selection of the points of articulation of the butt-links. The article referred to above includes a discussion of the theory of the method of balancing and a statement of the advantages claimed for the type.

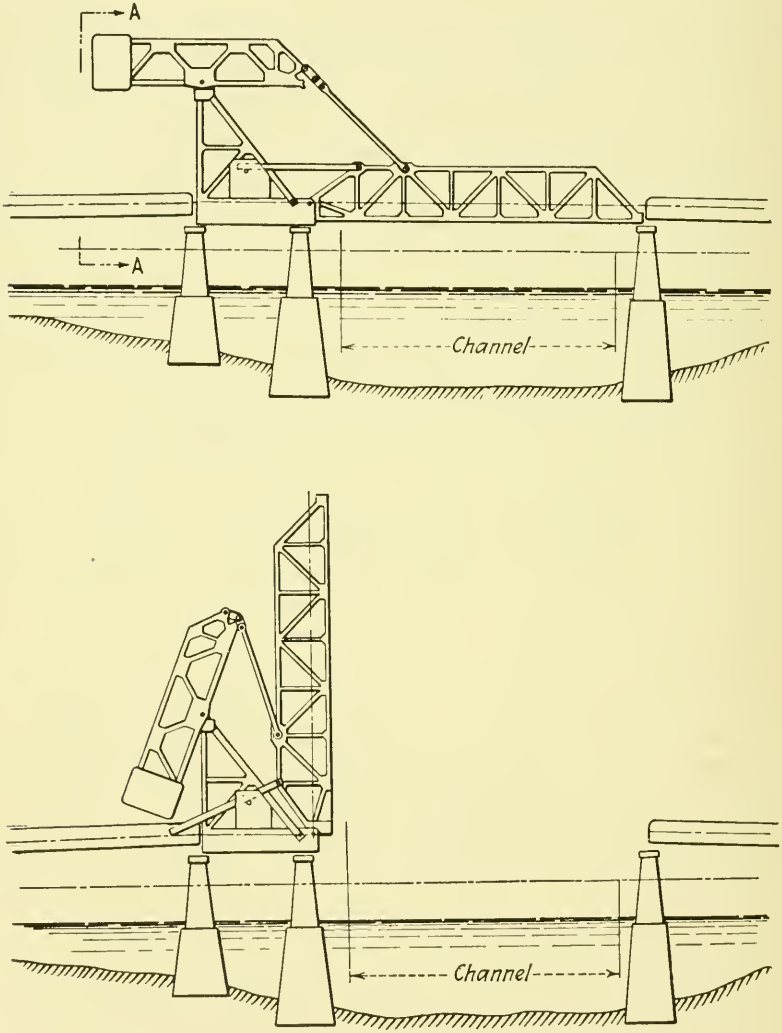


Fig. 4CC.

12. American Bridge type bascule bridge.—During 1920 there was developed a new type of trunnion-balanced bascule bridge, in which the counterweight moves in the opposite direction to the bascule leaf.

This type has been patented by Mr. Hugo A. F. Abt, Assistant Engineer with the American Bridge Co., at Chicago. The first bridge of this type to be built was that of the Wabash Railway, across the River Rouge, in Detroit, Mich.[19] Figure 4DD is from a general elevation and shows the method of counterbalancing. Figure 4EE shows the bridge closed, and Fig. 4FF shows it fully open. The bridge has two railway tracks and was designed for Cooper's Class E50 loading and in accordance with the Specifications of the American Railway Engineering Association, of 1910; and the machinery as required by their Specifications for Movable Bridges, of 1922.

The channel span is 162 ft. between centers of end bearings, and the counterweight span is 73.5 ft. long. From upper chord panel point 2 of the main truss a tension member extends diagonally upward and rearward. A rocker link connects it to the heel of the truss at 1. The counterweight is in a steel box with arms which are hinged to the top of the tower. A compression link, pin-connected to the counterweight box, is articulated to a tension operating link which is hinged to the rocker-link joint. A track in the tower, inclined about 30° with a horizontal plane, bisects the angle between the compression and tension links forming a scissors joint. This joint is pin-connected to a truck adapted to roll down the inclined track. The trucks at the two sides of the bridge are connected by a box girder, which carries the machinery and operator's house. Power is applied by pinions engaging racks on the inclined track girders. As the counterweight and the operating-link joint rotate in arcs of circles, and power is applied along the bisector of the scissors-joint angle, an almost exact balance is attained in all positions of the moving leaf. The scissors-joint link, trucks, and the ends of the counterweight are in the plane of the moving-leaf trusses and move within the tower, which must be wide enough to allow the necessary clearance.

The operating machinery is driven by two 100-h.p. electric motors, through an equalizer and reduction gearing located on the machinery girder. Auxiliary power is supplied by a 58-h.p. gasoline engine. The operating machinery may be fully mounted on the transverse girder and tested in the shop, and shipped and erected as a unit. The bridge is fully opened or closed by one motor in $1\frac{1}{2}$ minutes. The horsepower developed was from 74 to 98, with an average of 85. The maximum and minimum varied from the average by about 12 per cent of the nominal horsepower of the motors. In ordinary operations, only one motor is used. The auxiliary requires about 8 minutes for full opera-

[19] *Eng. News-Rec.*, 1922, I, p. 363.
Journal of the Western Society of Engineers, 1924, Vol. XXIX, p. 177.

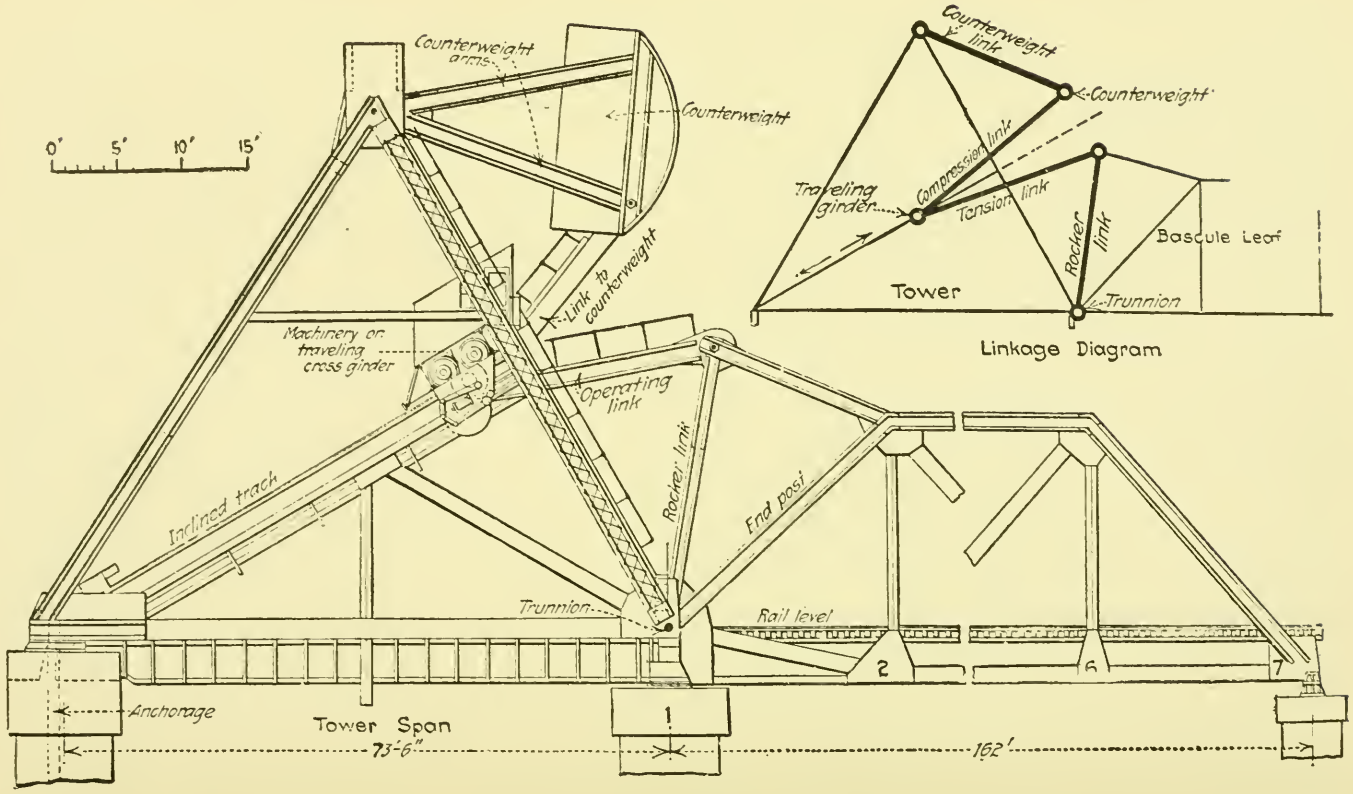


FIG. 4DD.

Eng. News-Rec.

tion. Each motor is fitted with a full-torque solenoid brake, and a pneumatic emergency brake grips special I-beams along the sides of the track girders. Each emergency brake develops a braking effect of 88 tons. One of the solenoid brakes can be applied by hand, when the auxiliary gasoline equipment is being used. All brakes are arranged to set slowly and release quickly. A compressor, with a capacity of 25 cu. ft. of free air per minute, keeps the receiver pressure automatically at 100 lbs. per square inch.

The free ends of the trusses are locked to the pedestals on the rest pier when the bridge is closed. The lock is a square bar moving longitudinally in guides under the center of the end bottom chords and engaging rollers in the pedestals. It is operated by a small motor.

The bridge mechanism is electrically interlocked with the track connections and signals, so that the bridge cannot be operated until the signals have been set properly, and the several operations must be performed in correct sequence.

When the bridge is open the counterweight trunnions carry their maximum loads, made up of the weight of the counterweight and its box and connections. When the bridge is closed these trunnions are only lightly loaded. The design lends itself to easy construction in the open position of the leaf. The counterweight box then hangs like a pendulum and no forms are needed. The concrete can be gradually placed, as required, as the erection of the leaf proceeds. A bridge of this type can be completely erected in open position and the machinery tested, by actual operation for small angles, without in any way interfering with the traffic over an old structure.

The floor-break is in front of the trunnions.

The weight of structural steel is 1,783,000 lbs., and of the machinery, 301,000 lbs., a total of 2,084,000 lbs. The weight of the machinery is 16.9 per cent of that of the structural steel. The counterweight weighs about 1000 tons. Provision is made for adjustment by the use of separate concrete blocks weighing 2500 lbs each.

The bridge was put in service in January, 1922, and its operation is smooth and satisfactory.

Type with horizontal track.—In a modified form, the same principles are applied and the track in the tower is made horizontal. Figure 4GG illustrates this construction. In the particular case illustrated there is a short span, adjacent to the bascule, which serves as the tower span. The angle of the scissors joint is made small in this design and the normal reaction on the traveling truck and its track girder is much less than in the type with the track at a 30° slope. The vertical reaction of the operating strut is taken by a separate truck, thus distributing

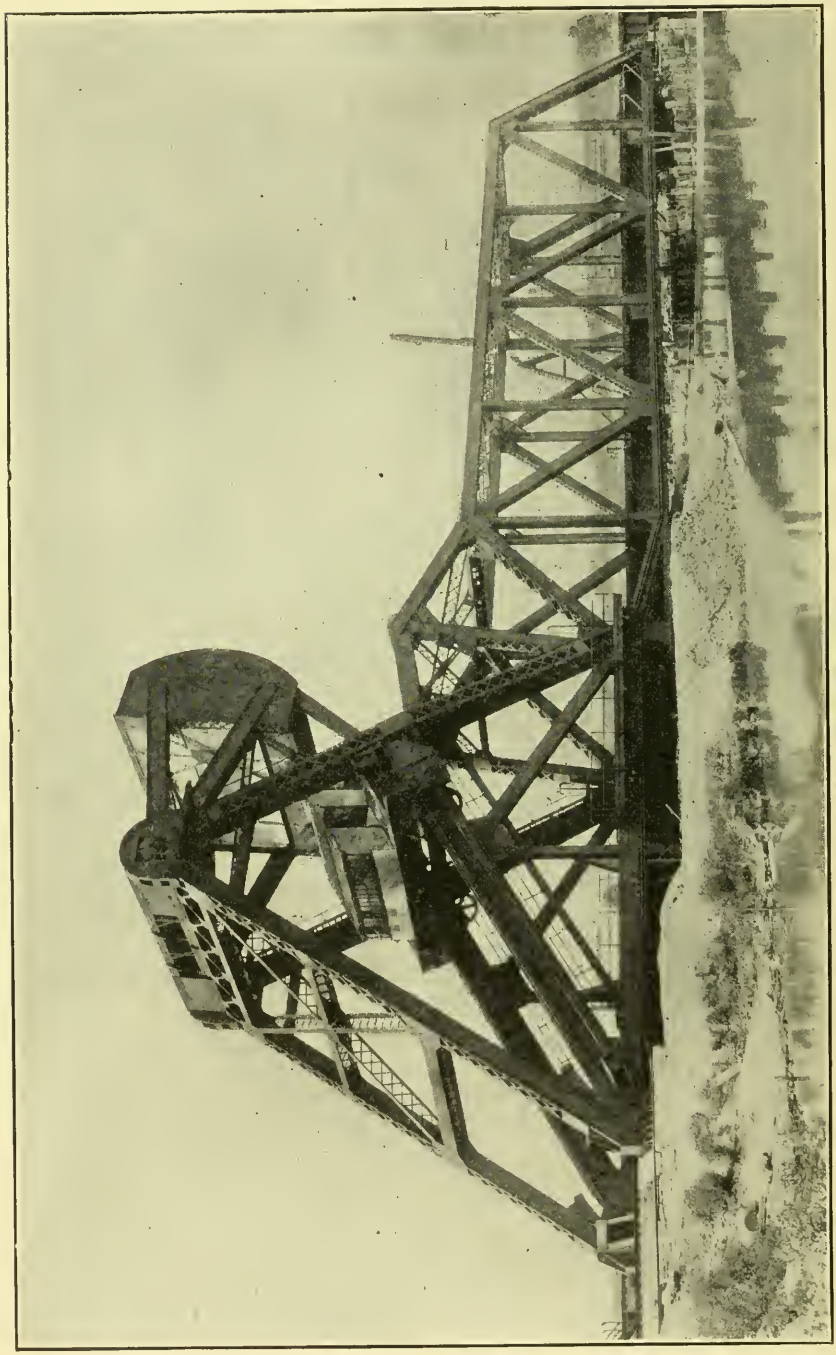


FIG. 4EE.

the loads on the track girder. The horizontal track girder improves the appearance and operation of the bridge. The rack is on the bottom of a horizontal tension link, and the machinery is in fixed position on the tower and can readily be located and housed under the link. The

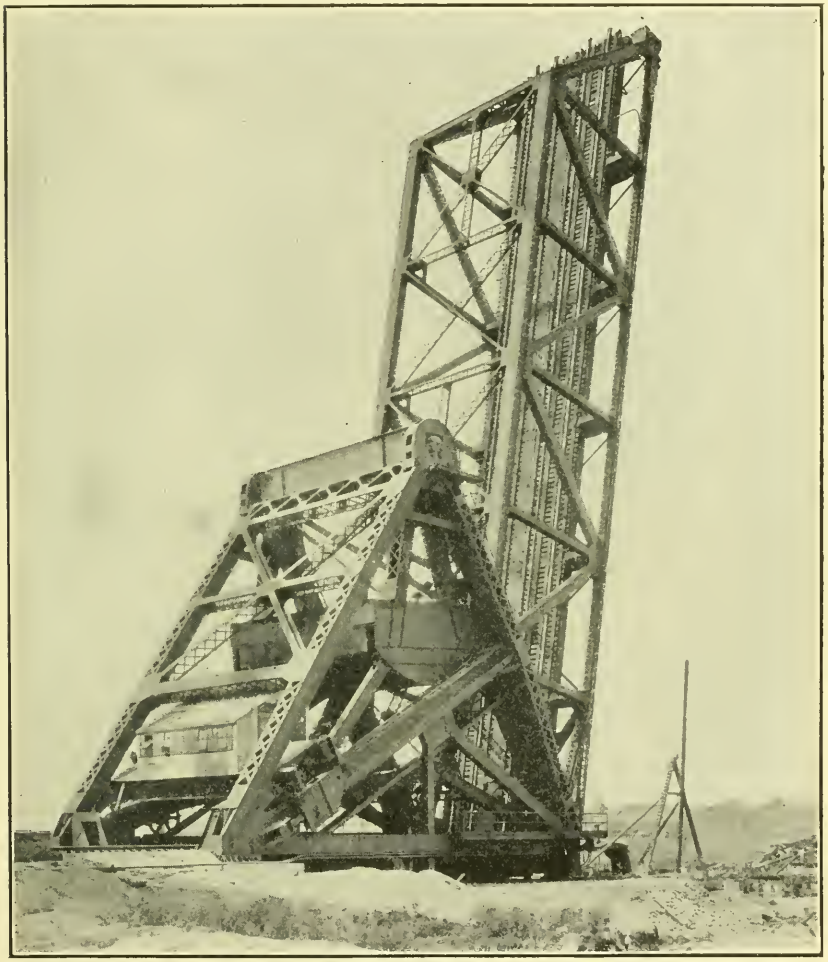


Fig. 4FF.

operator's house may be at any convenient point, as at the extended end of the pier. The motors would then be operated by remote-control panels.

When the conditions at the site render it economical, the counterweight tower may be mounted on an adjacent fixed span of any desired

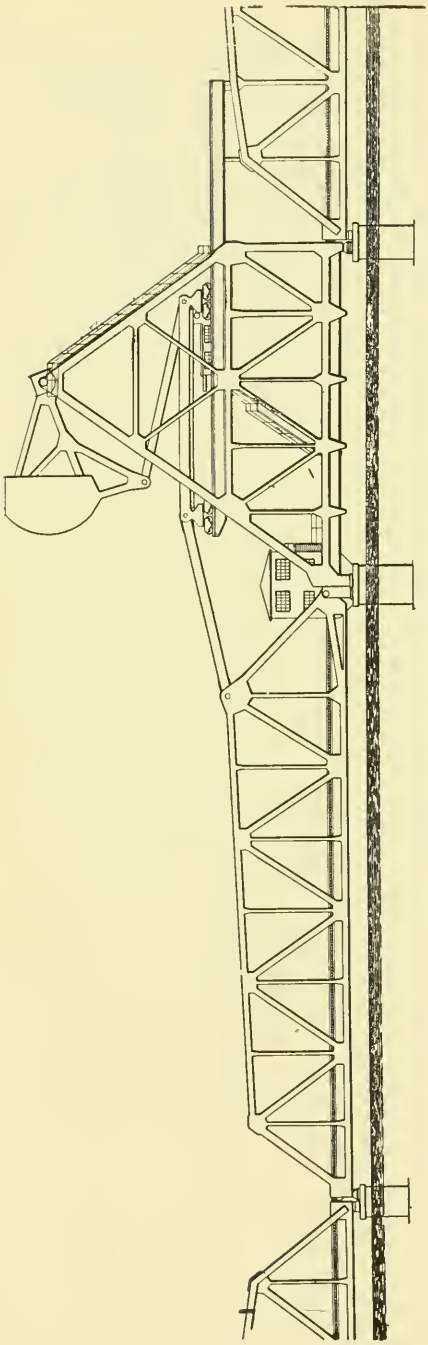


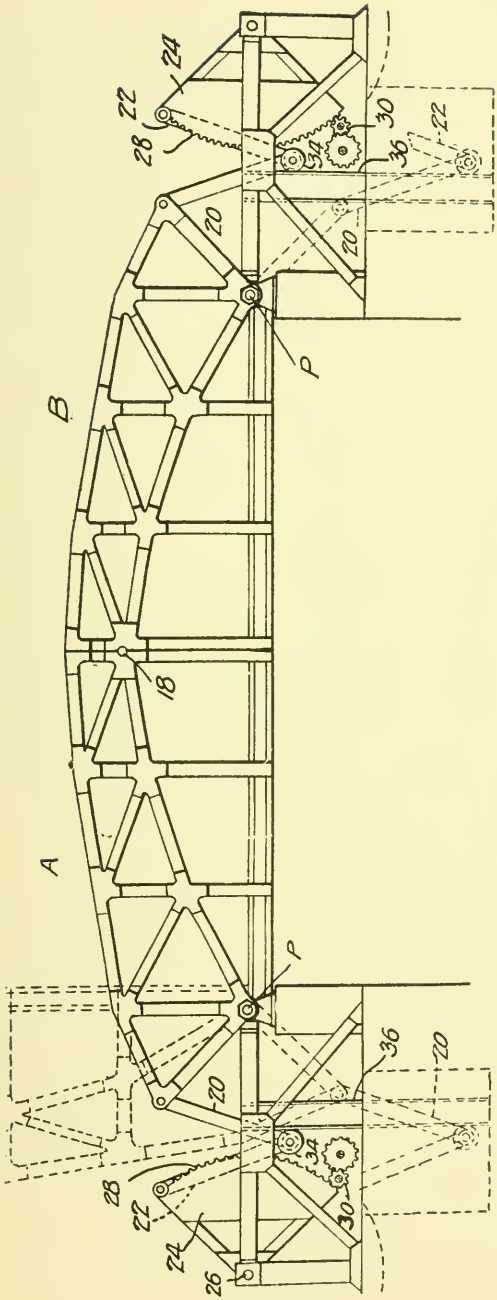
Fig. 4GG.

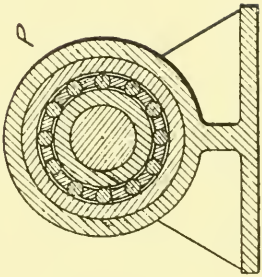
length, or the track girders may be extended on top of the span, as shown in Fig. 4GG.

This design affords the same simplicity of erection that is a feature of the earlier design. On account of the geometric proportions of the linkage, the reaction at the main trunnion is positive in all positions of the moving leaf.

Type with vertical track.— Figure 4HH is a sketch illustrating an application of the same principles in such a way as to provide for vertical movement of the joint between the operating and counterweight links. The truss trunnions are at *P* and those of the counterweight at 26. The counterweight 24 rotates in a direction opposite to that of the bascule leaf *A*. The roller 34 at the scissors joint bears against a vertical track 36, and is always held in contact by the horizontal components of the stresses in the counterweight link 22 and the operating link 20.

In the design shown, the two bascule leaves *A* and *B* close on the pin 18 and act as an arch under live loads, the end reaction being at the main trunnions *P*. A rack 28 is attached to the counterweight, which is in mesh with the pinions 30. Any desired type of motive power may





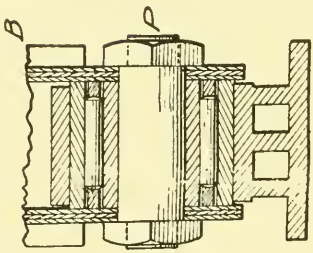


Fig. 4HH.

be applied to these pinions through the medium of trains of reduction gearing. This general arrangement allows the operating machinery and the vertically moving joint and roller to be placed below the bridge floor when this arrangement is preferable to the overhead construction of the two preceding types. It is well adapted for use in highway and city-street bridges, and good architectural treatment is feasible.

Properly designed roller bearings are desirable for the main trunnions. The movement is only 90° at the most, and good lubrication of plain trunnion bearings under heavy pressure is uncertain. Roller bearings may be packed full of grease, and any movement distributes it and insures thorough lubrication.

13. Miscellaneous bascule types.—It is not feasible to describe all of the designs that have been proposed for balanced bascule bridges. Probably no phase of movable bridge construction has received as much study as the design of such structures. Several types have been designed but not built, and others are represented by only one, or a very few, bridges. Within the limits of available space, it is only possible to mention the types most frequently used. Some designs have not been promoted vigorously by their inventors; others, for various reasons, have not been received with favor.

An engineer proposing to use what appears to be a new type should exercise much caution, for the many patents already granted limit the possibilities of new invention in this field. In order to assist investigators, a list of the United States patents issued is given in Table 4D.

TABLE 4D

United States Patents Granted for Bascule Bridges

Name	Number	Date	Description
Abt, H. A. F.	1,394,519	Oct. 25, 1921	Bascule bridge
Abt, H. A. F.	1,412,327	April 11, 1922	Bascule bridge
Abt, H. A. F.	1,412,328	April 11, 1922	Bascule bridge
Adams, M. B. and Krause, F. L.	173,253	Feb. 8, 1876	Bascule bridge
Agnew, R. M.	983,194	Jan. 31, 1911	Locking device
Anderson, H.	1,337,187	Aug. 20, 1920	Bridge
Benni, F. J.	1,048,440	Dec. 24, 1912	Bascule bridge
Borg, F. J.	828,873	Aug. 21, 1906	Bridge
Brayton, W. L.	632,985	Sept. 12, 1899	Bridge
Breithaupt, W. H.	587,926	Aug. 10, 1897	Draw bridge
Brown, T. E.	590,787	Sept. 28, 1897	Draw bridge
Brown, T. E.	1,151,657	Aug. 31, 1915	Bascule bridge
Brown, T. E.	1,203,695	Nov. 7, 1916	Bascule bridge
Brown, T. E.	1,210,410	Jan. 2, 1917	Bascule bridge
Brown, T. E., Jr.	1,251,634	Jan. 1, 1918	Bascule bridge
Brown, T. E., Jr.	1,254,772	Jan. 29, 1918	Bascule bridge
Brown, T. E., Jr.	1,254,773	Jan. 29, 1918	Bascule bridge
Brown, T. E.	1,270,925	July 2, 1918	Draw bridge
Brown, T. E.	1,302,302	April 29, 1919	Bascule bridge
Brown, T. E., Jr.	1,406,259	Feb. 14, 1922	Bascule bridge
Brown, T. E., *et al.*	1,470,140	Oct. 9, 1923	Bascule bridge
Brown, T. E.	1,519,189	Dec. 16, 1924	Draw bridge
Cowing, J. P.	633,811	Sept. 26, 1899	Bascule lift bridge
Cowing, J. P.	644,405	Feb. 27, 1900	Bascule lift bridge
Cowing, J. P.	665,405	Jan. 8, 1901	Bascule bridge
Cowing, J. P.	672,848	April 23, 1901	Revolving-lift bridge
Cummings, E. D.	689,856	Dec. 31, 1901	Lift bridge
Cunningham, A. O.	1,510,043	Sept. 30, 1924	Bascule bridge
Gardner, R. D.	1,224,629	May 1, 1917	Counterbalance device
Hall, C. F.	694,744	March 4, 1902	Bascule bridge
Hall, C. F.	708,348	Sept. 2, 1902	Bascule bridge
Harmon, W.	383,880	June 5, 1888	Bridge
Jennings, E. B.	554,390	Feb. 11, 1896	Draw bridge
Joyce, J. A.	780,193	Jan. 17, 1905	Bascule bridge
Keller, C. L.	685,767	Nov. 5, 1901	Bridge
Keller, C. L.	685,768	Nov. 5, 1901	Bridge
Keller, C. L.	752,563	Feb. 16, 1904	Bascule bridge
Keller, C. L.	1,047,950	Dec. 24, 1912	Means for operating bascule bridge
Krase, H. C.	1,042,238	Oct. 22, 1912	Lock for bascule bridge
Lamont, R. P.	503,377	Aug. 15, 1893	Counterbalance for bridges

TABLE 4D—*Continued*

Name	Number	Date	Description
Lamont, R. P.	503,378	Aug. 15, 1893	Driving mechanism for bridges
Lamont, R. P.	544,733	Aug. 20, 1895	Bridge
Leslie, B.	1,078,293	Nov. 11, 1913	Draw bridge
La Pointe, F.	657,122	Sept. 4, 1900	Lift bridge
Larsson, C. G. E.	1,124,922	Jan. 12, 1915	Lift bridge
McKibben, C.	1,128,478	Feb. 16, 1915	Bascule bridge
Mercer, C. H., and Woehle, C. H.	1,241,237	Sept. 25, 1917	Bascule bridge
Moody, G.	180,491	Aug. 1, 1876	Draw bridge
Moreell, S.	1,311,284	July 29, 1919	Bascule bridge
Newton, R. D.	824,135	June 26, 1906	Bascule bridge
Nikonow, J. P.	843,167	Feb. 5, 1907	Bridge structure
Osborn, F. C.	1,156,546	Nov. 2, 1915	Lifting mechanism for bridges
Page, J. W.	673,923	May 14, 1901	Bascule bridge
Page, J. W.	731,321	June 16, 1903	Bascule bridge
Page, J. W.	731,322	June 16, 1903	Bascule bridge
Rall, T.	669,348	Mar. 5, 1901	Movable bridge
Rall, T.	817,516	April 10, 1906	Movable bridge
Rall, T.	1,094,473	April 28, 1914	Bascule bridge
Roake, S. A.	1,471,282	Oct. 16, 1923	Bridge construction
Scherzer, W. dec'd.	511,713	Dec. 26, 1893	Lift bridge
Scherzer, A. H., and Kandeler, C. F. T.	721,918	Mar. 3, 1903	Bascule bridge
Scherzer, A. H., and Kandeler, C. F. T.	735,414	Aug. 4, 1903	Bascule bridge
Scherzer, A. H.	963,399	July 5, 1910	Bascule bridge
Scherzer, A. H.	968,987	Aug. 30, 1910	Locking means for bascule bridge
Scherzer, A. H.	968,988	Aug. 30, 1910	Center lock for bascule bridge
Scherzer, A. H.	978,483	Dec. 13, 1910	Rear-end lock for bascule bridge
Scherzer, A. H.	1,021,488	Mar. 26, 1912	Bascule bridge
Scherzer, A. H.	1,041,885	Oct. 22, 1912	Bascule bridge
Scherzer, A. H.	1,104,318	July 21, 1914	Bascule bridge
Scherzer, A. H.	1,109,792	Sept. 8, 1914	Bascule bridge
Scherzer, A. H.	1,114,535	Oct. 20, 1914	Bascule bridge
Scherzer, A. H.	1,370,999	Mar. 8, 1921	Bascule bridge
Schinke, M. G.	517,809	April 3, 1894	Draw bridge
Schinke, M. G.	551,004	Dec. 10, 1895	Draw bridge
Shaw, E. S.	564,164	July 14, 1896	Bascule bridge
Shoemaker, L. H.	887,131	May 12, 1908	Counterbalance for liftbridge

TABLE 4D—*Continued*

Name	Number	Date	Description
Smetters, S. T.	719,153	Jan. 27, 1903	Rolling-lift bridge
Strauss, J. B.	668,232	Feb. 19, 1901	Bridge
Strauss, J. B.	738,954	Sept. 15, 1903	Bridge
Strauss, J. B.	894,239	July 28, 1908	Bascule bridge
Strauss, J. B.	974,538	Nov. 1, 1910	Bridge
Strauss, J. B.	995,813	June 20, 1911	Bridge
Strauss, J. B.	1,124,356	Jan. 12, 1915	Bascule bridge
Strauss, J. B.	1,150,643	Aug. 17, 1915	Bascule bridge
Strauss, J. B.	1,150,975	Aug. 24, 1915	Bridge
Strauss, J. B.	1,157,449	Oct. 19, 1915	Bridge
Strauss, J. B.	1,157,450	Oct. 19, 1915	Bridge
Strauss, J. B.	1,170,703	Feb. 8, 1916	Double-deck bascule bridge
Strauss, J. B.	1,171,553	Feb. 15, 1916	Bridge
Strauss, J. B.	1,211,639	Jan. 9, 1917	Bascule bridge
Strauss, J. B.	1,211,640	Jan. 9, 1917	Bridge
Strauss, J. B.	1,241,052	Sept. 25, 1917	Bridge
Swartz, S.	136,278	Feb. 25, 1873	Lifting bridge
Swartz, S.	172,204	Jan. 11, 1876	Lifting bridge
Thomson, G. H.	496,074	April 25, 1893	Draw bridge
Vent, F. G.	648,447	May 1, 1900	Bascule bridge
Vent, F. G.	652,201	June 19, 1900	Rocking bascule bridge
Vent, F. G.	683,627	Oct. 1, 1901	Bridge
Von Babo, A. F. L.	1,001,800	Aug. 29, 1911	Trunnion bascule bridge
Waddell, J. A. L. and Harrington, J. L.	952,485	Mar. 22, 1910	Bascule bridge
Waddell, M.	598,167	Feb. 1, 1898	Draw bridge
Waddell, M.	598,168	Feb. 1, 1898	Draw bridge
Waddell, M.	621,466	Mar. 21, 1899	Bridge
Waddell, M.	637,050	Nov. 14, 1899	Bridge
Waddell, M.	660,827	Oct. 30, 1900	Bridge
Waddell, M.	661,113	Nov. 6, 1900	Bascule bridge
Waddell, M.	693,467	Feb. 18, 1902	Lifting bridge
Waddell, M.	890,947	June 16, 1908	Lifting bridge
Waddell, M.	908,713	Jan. 5, 1909	Draw bridge
Watson, W. J.	789,398	May 9, 1905	Bascule bridge
Wiedenmayer, G. A. and Bergman, S.	442,847	Dec. 16, 1890	Draw bridge
Wilkins, J. D.	691,035	Jan. 14, 1902	Counterbalance for lift bridges
Worden, B. L.	534,704	Feb. 26, 1895	Draw bridge
Worden, B. L.	536,313	Mar. 26, 1895	Draw bridge

CHAPTER V

VERTICAL-LIFT BRIDGES

1. Early types.—Vertical-lift bridges were developed at a much later period than the bascule, retractile, and swing types. There were only a few small bridges of this type built during the nineteenth century. These were mainly over canals and small navigable streams in cases where it was only necessary to lift the spans a few feet to clear traffic in the channels. The application of the general principles of the type to large structures began in the decade from 1890 to 1900.

Surrey Canal Bridge.—One of the earlier English vertical-lift bridges is shown in elevation and section in Fig. 5A, (*a*) and (*b*).[1] This bridge was across the Surrey Canal, on the London, Brighton, and South Coast Railway. It was designed by Mr. R. Jacomb Hood. The clear span is 22 ft. 6 in., and the towers are 31 ft. 6 in. between centers. The bridge carries a single-track railway in the middle of a 12-ft. roadway. The vertical movement of the span is about 10 ft.

The lift span is of wrought-iron deck-plate girder construction, and the towers are of cast iron. Plate girders join the opposite tower-tops. Counterweights, at each end and on each side of the structure, are connected to chains passing over sheaves on the towers, thence down to attachments to the four corners of the lifting span. Longitudinal shafts connect the two sheaves on the sides of the bridge. At diagonally opposite corners, a large gear is keyed to these shafts and meshes with small pinions driven by man-power by means of ropes laid around large grooved sheaves on the pinion shafts, thence down to the level of the bridge floor. Two men are required to operate the bridge. The cast-iron towers are notched along one edge and engage pawls at the four corners of the span. The lifting span is held securely at any height. The pawls are disengaged when lowering the bridge.

There are other examples of similar early vertical-lift bridges in England, and this design is quite typical of them.

Erie Canal bridges.—During the decade from 1870 to 1880, Mr. Squire Whipple designed, patented, and built several vertical-lift

[1] "Cast and Wrought Iron Bridges and Girders," by William Humber, London, 1857.

bridges along the Erie Canal, in New York. Figure 5B is an outline sketch of a bridge of this type.[2] This bridge was built in 1874.

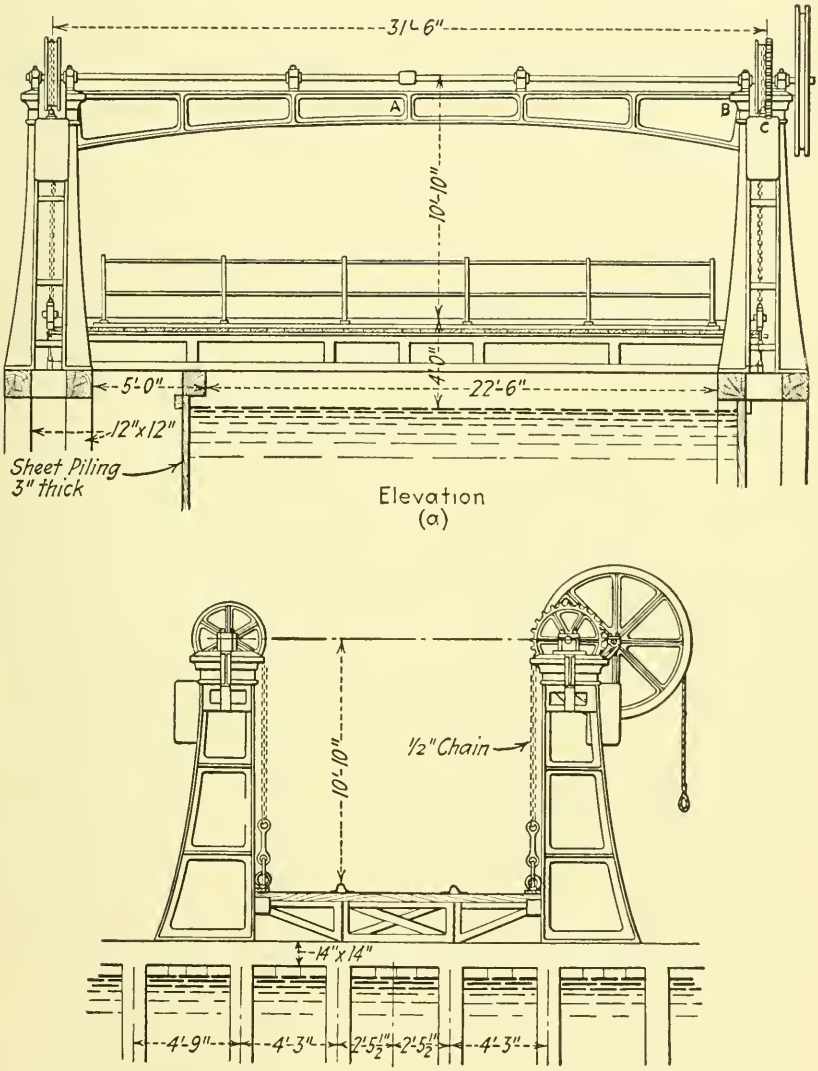
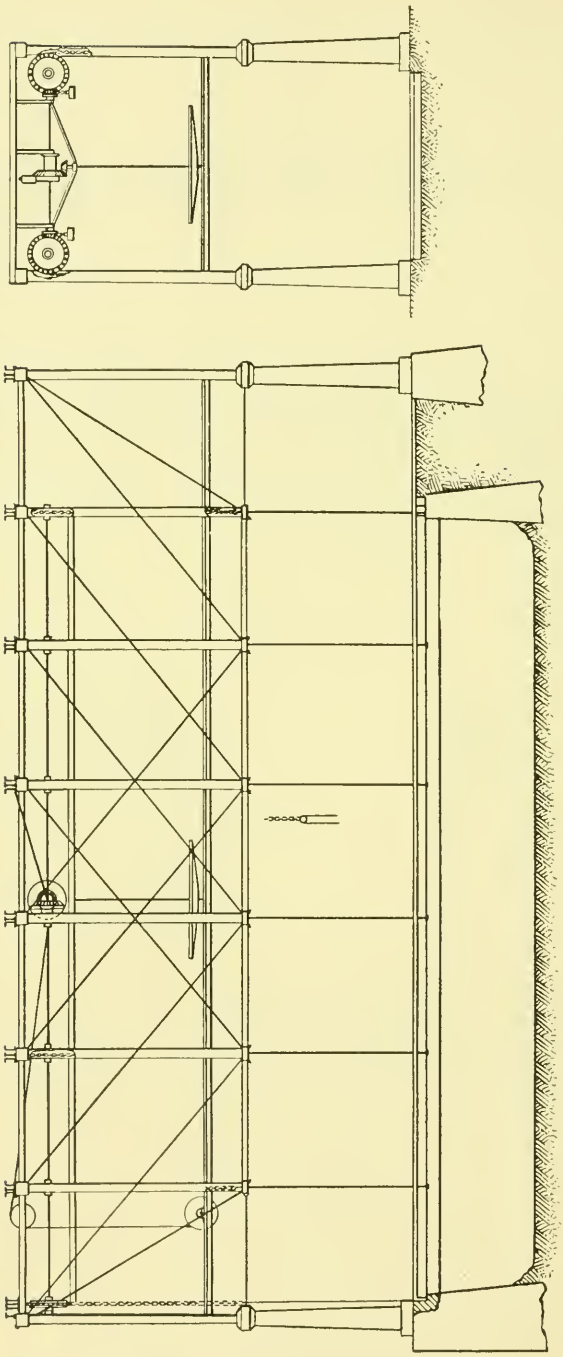


Fig. 5A.

[2] *Trans. Am. Soc. C. E.*, Vol. III, p. 190. The Utica Lift Draw-Bridge. A paper by Squire Whipple, Hon. M. Am. Soc. C. E.



Trans. Am. Soc. C. E.

Fig. 5B.

Two parallel Whipple trusses spanned the canal and the towpath. They were braced by lateral systems and rested on cast-iron posts on the abutments. The roadway platform was an independent iron structure, 60 ft. long and 18 ft. wide, and was suspended at each panel point by rods passing upward from the ends of the floorbeams and telescoping inside of the hollow vertical posts of the trusses overhead. The vertical lift of the roadway was 11.5 ft. Wire ropes, $\frac{3}{4}$ in. in diameter, were attached to the upper ends of the suspender rods and passed over sheaves, 3 ft. in diameter, near the upper chords of the trusses. The other end of each rope carried an iron counterweight, weighing about 1600 lbs., balancing the weight of the roadway deck.

The counterweight sheaves were keyed to longitudinal shafts, of Bessemer steel, $2\frac{1}{2}$ in. in diameter. The tops of the posts were slotted to allow the sheaves to project inwardly to the center of the posts. "The two line-shafts have each a bevel gear-wheel near the center and at opposite points, into which work pinions mounted on the ends of a transverse shaft that serves to connect the two and secure their uniform rotation, as well as that of the sheaves mounted thereon." All journal bearings were babbitted and their caps arranged to hold waste and oil, to secure continuous lubrication.

The method of operating the bridge was characteristically ingenious. It was thought that direct operation by man-power would be too slow. The mechanism was therefore arranged so that two weights could be wound up in advance of an opening of the bridge. One served for opening and the other for closing the span. The weights were designed to afford twice the power required to overcome friction. A large tread wheel was installed in mid-span, between the trusses and above the traffic clearance. The vertical shaft of this wheel carried a pinion engaging a bevel gear on a transverse shaft at the top, and first one and then the other operating weight could be wound up. The mechanism contained pawls engaging ratchet wheels, similar to the arrangement of clock-weights. About one minute was required to wind each weight, and the time of opening or closing was about ten seconds.

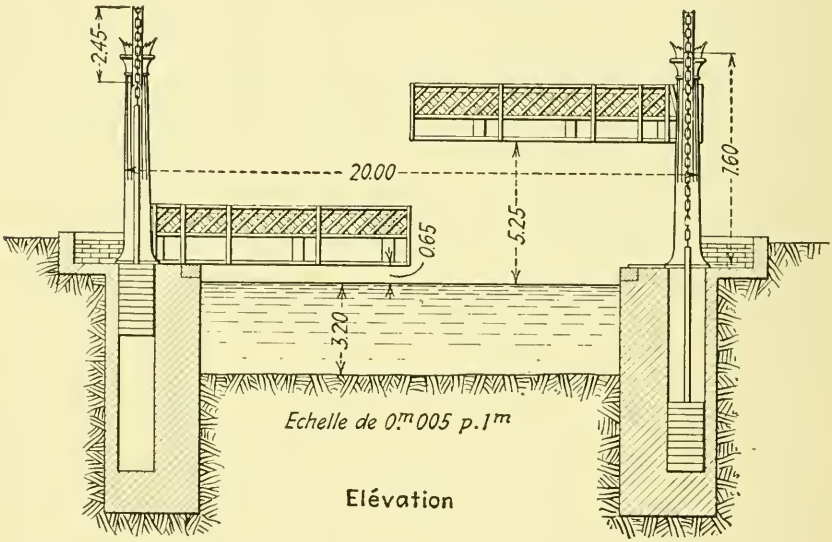
It is interesting to note that this very successful type of lift bridge was designed by Squire Whipple, who was the first American engineer to analyze the stresses in ordinary bridge trusses under passing loads. His able work did much to advance the art of iron-bridge building in the United States.[3]

French bridges.—Several successful vertical-lift bridges were built

[3] "Practical Treatise on Bridge Building," by S. Whipple, C. E. D. Van Nostrand Co., New York, 1873. Revised and enlarged edition, 1899. His original essays were published in 1847.

in France during the nineteenth century. They were mainly canal bridges, as was the case in England and the United States. Figures 5C and 5D show an elevation and section of bridges to carry the Rue de Cremée across the Bassin de la Villette, in Paris, and on the Canal St. Denis.[4] These bridges were designed under the direction of M. L. Le Chatelier, Ingénieur des Ponts et Chaussées.

The movable span is 65.5 ft. long and 26.25 ft. wide. The height of the lift is 15.09 ft. The moving span weighs 93.6 tons. At each



Ann. des Ponts et Chaussées.

Fig. 5C.

corner of the bridge a chain is attached; this chain passes up and over a sheave 8 ft. in diameter and carries a counterweight of cast-iron discs, which moves in a cylindrical well in the abutment masonry. A transverse shaft, extending across each end of the span, has spur gears on its ends, which engage racks on the tower posts. Each shaft has a bevel-gear meshing with its mate on a longitudinal shaft extending the whole length of the span. Thus the four corners of the span are forced to move together.

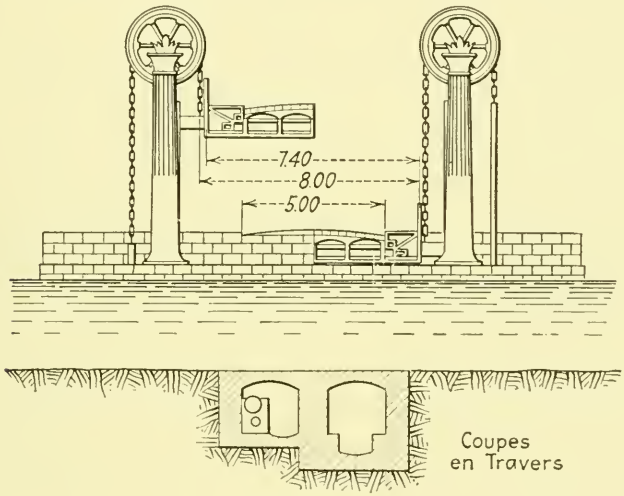
Power is applied by vertical hydraulic plungers in cylinders which are installed in wells in the abutments. These plungers are connected to the span on its longitudinal center line. An ingenious compound valve, in the supply pipe at each end, regulates the flow of water so that

[4] *Annales des Ponts et Chaussées*, 6 Series, Tome XI, 1886.

the movements of the span are always under full control. Both plungers are used for ordinary movements, but the span may be operated by a single ram at either end, whenever it is necessary to repair the other.

French engineers appear to favor hydraulic machinery for operating such movable bridges. Its simplicity and perfect control of all movements are valuable features. Similar mechanism to that of the bridges described above was also used on the bridge at Larrey.[5] The details of the machinery of both of these bridges are worthy of careful study.

2. Halsted Street Lift Bridge.—The first of the large modern vertical-lift bridges to be built was that carrying South Halsted Street,



Am. des Ponts et Chaussées.

Fig. 5D.

in Chicago, across the Chicago River.[6] In 1892 an old swing bridge was knocked down by a colliding vessel. The center pier formed a serious obstruction to navigation, and the engineers of the War department declined to approve a project for rebuilding the bridge as a swing span. It was finally decided to use a vertical-lift bridge in accordance with plans made by Dr. J. A. L. Waddell, M. Am. Soc. C. E., and contracts were let during 1893.

A general elevation and end view is shown in Fig. 5E. The main span is 130 ft. between centers of end bearings. The bridge carries a roadway 34 ft. wide and two sidewalks of 7 ft. each. When it is fully

[5] *Annales des Ponts et Chaussées*, August, 1893. An article by M. Galliot.

[6] *Trans. Am. Soc. C. E.*, Vol. XXXIII, p. 1, 1895. A paper by J. A. L. Waddell, M. Am. Soc. C. E.

open, the clear headway for vessels is 155 ft. above mean low water. At each side of the river is a steel tower 217 ft. high, with vertical front and battered back legs. Screw adjustments are provided in the feet of the rear tower legs, to take up any settlement of the piers. The tower-tops are connected by light longitudinal trusses spanning the channel. They serve to brace the towers, to support idlers for the operating ropes, and to carry footwalks for the bridge tenders.

Eight wire ropes, $1\frac{1}{2}$ in. in diameter, are connected to the trusses at each top corner of the lift span. They pass upward and over eight grooved sheaves, 12 ft. in diameter, on the tower-tops, thence to connections with the four cast-iron counterweights, which move vertically within the towers. The counterweight ropes are in sixteen pairs. Their ends are brought together at the span connection-pins and clamped. The bights of the ropes pass around 15-in. sheaves in the counterweight connections, to provide for movements due to unequal stretch of the ropes. Each sheave carries four ropes. The span moves between guides on the tower posts, and antifriction rollers are used at the points of contact. The counterweights are guided by vertical angles. The weight of the counterweight ropes is balanced by iron chains with one end attached to the bottom of the span and the other to the bottom of the counterweight. Four hydraulic buffers are attached to the top of the towers and four to the span under its ends, to assist in bringing the bridge gently to rest at its open and closed positions.

A steam-power plant is installed under one of the bridge approaches. Two 70-h.p. reversing steam engines drive an 8-in. transverse shaft through gear trains. On this shaft are fitted four 6-ft. spirally grooved drums. The ends of one set of the $\frac{7}{8}$-in. wire operating ropes are attached to the top corners of the span and the ends of another set to the counterweights. The other ends are fastened to the drums so that when they rotate in one direction the bridge-lifting ropes are wound on the drums and the counterweight-operating ropes are paid out, lifting the span. In lowering the bridge, the direction of drum-rotation is reversed, the counterweights being lifted and the operating ropes of the span being paid out. The ropes are in pairs, eight ropes for raising, and eight for lowering the span. As all of the ropes pass over the sheaves at the tops of the towers, and one-half of them across the channel to the end of the span opposite the machinery, there are about 8000 ft. of operating ropes on the bridge. The maintenance of these ropes and their sheaves and bearings is one of the disadvantages of this method of operation. Later designs by the same engineer have simplified the arrangement and reduced the length of the operating ropes.

The bridge was completed in 1894 and has given satisfactory service.

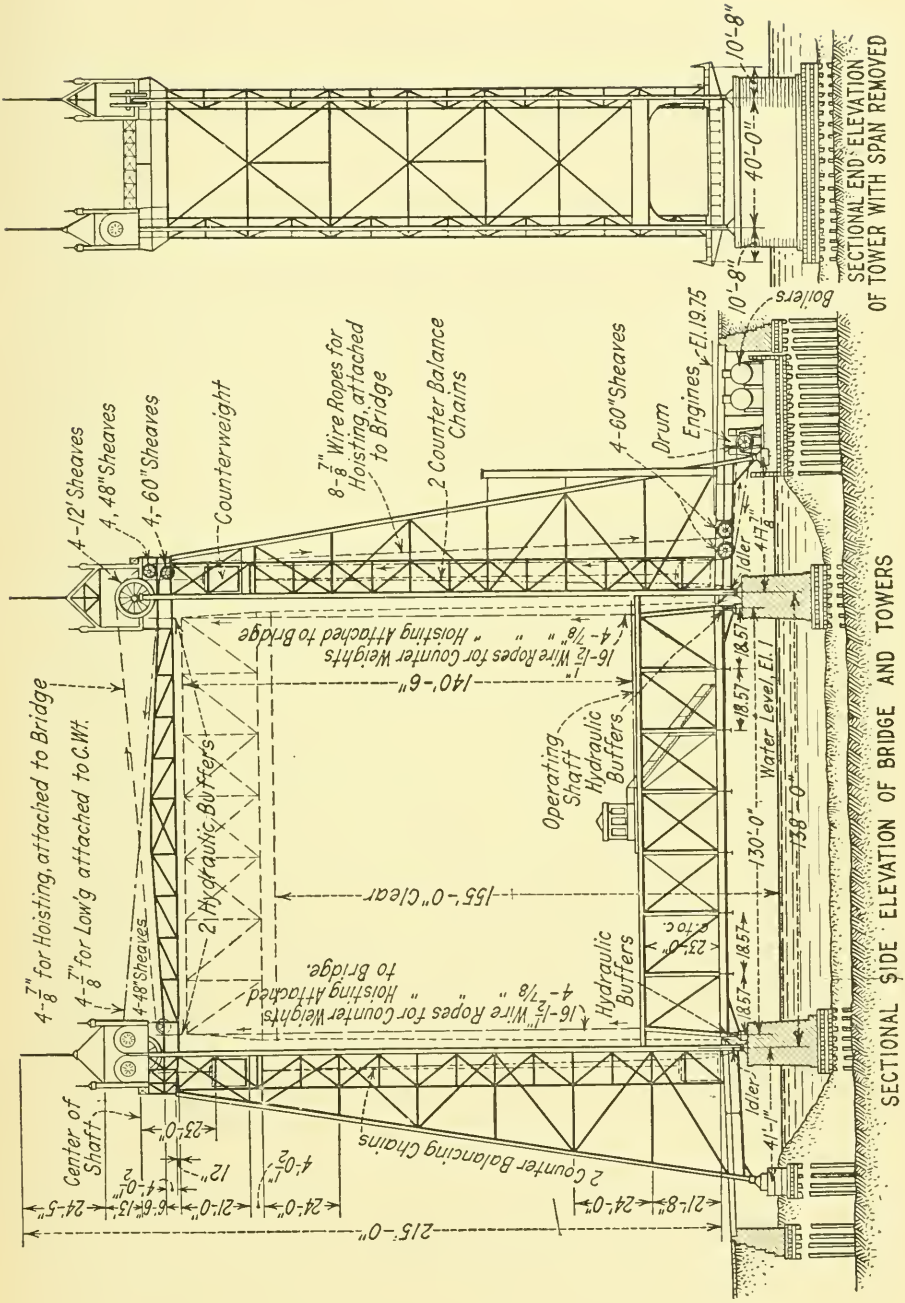

SECTIONAL END ELEVATION OF TOWER WITH SPAN REMOVED
SECTIONAL SIDE ELEVATION OF BRIDGE AND TOWERS
4-12" Sheaves
4, 48" Sheaves
4-60" Sheaves
Counterweight
8-7/8" Wire Ropes for Hoisting, attached to Bridge
2 Counter Balance Chains
4-60" Sheaves
Drum
Engines
El. 19.75
Boilers
10'-8"
40'-0"
Idler
4-7/8" for Hoisting, attached to Bridge
4-7/8" for Low'g attached to C.Wt.
4-48" Sheaves
2 Hydraulic Buffers
16-1 1/2" Wire Ropes for Counter Weights
4-7/8" " " Hoisting Attached to Bridge.
140'-6"
Operating Shaft
Hydraulic Buffers
155'-0" Clear
Water Level, El.
130'-0"
136'-0"
18.57
23'-0" c. to c.
Center of Shaft
23'-0"
12"
4'-0"
21'-0"
24'-0"
215'-0"
2 Counter Balancing Chains
41'-1"

FIG. 5E.

3. Waddell type.—Dr. J. A. L. Waddell and the former firm of Waddell & Harrington have simplified and improved the design of vertical-lift bridges. Many of the features of their later designs are covered by United States patents. The type of design now preferred and advocated by Dr. Waddell is shown in sketch form in Fig. 5F. The various parts are noted. This design differs, in some respects, from those of other engineers, and a few features are characteristic.

The operating machinery and operator's house are at the middle of the lift span, and move with it. The counterweights must be increased to balance their weight, and this increased weight augments the trunnion friction and adds to the power required for operation.

Ordinary gear trains connect the motors with a transverse shaft, geared to drums in, or near, the planes of the main trusses. Each drum is spirally grooved and has two operating ropes. These ropes are anchored to the drums and extend to the end corners of the span, over or under idler sheaves, and the ends of one set are anchored to the top and those of the other set to the bottom of the vertical front legs of the towers. Each rope is provided with an adjusting device near its anchorage. Rotation of the drums in one direction winds the up-haul ropes and pays out the down-haul ropes, lifting the span; reversal of the direction of rotation of the drums pulls the span down. This arrangement of operating ropes causes the span to move parallel to its closed position throughout its movement.

The counterweights are usually of ordinary concrete. In rare cases, concrete loaded with scrap iron or steel punchings may be needed when the clearances are small.

Wire ropes connect the counterweights to the span, and pass over large grooved sheaves on the towers. When the span and the counterweights are heavy, the design of these sheaves, their bearings, and their supports is a real engineering problem. The size and number of ropes require sheaves of large diameter and considerable length, and in some cases it has not been feasible to cast such sheaves as single pieces. Various designs of sheaves built up from structural steel with cast rim segments, and sometimes with cast hubs, have been used, and they have not always been satisfactory. Special attention should be directed to this detail, which is one of the most important in this type of structure.

The counterweight ropes do not always stretch uniformly under load and movement, and therefore equalizers should be used at one end of the ropes. Triangular plates with top pin attachments for two ropes, and with a single pin attachment at the bottom apex of the triangle, have been quite generally used. While these pieces can

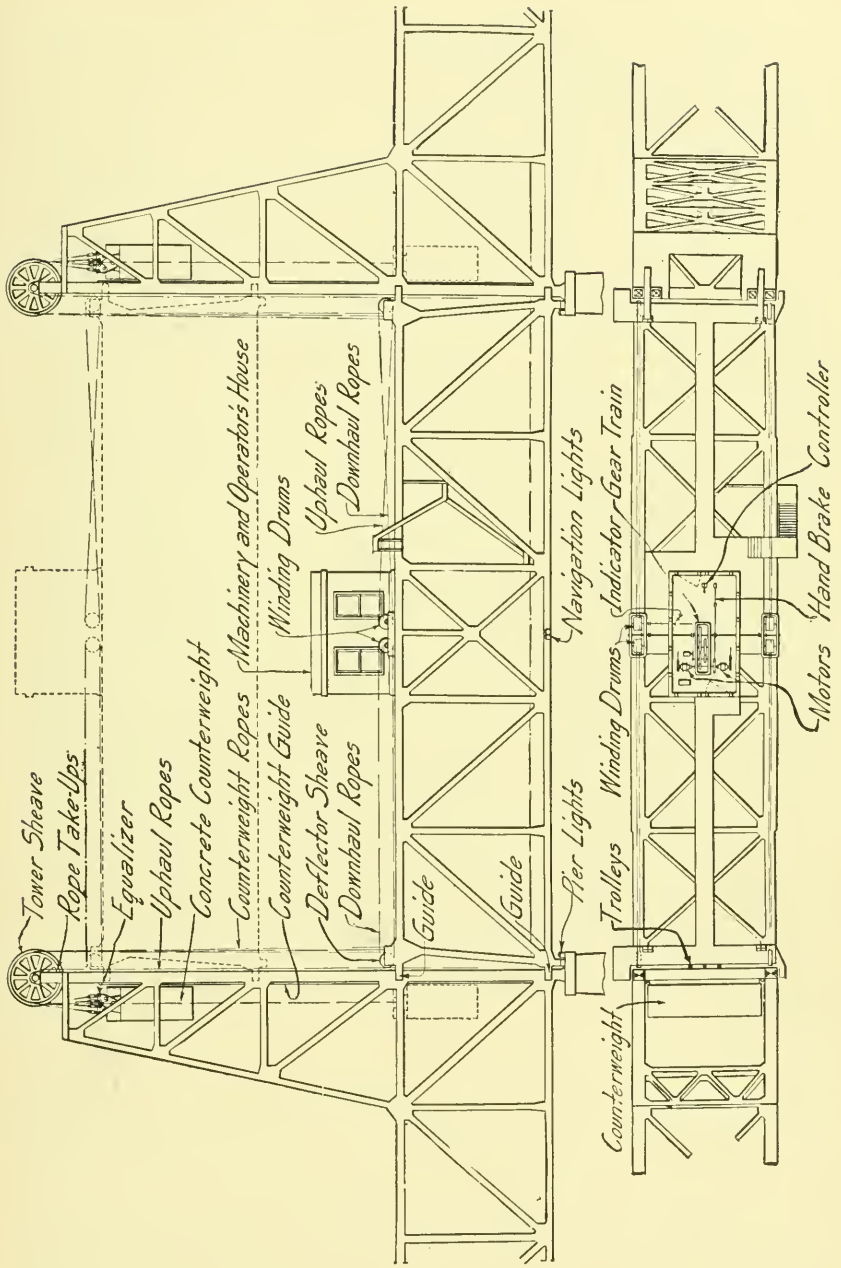

Tower Sheave
Rope Take-Ups
Equalizer
Uphaul Ropes
Concrete Counterweight
Counterweight Ropes
Counterweight Guide
Deflector Sheave
Downhaul Ropes
Machinery and Operator's House
Winding Drums
Uphaul Ropes
Downhaul Ropes
Guide
Guide
Navigation Lights
Pier Lights
Trolleys
Winding Drums
Indicator-Gear Train
Hand Brake
Controller
Motors
Counterweight

Fig. 5F.

rotate when the length of the two ropes differs, they do not equalize the loads on the ropes, for any rotation about the lower pin increases the lever arm of the longer rope and diminishes that of the shorter.

The counterweight ropes should not all be carried by a single sheave at each corner. When, for any reason, repairs or renewals of the sheaves, trunnions, or bearings become necessary, it is a difficult and expensive operation to make the changes and maintain the bridge in service. Two, three, or more sheaves can be used at each corner, the ropes passing over them in small groups. When properly designed, any single sheave with its trunnions, bearings, and ropes may be removed and repaired or replaced without stopping operation of the bridge. The remaining parts would temporarily be over-stressed, but the unit stresses used in proportioning most structures are so small that the over-stress would not be serious. Another advantage secured by the use of multiple sheaves is the reduced diameter and length of the trunnion bearings, with the resulting reduction of power required to turn them.

Typical small vertical-lift bridge.—Small vertical-lift bridges may be of simple construction. This is illustrated by Fig. 5G, which shows the general design of the Big Choctaw Bayou bridge on the line of Morgan's Louisiana and Texas Railroad.[7] It is a single-track structure. The draw is an ordinary deck-plate girder span 50 ft. long. The clear channel is 46 ft. The bottom of the girders is 4 ft. above high water and the lift is 47 ft. At each end of the draw there is a cross girder 18 ft. 9 in. long, carrying the drums for the operating cables. These drums are in line with the tower posts. Each tower consists of two columns 16 ft. 3 in. between centers, and 67 ft. 6 in. high. They are of H section, made up of plates and angles with Z-bar guides added to the outer flanges. Stability of the towers is secured by portal bracing with lattice girders at the top. Two longitudinal lattice girders 8 ft. apart, with laterals, connect the two towers.

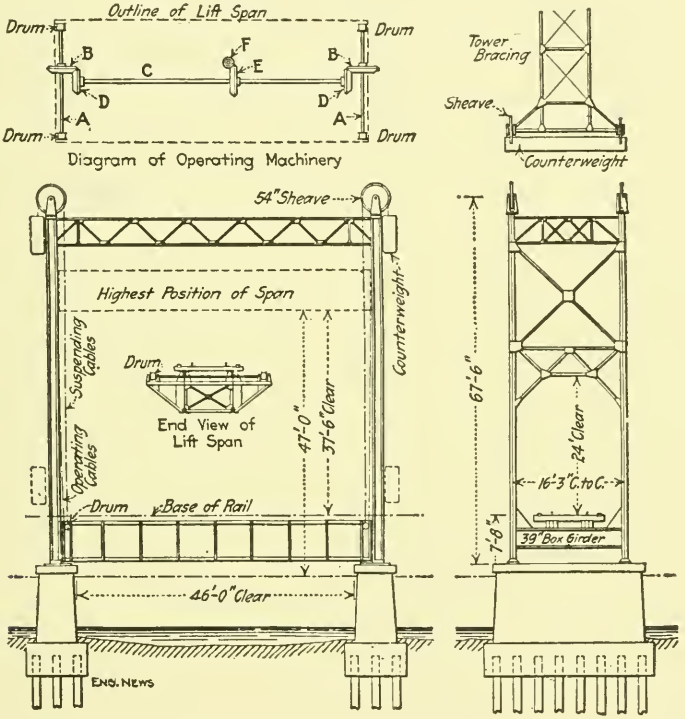
The counterweights are of concrete weighing 145 lbs. per cubic foot, built around steel frames. Loose concrete blocks provide for the exact balancing of the draw span.

A double-grooved cast-steel sheave, of 54-in. pitch diameter, is mounted on the top of each tower column. Two 1-in. wire ropes pass over each sheave, one end being attached to the end girders of the draw and the other to the counterweights. Adjustment and equalizers are used at the counterweight ends of the ropes.

The bridge is operated by man-power. A capstan, at the middle of the lift span, is attached to a vertical shaft, which drives a longitudinal shaft by bevel gears. This shaft drives a transverse shaft at

[7] *Eng. News*, 1912, II, p. 99.

each end of the span by the same means. Four 18-in. spirally grooved drums, each carrying two $\frac{1}{2}$-in. wire operating ropes, are keyed to the ends of the shafts. Each rope has one end anchored to the drum, and the other end of the lifting cable is attached to the top of the tower column. Similarly, one end of the lowering cable is attached to the bottom of the column. When the capstan is turned in one direction the bridge is raised, and when in the other it is lowered. The bridge can be opened or closed in ten minutes.



Eng.-News.

Fig. 5G.

When it is closed, hook-shaped end locks automatically engage lugs on the towers. These are unlocked by wire ropes connected to a hand lever on the span.

This bridge was built in 1911.

Bridge over the Ohio River at Louisville, Ky.—Figure 5H shows a typical Waddell vertical-lift bridge in closed position. This bridge was built for the Pennsylvania Lines West of Pittsburgh, and crosses the Louisville and Portland Canal, which is adjacent to the Ohio River

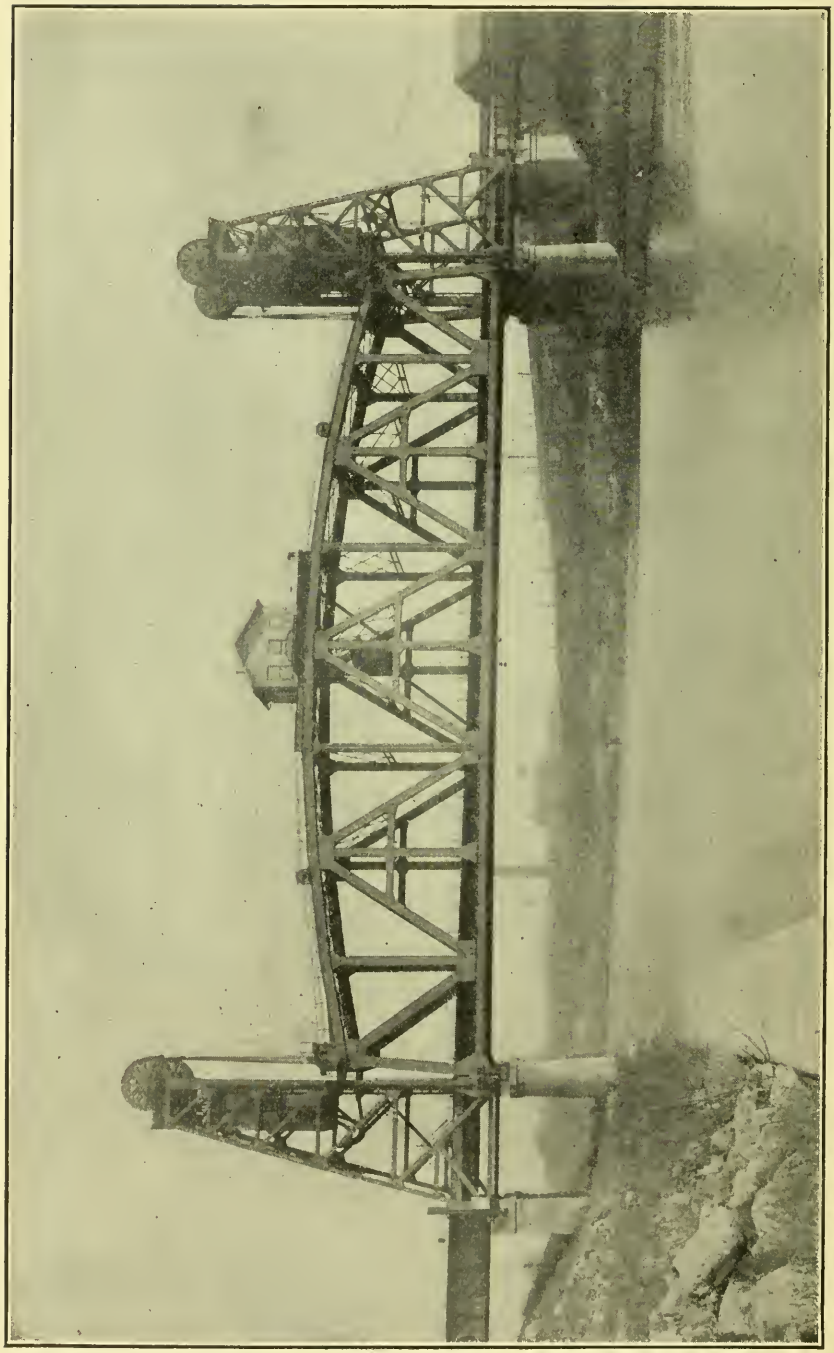


Fig. 5H.

at Louisville, Ky.[8] It is a double-track structure, 260 ft. between centers of end bearings. The railway crosses the canal at such an elevation that a lift of 32.4 ft. is sufficient. The centers of the counterweight sheaves are 92 ft. above the base of the rails on the bridge.

The lift span was proportioned in accordance with the specifications of the Pennsylvania Lines West of Pittsburgh, for live loads based upon 66,000-lb. loads on the driven axles of the locomotives. It weighs about 3,000,000 lbs., complete as operated.

There are sixty-four counterweight ropes $2\frac{1}{8}$ in. in diameter, passing over the four 15-ft. sheaves. These sheaves are built up of steel plates, shapes, and castings, and weigh about 38 tons each. The rim segments have $\frac{1}{2}$-in. spaces between their ends, and each segment is fully riveted to the webs to provide for the entire load from the ropes. The web and reinforcing plates of the hub bear directly on the trunnion shafts, to which they are keyed by three keys anchored in position by set-screws. The trunnion journals are $22\frac{1}{2}$ in. in diameter and 24 in. long. The bearings are lined with hard phosphor-bronze. Oil grooves are cut in these bushings, the lubricant being supplied from marine-type, screw-feed, compression grease cups.

The power plant is in a house in the middle and on top of the lift span. The motive power consists of two 150-h.p., 220-volt, 3-phase, 60-cycle alternating-current motors, running at 580 r.p.m. Each motor is equipped with a solenoid brake. The arrangement of gearing, shafts, and drums is about the same as shown on Fig. 5F.

There are 16 plow-steel operating ropes, each 1 in. in diameter, the drums and sheaves over which they run being 36 ins. in diameter. These ropes work in pairs, i.e., there are two down-haul ropes at each corner of the span. The take-up devices for the ropes are eyebolts threaded over the entire length, with anchorage attachments at top and bottom of towers.

The rail locks are of sliding-tongue type, standard with the Pennsylvania Lines. The four tongues at each end of the span are driven by a 5-h.p. motor. Limit switches are provided to cut off the current at each end of the travel. The controllers for the rail-lock motors are interlocked with the signal system so that the locks cannot be opened until the signals are set against train movements over the bridge, and so that clear signals for train operation cannot be given until the locks are closed. The controllers are also interlocked with those for the main operating motors so that current cannot be supplied to the latter until the locks have been opened, and so that the locks cannot be closed until the bridge has been seated.

The span is kept in correct position during motion by guide rollers, which roll on vertical guides on the outsides of the tower columns.

[8] *Trans. Am. Soc. M. E.*, Vol. 40, p. 1035. Discussion of a paper by H. P. Van Cleve, M. Am. Soc. M. E., by Dr. J. A. L. Waddell.

There are eight rollers for transverse guiding, one at each L_0 point and one at each U_0 point. Longitudinal guiding is effected by two rollers at each L_0 point at the fixed end of the span. There is considerable play in the guides so as to eliminate any possibility of binding. On account of this play they do not center the span closely enough for the rail locks, which have very little play. For this reason there is placed a transverse centering casting, having very little play, at the middle of each end floor beam.

The train thrust is cared for by two thrust castings, one at each L_0 point at the fixed end of the span.

In order to eliminate jar when the span seats, there are provided air buffers near each end of each of the end floor beams. Adjustable needle valves on the exhaust ports of the buffers enable the resistance of the said buffers to be varied at will.

Bridge locks were not used, but the counterweights were made about six tons lighter than the span, the excess weight of the latter overcoming any tendency for it to rise.

As was stated previously, each motor is equipped with a solenoid brake. In addition there are two other brakes, one operated by hand and the other by motor. The lever of the hand brake is in the machinery house on the span, and gives the operator graduated braking power, so that he can stop the moving mass without jar. The controller of the motor-operated brake, which has three degrees of braking power, is located in the interlocking tower.

The time of raising or lowering the lift span is about 45 seconds.

Hand operation is provided for by two four-arm capstans.

Willamette River double-deck bridge at Portland, Ore.—The most unusual vertical-lift bridge is the double-track, double-deck structure across the Willamette River, at Portland, Ore. It was built for the Oregon-Washington Railway and Navigation Co., and was finished during 1912. Figure 5 I shows the general arrangement.

The new bridge replaced a drum-bearing swing span, designed by the late George S. Morison and built in 1888. This was the first steel bridge across the Willamette River and was proportioned for the heaviest loads of the time, but for several years previous to 1910 had been much overloaded. The railway approaches were also inconvenient, as it was possible to turn in only one direction at the east end. The old bridge, like the new, was a double-deck structure, providing for a double-track railway on the lower deck and for a highway on the upper. The new bridge is located about 600 ft. above the old, at a site where there is room for a direct turn in either direction at the east end; but it was necessary to build the east fixed span with diverging trusses and to use 16° curves to accomplish this result.

The river portion of the new bridge consists of two fixed spans, each 287 ft. between end bearings, and a central lift span of 220 ft., all Pratt trusses.

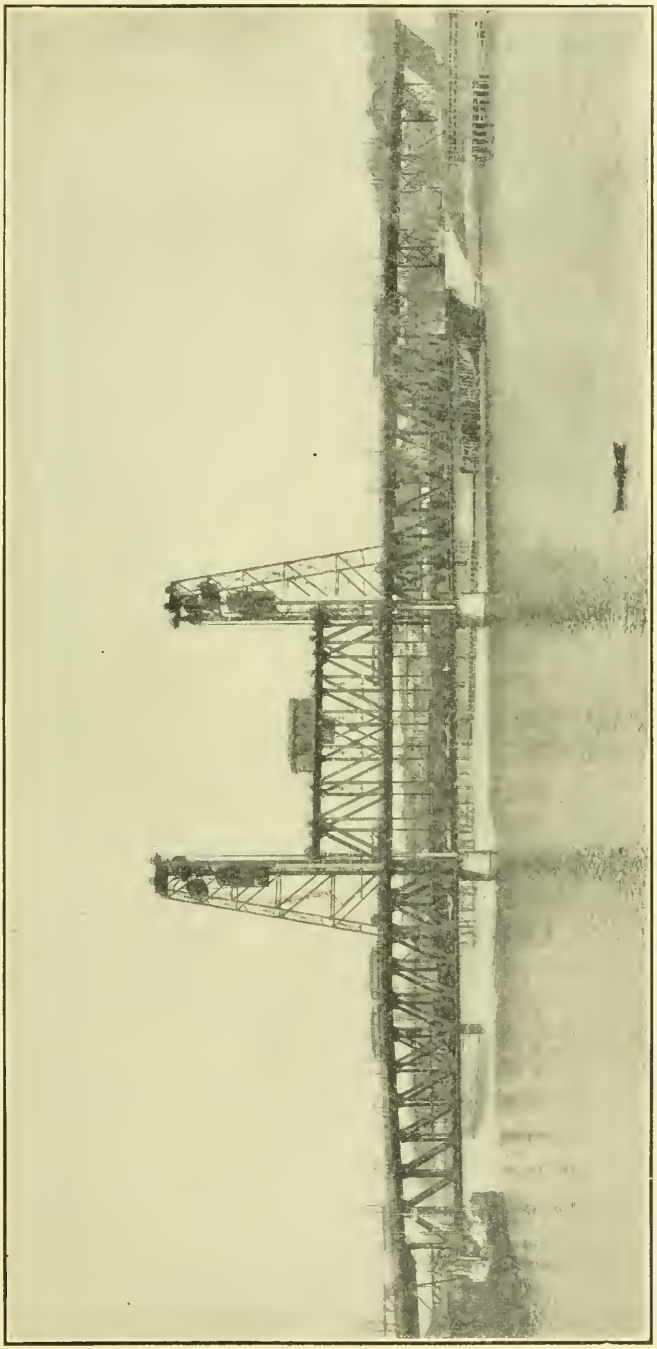


Fig. 51.

The lower deck has a clearance of 26 ft. above low and 5 ft. above high water. The upper deck is 52.5 ft. above this. The lower deck can be lifted 46 ft. without moving the main lift span. Both the lower and upper decks may then be lifted a further distance of 93 ft. The total clearance is then 144 ft. above high and 165 ft. above low water. This clears the highest-masted vessels that enter Portland. As a rule, it is only necessary to lift both decks from four to six times daily, most vessels passing under the upper deck when the lower deck is lifted.

The lower deck was designed for double-track railway traffic in accordance with Common Standard Specifications 1006 of the Railway Co., and the upper for heavy city traffic. The upper deck of the lift span provides a central roadway of 28 ft., for a double-track street railway and heavy trucks; an 11-ft. roadway on each side for horse-drawn vehicles; and a 6-ft. clear sidewalk on each side, outside of the roadways.

The vertical-lift span is the heaviest yet built. The estimated weight, fully equipped, is 3,800,000 lbs. for the upper deck and 975,000 lbs. for the lower, a total of 4,775,000 lbs.

The vertical hangers of the lower deck telescope within the vertical posts of the truss above, and each panel point is separately counterweighted. The counterweight ropes pass up within the truss posts, over and under deflecting sheaves on the top chords, thence to the tops of the towers, over a pair of sheaves and down to the separate panel-point counterweights, of which there are four in each tower, located behind the large main counterweights.

The upper deck span has end, top, transverse box girders, in which the main counterweight ropes are anchored with adjustable attachments. At each corner of the span there are sixteen Monitor steel ropes, $2\frac{1}{4}$ in. in diameter. They pass upward nearly to the tops of the towers and over grooved sheaves, of 14-ft. pitch diameter, thence downward to equalizers which are connected to the steel counterweight frames by pins. The equalizers are triangular, with the apex downward, and fully equalize the stresses in the ropes only when the ropes are all of exactly the same length.

The main counterweights are of ordinary concrete, built around steel frames, and weigh about 1,900,000 lbs. each. They move between angle guides attached inside of the towers.

The front columns of the towers are box sections with flanges arranged to guide the moving span.

The operating machinery is in a house at the middle and on top of the lift span. The operator's room is suspended under the middle of this house, so that he can observe the main deck and the river traffic.

The machinery for operating the lower deck is driven by two 200-h.p. motors placed on the downstream side of the house, and that for the upper deck by two 200-h.p. motors on the upstream side. By means of a common shaft, either set of motors can be made to operate either deck. The general arrangement of the machinery that operates the upper deck is similar to that of the typical design described earlier in this article. A train, of three pairs of reduction gearing, connects the motors to the main transverse drum shaft. The operating ropes are $1\frac{1}{8}$ in. in diameter. They are in pairs and pass from the drums to the ends of the span, under or over deflecting sheaves, and thence to anchorages near the tops and bottoms of the towers. Rotating the drums in one direction lifts the span, and rotating them in the other direction lowers it.

The motors for operating the lower deck are connected by bevel gears to $5\frac{3}{4}$-in. longitudinal shafts extending to the ends of the span. Bevel gears connect them to end transverse shafts having spur pinions on their ends which mesh with gears on grooved drums. The panel-point operating ropes pass under these drums, up the towers, over pairs of top sheaves, and down to the respective counterweights. Their other ends pass from under the driving drums over deflecting sheaves at the various panel points and down inside of the truss posts to connections with the lower deck hangers. Rotation in one direction lifts the lower deck, and rotation in the opposite direction lowers it.

The mechanism was built to lock down automatically each vertical hanger of the lower deck, thus providing against continuous-girder deflections of the railway deck. On account of its complex character, small parts, and close clearances within the vertical posts of the upper-deck trusses, this mechanism has never operated satisfactorily and is not used at the present time. The lower deck also has four end locks, driven by motors on the piers, and controlled from the operator's room. When closed, the upper deck is automatically locked down at each end and is released by the operator by turning a wheel on a vertical stem.

Electric current is supplied to the motors on the lift span by two conductors, about 3 ft. apart, on one of the fixed spans and its tower, on which run short trolley arms attached to the end of the lift span.

The erection of this bridge was difficult, as river traffic had to be maintained at all times. The current in the river was too rapid for erection by floating. The side spans were placed on ordinary wooden falsework. The lift span was erected on wooden cantilever frames, extending outward from the piers and supporting a wooden Howe truss channel span 120 ft. long. A description of the erection methods is

given in an article by Mr. W. P. Hardesty, in *Engineering News*, 1912, II, p. 1100.

Piscataqua River highway bridge at Portsmouth, N. H.—The Piscataqua River bridge, at Portsmouth, N. H., is among the most typical vertical-lift highway structures. Figure 5J shows the bridge with the draw lifted. As the water is deep, the foundations difficult, and the navigation requirements severe, it proved economical to build the bridge with three equal spans of 297 ft. 1 in. between centers of end bearings, the central span being a vertical-lift draw.

The centers of the counterweight sheaves on the towers are 176.33 ft. above the elevation of the roadway. Their front legs are supported by the piers, and the rear legs on the trusses of the fixed spans at the second upper chord panel points. The span can be lifted 129 ft., affording an overhead clearance of 150 ft. for navigation. There is one roadway 28 ft. wide, and the two sidewalks are each 6 ft. in clear width.

The bridge was designed for 20-ton trucks or 100 lbs. per square foot on the floor, and the trusses for a system of uniform loads corresponding to 94 lbs. per square foot on the roadway. The specifications were those given by Dr. J. A. L. Waddell in "Bridge Engineering," Vol. II.

The fixed spans have sheet asphalt pavements on reinforced concrete slabs, but on the lift span the asphalt is laid directly on 3-in. planks, supported by transverse wooden ties on the steel stringers.

The estimated weight of the lift span, as operated, is 1,710,000 lbs. The counterweights are of ordinary concrete in steel boxes. Sixty-four Monitor plow-steel ropes, $1\frac{5}{8}$ in. in diameter, connect the counterweights to the bridge, sixteen at each corner of the lift span. Each counterweight sheave is a single steel casting, and they are 10.5 ft. in pitch diameter. The main trunnion journals are $15\frac{1}{2}$ in. in diameter and $19\frac{1}{2}$ in. long, fitted with the usual hard phosphor-bronze bushings, in which are cut suitable grooves for the lubricant.

The operating machinery is in a house at the middle of the span, supported by girders between the posts of the trusses below the top chords. The arrangement of the machinery and operating ropes is practically the same as that shown by the typical plan, Fig. 5F. The power plant consists of two motors of 100 h.p. each. The electric supply is 3-phase, 60-cycle, 440-volt, alternating-current.

The water under the lift span is deep and the bed rock is covered by only a thin overburden of earth and mud. The tidal current is rapid and it was necessary to maintain navigation. On account of these conditions, all three spans were erected on falsework near the bridge site and then floated into position at slack high tide. This operation was carried out without difficulty or delays.

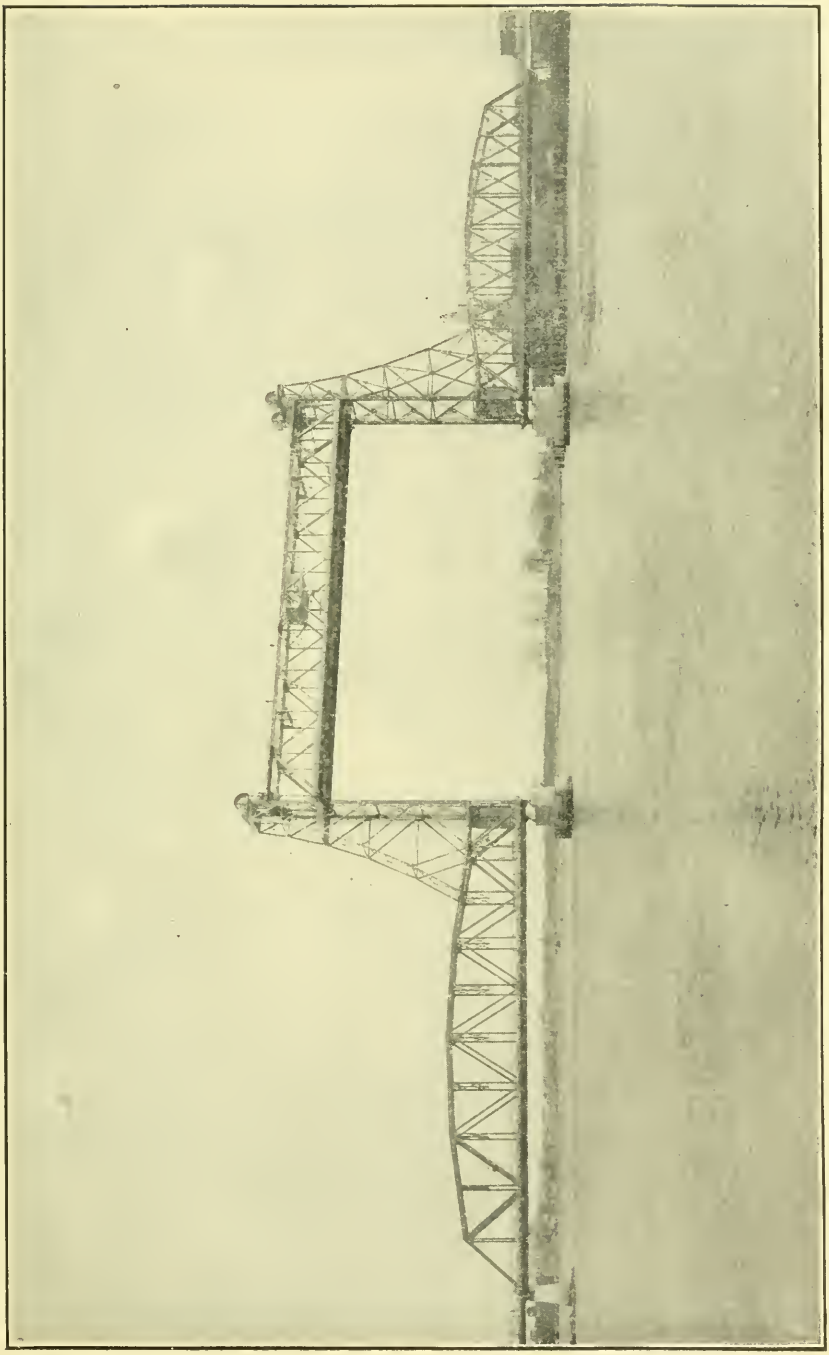


FIG. 5J.

The approximate quantities are as follows:

Classification	Structural Steel, Pounds	Machinery, Pounds	Machinery to Structural, Per Cent
One lift span	1,125,000		
Two towers	775,000		
Sheaves, trunnions, and bearings	.....	100,000	5.26
Wire ropes and fittings	.....	60,000	3.16
Equalizers	.....	20,000	1.05
Operating machinery	.....	60,000	3.16
Totals	1,900,000	240,000	12.63
Total machinery	240,000		
Grand total	2,160,000		

The bridge was opened for traffic on Saturday, August 18, 1923.

4. Strauss type.—Figure 5K diagrammatically represents the Strauss type of vertical-lift bridge. In this system of construction the use of wire ropes and sheaves is avoided, but twenty auxiliary pin joints are introduced in addition to the four main trunnions. This type is suitable only where the vertical lift for clearance required above the water is relatively small. The lift span S is connected to a counterbalancing mechanism, mounted on the two towers R. The method of counterbalancing is described by the Strauss Bascule Bridge Co., as follows:

The counterbalancing device at each end comprises a truss F mounted on the main trunnion at the top of post R, one end being pivotally connected to the hip of the main span S, through the agency of the hanger H, and carries at its other extremity two independent concrete counterweights. The larger counterweight, above, is pivotally connected to F at the base, and at the top is pivotally connected to the upper extremity of H by means of the member BD, termed the counterweight link L. The pivotal points $ABCD$ form a parallelogram and the point T is located in the side AC of this parallelogram. The secondary counterweight is rigidly connected to F and its only function is to bring the center of gravity of the truss F into the fulcrum T, so that F within itself is always in equilibrium about T.

Considering the bridge in a partially open position (shown at the right of the diagram), the forces acting vertically to the left of the fulcrum T, which must be counterbalanced, are:

P_1 = weight of one-half of the main span S, applied at E.

P_2 = weight of the hanger H, applied at its center of gravity h;

P_3 = weight of one-half of the counterweight link, applied at B.

Consider the three vertical forces P_1, P_2, and P_3 replaced by a single force P, equal in magnitude and applied at G, so located that the moment of P about T will equal the summation of the moments of P_1, P_2, and P_3 about T, and it then remains to locate a vertical force at a point on the side CD of the parallelogram $ABCD$, whose moment about T will be opposite and equal thereto. To find the point on CD where this counterbalancing force should be applied, draw a straight line from point G through T until it intersects the line CD, which point is marked g; however, there is a vertical force W_2, being one-half the weight of L, applied at D, and therefore a vertical force W_1 must be applied at g^1, so located on CD that the resultant force W, of W_1 and W_2 will fall at point g, and of such magnitude that the moment of W about T will be equal and opposite to the moment of P about T. The center of gravity of the concrete counterweight is therefore located at g^1. The similar triangles TAG and TgC remain similar during the operation of the bridge by virtue of the parallelogram $ABCD$ and, therefore, the horizontal moment arms x^1 and y^1 of the constant vertical

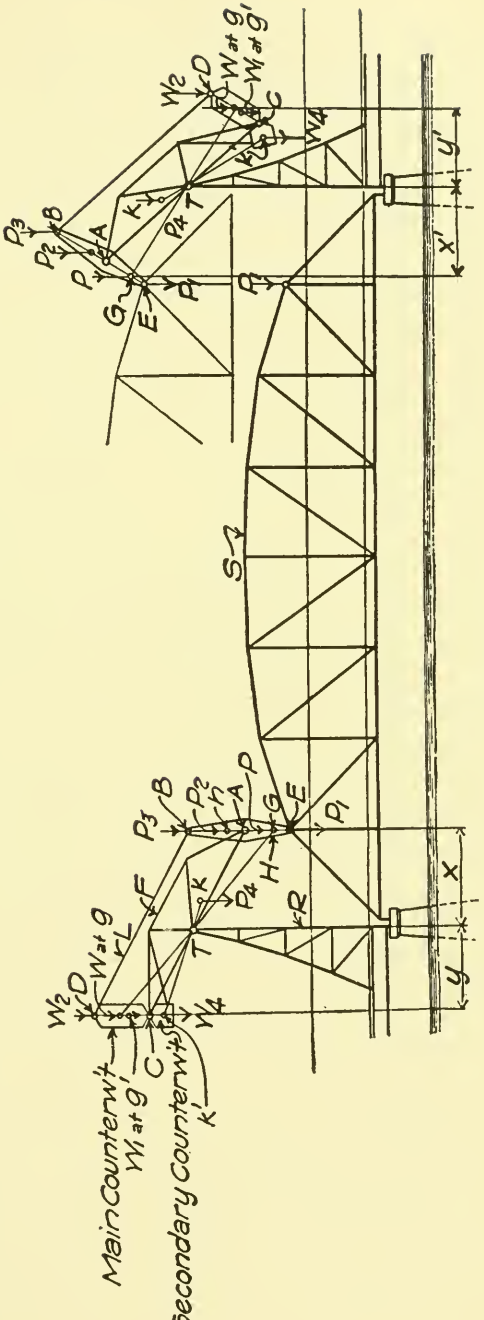


FIG. 5K.

forces P and W will always bear the same relation to each other and the bridge is in perfect balance for any position.

The operating machinery is usually in duplicate at each end of the lift span. A motor drives a transverse shaft on which are mounted three bevel gears. Two of these are bevel pinions driving short longitudinal counter shafts. These carry spur pinions which drive a second set of longitudinal shafts, on which are the main pinions, meshing with vertical racks on the tower posts. The third bevel gear on the transverse shaft connects with the hand-operating train, terminating in an ordinary capstan. To insure uniform motion at both ends of the span, two of the counter shafts are connected by a longitudinal equalizer shaft. On this is a motor-driven emergency brake. Each motor should be supplied with a solenoid brake.

Bridge across the Louisville and Portland Canal at Louisville, Ky.—The direct-lift highway bridge, built for the United States Government, across the Louisville and Portland Canal, at 18th Street, Louisville, Ky., is a typical example of this system of construction. The lift span is 210 ft. long and is flanked by approach spans of 85 and 75 ft., of nearly triangular shape in elevation. They serve also as towers for the counterbalancing mechanism. The roadway is 16 ft. wide. The vertical movement of the lift span is 40 ft., which gives 90 ft. clearance above normal water stage.

This bridge was finished in 1915.

Rideau Canal bridge at Ottawa.—The Praetoria Avenue Bridge across the Rideau Canal, at Ottawa, is an ingenious adaptation of the type. The counterbalancing and operating mechanisms are below the bridge floor. The counterweight trusses are placed at right angles to the longitudinal center line of the bridge, thus utilizing the piers to support and conceal the counterweights and mechanism. The lift span is 88 ft. long, and the entire length, including the approaches, is 200 ft. The movement of the vertical lift is 20 ft.

This bridge was completed during 1918.

5. Strobel type.—Figure 5L illustrates a method of balancing a vertical-lift draw span without cables, as designed by the Strobel Steel Construction Co., of Chicago. The principles are similar to those of the Rall bascule design. Referring first to Fig. 5L (*a*), a pair of counterweight trusses is mounted on each of the two towers. Each counterweight A is proportioned to balance one-half of the weight of the lift span, when in closed position. At the end opposite the counterweight the truss is articulated to the top chord at a panel point of the bridge truss at B. The trunnion C turns in a roller adapted to move along a track on top of the tower. As the draw span is lifted, the trun-

nion rolls back, permitting point B to move in a vertical line. Thus, the counterweight A and the pin B each move in an arc of a circle about the movable trunnion C, and an exact balance is maintained throughout the vertical movement of the lift span.

When a new bridge is to be built, Fig. (*a*) shows the preferred arrangement. In some cases it becomes necessary to make an existing fixed span movable. Figure 5L (*b*) shows an adaptation of the principle which accomplishes this result. The counterweight trusses turn about fixed trunnions D. A roller encloses the pin at G. An auxiliary member EG is added to the upper chord of the span, with a horizontal track on its lower side along which the rollers G may travel. This permits the point G to rotate about D as a center, and at the same time the span can move vertically upward, and an exact balance is maintained.

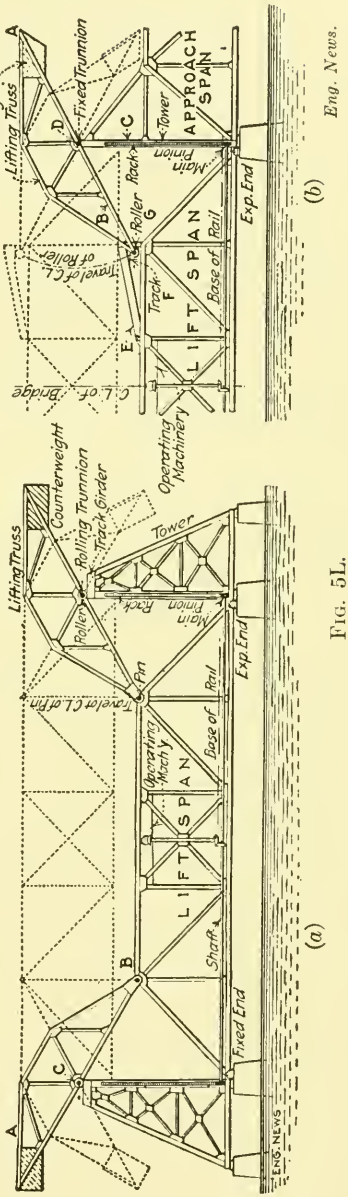


FIG. 5L.

Illinois River Bridge at La Salle.[9] —The Chicago, Burlington, and Quincy Railroad bridge over the Illinois River at La Salle, Ill., as originally built, consisted of five 148-ft. pin-connected truss spans. It is a single-track Pratt structure, each truss having six equal panels. The United States Government required that a movable span be provided with a clear headway of at least 40 ft. above high water level, when open. It was decided to make one of the fixed spans movable in the manner outlined in Fig. 5L (*b*) The maximum lift is 32 ft. Towers, having front vertical columns, were built at the channel ends of the

[9] *The Daily Railway Age Gazette*, March 18, 1914.

TABLE
VERTICAL LIFT

Name or Location	Design	Span, c. to c., Feet	Height of Towers or Lift, Feet	Tracks, Number or Roadway, Feet	Sidewalks, Feet
Railway Bridges					
Morgan's La. & Tex. R.R. & S.S. Co., Breaux, La.	Waddell & Harrington	50	Towers... 65 Lift..... 40	1	.....
Oregon-Washington Ry. & Navigation Co., Willamette River, Portland, Ore.	Waddell & Harrington Double-deck, double-track R.R with upper highways. Lower deck telescopes upper deck	220	Towers. 178.75 Lift, lower deck. 46 Lift both decks 93 Total. 139	Upper, 1 @ 29.25 Upper, 2 @ 15.08 Lower, 2 R.R. tracks	Upper, 2 @ 6
Southern Pacific Co., Willamette River, Salem, Ore.	Waddell & Harrington	130.83	Towers.... 101 Lift..... 51	1	.....
Great Northern Ry., Missouri River, Snowdon, Mont.	Waddell & Harrington	296	Lift..... 40	1	
Great Northern Ry., Yellowstone River, Fairview, Mont.	Waddell & Harrington	271	Lift..... 40	1	.....
Northern Pacific Ry., Steilacoom Waterway.....	Strauss type with overhead balance trusses	96	Towers.. 77.17 Lift..... 50	2	.. .
Alabama Great Southern R.R. Co., Warrior River, Eutaw, Ala.	American Bridge Co. Gasoline engine	214	Towers..... 80 Lift..... 69	1	.....
Cin. N. O. & Tex. Pac. Ry., Tennessee Riv., Chattanooga, Tenn.	American Bridge Co. Gasoline Engine	310	Towers..... 90 Lift..... 46.5	2	.....
Cin. N. O. & Tex. Pac. Ry., Ohio River, Cincinnati, Ohio	American Bridge Co. Hydraulic Machinery	365	Towers.. 80.77 Lift..... 13	2	.....
Mo. Pacific R.R., Osage River, Osage, Mo.	Harrington, Howard, and Ash	100	Towers.. 79.29 Lift..... 35	1	.....
Meridian Hy. Br. Co., Missouri River, Yankton, So. Dakota	Harrington, Howard, and Ash	250	Towers, Lift 27	Upper deck, 18' highway Lower deck, 1 railway track	.....

5A

BRIDGES

Weight of Structural Steelwork, Pounds		Weight of Machinery, Pounds		Machinery to Structural, Per Cent	Total Weight, Pounds	Date Built
Lift span	45,500	Sheaves, shafts, and bearings	2,000	1.57		
Towers	82,000	Wire ropes and fittings	1,200	0.94		
		Operating machinery	6,800	5.33		
	127,500		10,000	7.84	137,500	1911
Structural	4,920,000	Sheaves, shafts, and bearings	338,000	6.87		
Miscellaneous	402,000	Equalizers	100,000	2.03		
		Wire ropes and fittings	264.000	5.36		
		Operating machinery	532,000	10.82		
	5,322.000		1,234,000	25.08	6,556,000	1912
Lift span	310,000	Wire ropes and fittings	9,500	1.67		
Towers and cwt. frames	215,500	Sheaves, shafts, bearings, and operating machinery	82,000	14.58		
Miscellaneous	37,000					
	562,500		91,500	16.27	654,000	1912
Lift span	1,298,000	Sheaves, shafts, and bearings	100,000	5.82		
Towers and cwt. frames	419,000	Equalizers	17,500	1.02		
		Wire ropes and fittings	35,000	2.04		
		Operating machinery	75,000	4.37		
	1,717,000		227,500	13.25	1,944,500	1913
Lift span	1,114,000	Sheaves, shafts, and bearings	84,000	5.59		
Towers and cwt. frames	390,000	Equalizers	16,500	1.10		
		Wire ropes and fittings	31,000	2.06		
		Operating machinery	51,000	3.39		
	1,504,000		182,500	12.14	1,686,500	
Structural	749,500	Machinery	78,000	10.41	827,500	1914
Lift span	668,500	Sheaves, shafts, bearings, wire ropes, and machinery	103,500			
Towers and cwt. frames	200,000					
	868,500		103,500	11.92	972,000	1918
Lift span	2,335,000	Sheaves, shafts, bearings, wire ropes, and machinery	287,500			
Towers and cwt. frames	1,207,500					
	3,542,500		287,500	8.17	3,830,000	1920
Lift span	2,551,000	All machinery	175,000			
Towers and cwt. boxes	334,000					
	2,885,000		175,000	6.07	3,060,000	1922
Lift span and towers	488,500	All machinery	38,500			
	488,500		38,500	7.88	527,000	1924
Lift span	996,000	All machinery	184,000			
Towers	449,000					
	1,445,000		184,000	12.73	1,629,000	1924

adjacent fixed spans. Roller track girders were added on top of panels U_1U_2 of the lift span. The horizontal movement of the rollers is about 4 ft.

For operating the lift span, racks are bolted to the vertical tower posts, which are engaged by pinions on a shaft mounted to the ends of the span. These pinions are driven from the center of the span by means of shafts and gears connecting to the motive power. In this case hand-power is used at the present time, with provision for installing electric motors in the future should conditions of river traffic demand. The bridge is locked when in the closed position to prevent vertical or lateral movement.

The track girders are connected to the old span in such manner that no dismantling of the bridge was required and traffic was maintained at all times. The entire moving load in this structure is brought directly through the towers to the piers, without any portion going to the adjacent spans. This bridge requires no cables, sheaves, or chains.

The span can be lifted to its full height of 32 ft. by four men on the slow-speed gears in about eight minutes.

This remodeling of the bridge was done late in 1913.

6. Statistics.—The principal dimensions, facilities afforded, and the weights of the structural steelwork and machinery of a few railway vertical-lift bridges are given in Table 5A. In some cases the complete classification of the weights of the machinery was not available, but it is hoped that the information given may be of some value for approximate estimates and investigations.

7. Miscellaneous vertical-lift types.—The types of vertical-lift bridges developed are less numerous than the types of bascules. In the limited space available, only a few of those most frequently used, or most urgently advocated, have been mentioned. As in the case of bascules, the field for original design and invention is limited by the patents already granted. In order to assist those wishing to develop new types, a list of the United States patents granted is given in Table 5B.

TABLE 5B

United States Patents Granted for Vertical Lift Bridges

Name	Number	Date	Description
Alden, J. F.	431,101	July 1, 1890	Lift bridge
Harrington, J. L.	1,027,477	May 28, 1912	Lift bridge
Harrington, J. L.	1,027,478	May 28, 1912	Lift bridge
Harrington, J. L., Howard, E. E., and Ash, L. R.	1,261,124	April 2, 1918	Guide for counterweight ropes
Harrington, J. L.	1,285,696	Nov. 26, 1918	Lift bridge
Hedrick, I. G., and Cochrane, V. H.	1,087,233	Feb. 17, 1914	Counterweight
Joosting, P.	1,422,717	July 11, 1922	Lift bridge
Paine, C. E.	1,093,202	April 14, 1914	Bridge
Post, A. J.	162,576	April 27, 1875	Lift bridge
Rall, T.	1,140,316	May 18, 1915	Vertical-lift bridge
Rutherberg, A.	635,394	Oct. 24, 1899	Lift bridge
Shoemaker, L. H.	1,210,715	Jan. 2, 1917	Operating mechanism for lift bridges
Soulerin, L.	153,729	Aug. 4, 1874	Lowering bridge
Strauss, J. B.	1,038,226	Sept. 10, 1910	Bridge
Strauss, J. B.	1,111,872	Sept. 29, 1914	Bridge
Strauss, J. B.	1,170,702	Feb. 8, 1916	Bridge
Strauss, J. B.	1,269,976	June 18, 1918	Suspension lift bridge
Waddell, J. A. L.	506,571	Oct. 10, 1893	Lift bridge
Waddell, J. A. L. and Harrington, J. L.	932,359	Aug. 24, 1909	Lift bridge
Waddell, J. A. L. and Harrington, J. L.	952,486	Mar. 22, 1910	Lift bridge
Waddell, J. A. L. and Harrington, J. L.	953,307	Mar. 29, 1910	Lift bridge
Waddell, J. A. L., and Harrington, J. L.	1,049,422	Jan. 7, 1913	Lift bridge
Whipple, S.	134,338	Dec. 24, 1872	Lift draw bridge
Whitmore, E. B.	225,775	Mar. 23, 1880	Lifting bridge.

CHAPTER VI

CONTINUOUS GIRDERS

1. General.—A continuous girder is a beam, or truss, that is supported at more than two points. Continuous girder bridges have been built in greater numbers in Great Britain and Europe than in America.

The Britannia Bridge, across the Menai Straits, was designed by Robert Stephenson, and completed in 1850. It was the first large wrought-iron girder bridge, and was of the tubular type. The double-track railway was carried in two independent, parallel, single-track tubes. The two end spans were 230 ft. and the two channel spans 460 ft. long in the clear of the masonry piers. Each tube was continuous throughout for a length of 1524 ft.

Many continuous bridges were built in France during the latter part of the nineteenth century. They were usually lattice trusses with parallel chords, and with from three to five spans.

The Fades Viaduct, over the Sioule River, finished in 1908, consists of two end spans of 378 ft. and a central span of 472 ft. It is a single-track, lattice, deck steel structure, with the rails 435 ft. above the water, and is continuous throughout.

The first large, continuous steel bridge in America was the Lachine bridge, over the St. Lawrence River. It was designed by the late C. Shaler Smith, and built in 1888. It was a single-track, pin-connected structure, with two deck side spans of 268 ft. and two through middle spans of 408 ft.

The Sciotoville bridge, over the Ohio River, was finished in 1917. It is of the through-riveted type, and consists of two double-track spans of 775 ft., continuous across the middle pier.

The Bessemer and Lake Erie Railroad bridge, over the Allegheny River, was finished in 1918. It is a double-track deck bridge with two groups of continuous trusses, one with spans of 272, 520, and 347 ft., the other with spans of 347, 350, and 272 ft.

The Hudson Bay Railway bridge, over the Nelson River, is a single-track, riveted, continuous structure, with spans of 300, 400, and 300 ft. It was finished in 1918.

The Cincinnati, New Orleans, and Texas Pacific Railway bridge, over the Ohio River at Cincinnati, was finished in 1922. At the north end, two spans of 300 ft. and one of 500 are double-track, continuous, through trusses of combined riveted and pin-connected construction.

These examples are given to show that continuous bridges have been widely used. They have proved safe and satisfactory in service, when properly designed and built. An extended discussion of them would be out of place here. It is necessary to include enough information to lay a foundation for such equations as are required for the analysis of the ordinary types of swing spans, which are usually continuous girders.

2. Stresses.—The reactions at the supports of continuous girders cannot be found from the principles of simple statics alone. They depend upon the design and materials of the structure itself, and its elastic behavior under loads.

When a beam, or truss, is of the same cross-section throughout and is built from one kind of material, the solution is simple. The values of the moment of inertia of the girder and the modulus of elasticity of the material are constant. When the girder section varies from point to point along its length, a direct solution is too complicated and uncertain to be feasible. The best procedure is then to assume that both I and E are constant. A preliminary design is then made, the elastic deflections calculated, and any necessary corrections made in the first design.

The shears and bending moments in a continuous girder, of uniform section and material and of any number of spans and loaded in any manner, can be found by simple statics after the moments at each support are known. These moments may be found by the application of the *Theorem of Three Moments.*

3. The Theorem of Three Moments.—The first suggestion of what is now known as the Theorem of Three Moments was made in 1825, by M. Navier, but in a complicated form, difficult of application. In 1848 M. Clapeyron developed a much simplified form of the theorem for use in his design for the reconstruction of the Pont d'Asinères, near Paris, and in 1857 he announced it in *Comptes Rendus,* Vol. 45, pp. 1076 to 1080, in the form:

$$l_0Q_0 + 2(l_0 + l_1)Q_1 + l_1Q_2 = \tfrac{1}{4}(p_0l_0^3 + p_1l_1^3),$$

in which l_0 and l_1 are the lengths of any two consecutive spans which are loaded with p_0 and p_1 units of weight per unit of length, respectively, and Q_0, Q_1, and Q_2 are the bending moments at the three supports, proceeding from the left. I and E are assumed here to be constant.

M. Clapeyron also gave the theorem for equal spans for the same loads and conditions as:

$$Q_0 + 4Q_1 + Q_2 = \frac{l^2}{4}(p_0 + p_1).$$

Many investigators have since extended the formulae to include concentrated and partial loads, and also the effects of the settlement of supports below their original elevations and those due to differences in the temperatures of the top and bottom chords.

General Theorem of Three Moments.—A general form of the Theorem of *Three Moments* is given by Herr Heinrich F. B. Müller-Breslau, in his treatise *Die neueren Methoden der Festigkeitslehre*, Leipzig, 1886, and

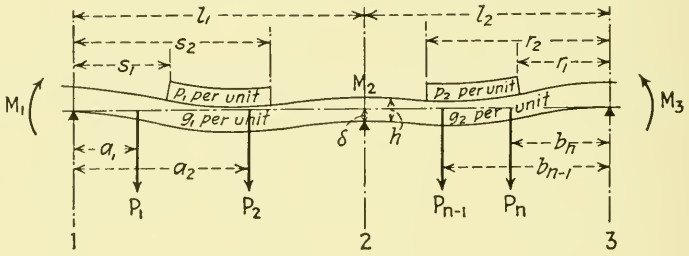


FIG. 6A.

in later editions. In this formula, Eq. (1), the quantities that are not clearly shown by Fig. 6A are:

δ = the downward settlement of pier 2 with reference to piers 1 and 3;
ϵ = the coefficient of expansion of the material for changes in temperature;
Δt = the difference in temperature between the top and bottom flanges or chords;
h = the effective depth of the beam or girder.

$$M_1 l_1 + 2M_2(l_1 + l_2) + M_3 l_2 + \frac{g_1 l_1^3}{4} + \frac{g_2 l_2^3}{4}$$

$$+ \frac{p_1(s_2^2 - s_1^2)(2l_1^2 - s_2^2 - s_1^2)}{4l_1} + \frac{p_2(r_2^2 - r_1^2)(2l_2^2 - r_2^2 - r_1^2)}{4l_2}$$

$$+ \frac{\sum_0^{l_1} P_a(l_1^2 - a^2)}{l_1} + \frac{\sum_0^{l_2} P_b(l_2^2 - b^2)}{l_2}$$

$$+ 3EI\frac{\epsilon \Delta t}{h}(l_1 + l_2) - 6EI\delta\frac{l_1 + l_2}{l_1 l_2} = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If the settlements of the supports 1, 2, and 3 are given separately with reference to their original positions, as in Fig. 6B, the last term of (1) may be written

$$-6EI\delta\frac{l_1+l_2}{l_1l_2} = -6EI\left[\frac{c_2-c_1}{l_1}+\frac{c_2-c_3}{l_2}\right]. \quad . \quad . \quad . \quad (2)$$

Application of Eq. (1).—The bending moments at the supports of any number of continuous spans, for any given loads, temperature condition, and final elevation of supports, may be calculated by applying Eq. (1) successively to each group of two spans. As many equations may be written as there are unknown moments at the supports. Thus, for a girder continuous over four supports there are three unknown moments. Equation (1) may be written for spans 1 and 2, 2 and 3, and 3 and 4, thus giving three equations.

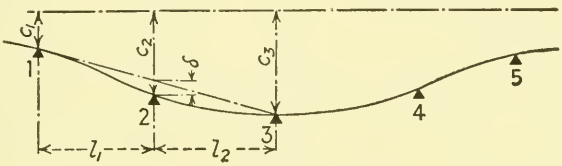


FIG. 6B.

In a series of continuous spans, 1 to S, inclusive, resting upon supports which are numbered from 1 to $S+1$, beginning at the left; it should be noted that M_1 and M_{s+1} are each zero, the first equation beginning

$$2M_2(l_1+l_2)+M_3l_2,+\text{ etc.}$$

The last equation begins

$$M_{s-1}+2M_s(l_{s-1}+l_s)+O+\text{etc.}$$

When applying Eq. (1) to any particular series of spans, omit all terms after

$$M_1l_1+2M_2(l_1+l_2)+M_3l_2,$$

which do not enter into the case under investigation. For example, the last term, taking account of settlement of the supports, will seldom be involved. The temperature condition will usually be investigated separately.

When the stress calculations are to be made by the influence-line method, the effect of a single concentrated load of a unit at any particular point will usually be found first, then that of a unit load at the next point. The various points for which influence lines are required are thus considered in order.

In cases where the extreme ends of the end spans are fixed, the theorem may still be applied, by considering a fixed end as equivalent to an additional span of zero length.

4. Common form of the equation of three moments.—The forces acting on two adjacent spans of a series of continuous spans are designated as shown in Fig. 6D. Any concentrated load, as one in span 2, is known as P_2, and its distance from support 2 as kl_2, k being a fraction less than unity. Using this nomenclature, the equation of three moments for a continuous girder, loaded only by a series of concentrated loads, will become

$$M_1l_1 + 2M_2(l_1 + l_2) + M_3l_2 = -\Sigma P_1l_1^2(k - k^3) - \Sigma P_2l_2^2(2k - 3k^2 + k^3). \quad . \quad (3)$$

When the load is uniform and equal to w per linear unit this reduces to

$$M_1l_1 + 2M_2(l_1 + l_2) + M_3l_2 = -\frac{w_1l_1^3}{4} - \frac{w_2l_2^3}{4}. \quad . \quad . \quad (4)$$

Equation (3) is in a form undesirable for direct application, as the process is long and tedious, and Eq. (1) will usually be preferred. Equation (3), however, does reduce to some simple relations, convenient for application to simple swing bridges, as will be shown later.

When it is desired to apply Eq. (3) to direct calculations, a table giving the values of $(k - k^3)$ and $(2k - 3k^2 + k^3)$ is convenient. Table 6A gives these values from $k = 0.01$ to $k = 0.99$. A table from $k = 0.001$ to $k = 0.999$ may be found in "The Continuous Girder," by Malverd A. Howe, Engineering News Publishing Co., New York, 1889.

5. Shear and bending moment in any span.—Assume any span of a continuous girder to be loaded by concentrated loads, as represented

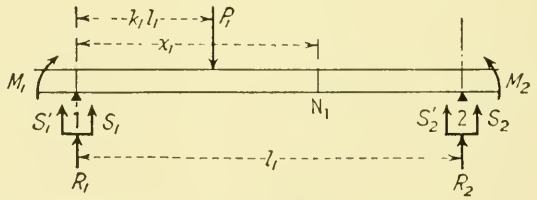


FIG. 6C.

in Fig. 6C. Let the forces acting upon the span be as shown. Consider one load, P_1, at a distance k_1l_1, from the left support, then:

End shears.

$$S_1 = \frac{M_2 - M_1}{l_1} + P_1(1 - k_1). \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$S'_2 = \frac{M_1 - M_2}{l_1} + P_1 k_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

TABLE 6A

$k=\frac{a}{l}$	$k-k^3$		$k=\frac{a}{l}$	$k-k^3$		$k=\frac{a}{l}$	$k-k^3$	
.01	.009999	.99	.35	.307125	.65	.69	.361491	.31
.02	.019992	.98	.36	.313344	.64	.70	.357000	.30
.03	.029973	.97	.37	.319347	.63	.71	.352089	.29
.04	.039936	.96	.38	.325128	.62	.72	.346752	.28
.05	.049875	.95	.39	.330681	.61	.73	.340983	.27
.06	.059784	.94	.40	.336000	.60	.74	.334776	.26
.07	.069657	.93	.41	.341079	.59	.75	.328125	.25
.08	.079488	.92	.42	.345912	.58	.76	.321024	.24
.09	.089271	.91	.43	.350493	.57	.77	.313467	.23
.10	.099000	.90	.44	.354816	.56	.78	.305448	.22
.11	.108669	.89	.45	.358875	.55	.79	.296961	.21
.12	.118272	.88	.46	.362664	.54	.80	.288000	.20
.13	.127803	.87	.47	.366177	.53	.81	.278559	.19
.14	.137256	.86	.48	.369408	.52	.82	.268632	.18
.15	.146625	.85	.49	.372351	.51	.83	.258213	.17
.16	.155904	.84	.50	.375000	.50	.84	.247296	.16
.17	.165087	.83	.51	.377349	.49	.85	.235875	.15
.18	.174168	.82	.52	.379392	.48	.86	.223944	.14
.19	.183141	.81	.53	.381123	.47	.87	.211497	.13
.20	.192000	.80	.54	.382536	.46	.88	.198528	.12
.21	.200739	.79	.55	.383625	.45	.89	.185031	.11
.22	.209352	.78	.56	.384384	.44	.90	.171000	.10
.23	.217833	.77	.57	.384807	.43	.91	.156429	.09
.24	.226176	.76	.58	.384888	.42	.92	.141312	.08
.25	.234375	.75	.59	.384621	.41	.93	.125643	.07
.26	.242424	.74	.60	.384000	.40	.94	.109416	.06
.27	.250317	.73	.61	.383019	.39	.95	.092625	.05
.28	.258048	.72	.62	.381672	.38	.96	.075264	.04
.29	.265611	.71	.63	.379953	.37	.97	.057327	.03
.30	.273000	.70	.64	.377856	.36	.98	.038808	.02
.31	.280209	.69	.65	.375375	.35	.99	.019701	.01
.32	.287232	.68	.66	.372504	.34	1.00	.000000	.00
.33	.294063	.67	.67	.369237	.33			
.34	.300696	.66	.68	.365568	.32			
	$2k-3k^2+k^3$	$k=\frac{a}{l}$		$2k-3k^2+k^3$	$k=\frac{a}{l}$		$2k-3k^2+k^3$	$k=\frac{a}{l}$

Shear at N_1.

$$\left.\begin{array}{ll} \text{When} & x_1 < k_1 l_1, \quad S_x = -S_1 \\ \text{When} & x_1 > k_1 l_1, \quad S_x = -S_1 + P_1 \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Bending moment at N_1.

$$\left.\begin{array}{ll} \text{When} & x_1 < k_1 l_1, \quad M_x = M_1 + S_1 x_1 \\ \text{When} & x_1 > k_1 l_2, \quad M_x = M_1 + S_1 x_1 - P_1(x_1 - k_1 l_1) \end{array}\right\} \quad . \quad . \quad . \quad (8)$$

The last terms in (5) and (6) give the shears due to the load P_1 at the ends of a simple span of length l_1. For several concentrated loads they become $\Sigma P_1(1 - k_1)$ and $\Sigma P_1 k_1$, respectively. For a uniform load of w_1 per unit of length they each become $\frac{w_1 l_1}{2}$.

Similarly, the last terms of (7) and (8) become

$$-\Sigma P_1 \quad \text{and} \quad -\Sigma P_1(x_1 - k_1 l_1).$$

For a uniform load of w_1 per unit of length, (8) becomes

$$M_x = M_1 + S_1 x_1 - \frac{w_1 x_1^2}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

From these equations the shears and moments in any section of the span can be calculated when the moments at the supports are known.

6. Reactions at the supports.—Let Fig. 6D represent any two consecutive spans of a continuous girder, loaded by concentrated loads; and assume the various forces acting upon the two spans to be as indicated in the figure.

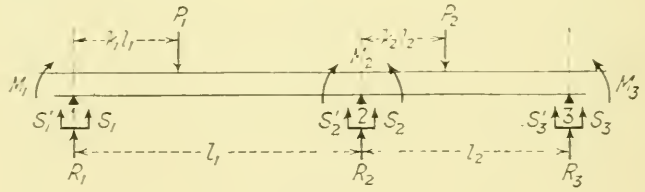


FIG. 6D.

Consider a concentrated load, P_1, at a distance of $k_1 l_1$ from support 1, to be in span 1, and a load, P_2, at a distance of $k_2 l_2$ from support 2, to be in span 2; then from eqs. (6) and (5)

$$S'_2 = \frac{M_1 - M_2}{l_1} + P_1 k_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$S_2 = \frac{M^3 - M_2}{l_2} + P_2(1 - k_2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

But

$$R_2 = S'_2 + S_2;$$

hence

$$R_2 = \frac{M_1 - M_2}{l_1} + \frac{M_3 - M_2}{l_2} + P_1 k_1 + P_2(1 - k_2). \quad . \quad (12)$$

The last two terms of (12) give the reaction at 2 for two simple spans. For several concentrated loads they become

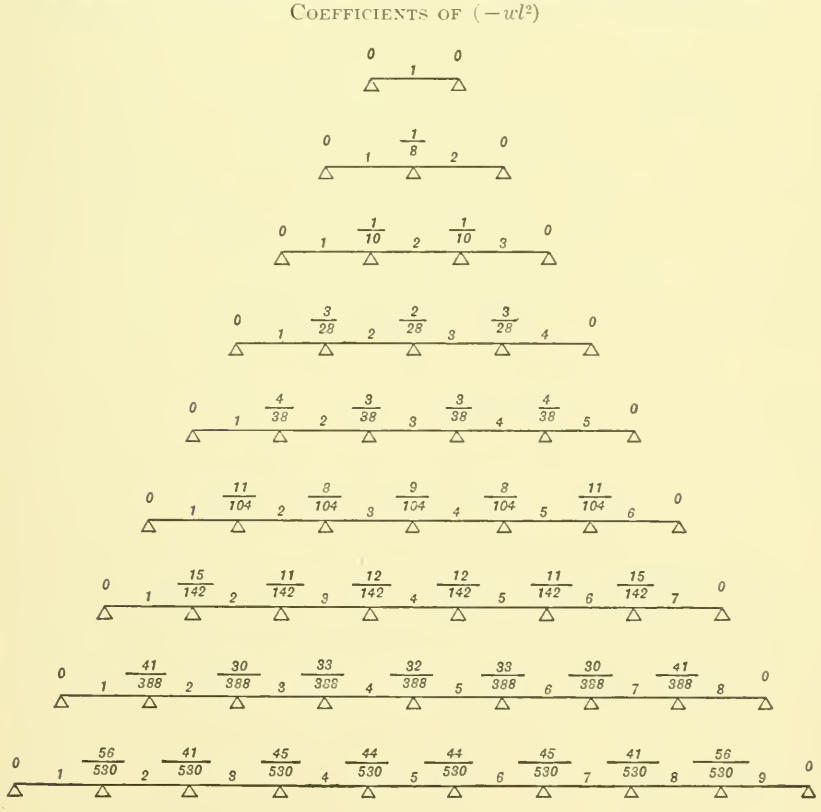
$$\Sigma P_1 k_1 + \Sigma P_2(1 - k_2);$$

and for uniform loads of w_1 and w_2 per unit of length in the respective spans,

$$\frac{w_1 l_1}{2} + \frac{w_2 l_2}{2}.$$

TABLE 6B

MOMENTS AT SUPPORTS; SPANS EQUAL; UNIFORM LOAD ON ALL SPANS
COEFFICIENTS OF $(-wl^2)$

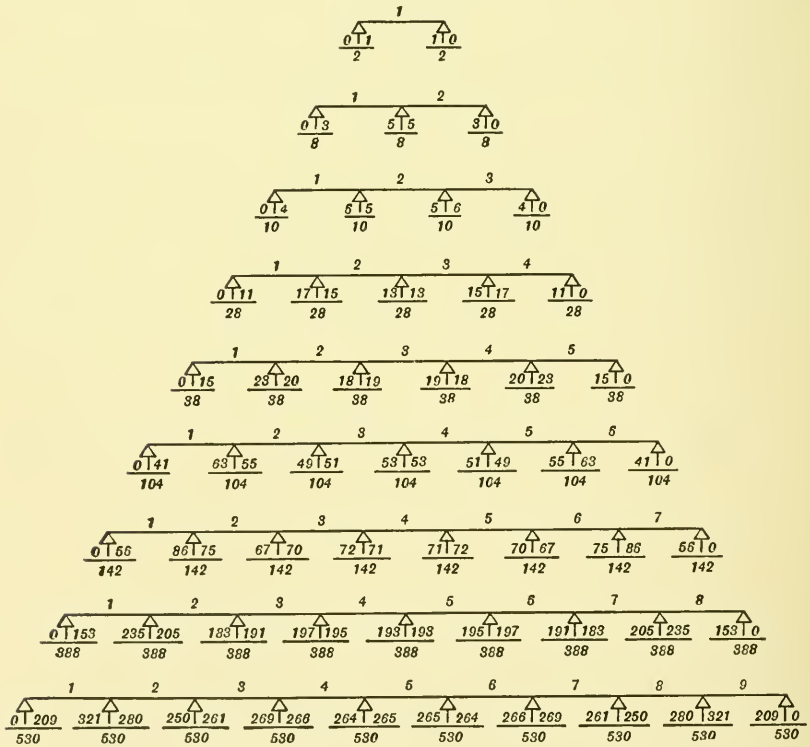


Equations (5) to (9) apply to any span, and (10) to (12) to any two consecutive spans of a series of continuous spans, when the proper numerical subscripts are inserted in place of subscripts 1 and 2 in the expressions.

7. Spans of equal length and uniformly loaded.—When all spans of a continuous girder are of equal length and uniformly loaded throughout, the expressions for M, R, and S become much simplified. Tables 6B and 6C give the coefficients of $-wl^2$ for moments, and of wl for shears at each side of each support, for any number of equal spans from one to nine.

TABLE 6C

SHEARS AT SUPPORTS; SPANS EQUAL; UNIFORM LOAD ON ALL SPANS
COEFFICIENTS OF (wl)



CHAPTER VII

ANALYSIS OF SWING BRIDGES

1. General.—Swing draw bridges usually have both arms of the same length and are opened by being revolved about a vertical axis. The simplest case is where the entire weight, when turning, is carried on a central pivot. In this case the girders, or trusses, are continuous over three supports when the span is closed and the ends are lifted.

Some swing bridges revolve on a cylindrical drum, supported by rollers running on a circular track on the center pier, the truss loads being delivered to the drum by a system of distributing girders. Such spans have four supports, when the bridge is closed and the ends are lifted. When the web bracing is continued across the middle panel, above the drum, the trusses are continuous over four supports. As the panel above the drum is much shorter than the two end spans, heavy negative reactions are caused at one side of the drum when one of the end spans is loaded. As it is difficult to obtain enough anchorage to resist these negative reactions, this type of draw bridge is seldom built and will not be treated in what follows.

In order to avoid the difficulty just mentioned, drum-bearing swing bridges are usually built with incomplete, or very light, web members above the drum, in the middle panel. Such a span is partially continuous. It is continuous across the middle panel for bending moments, and discontinuous for shears.

In some cases two adjacent swing spans are built with their ends connected at the center of the main navigation channel. The ends are locked together so that shears, but no bending moments, can be transmitted from one span to the other. Such a bridge is partially continuous.

The discussion will be taken up in the following order:

Section I.—The two-span, center-bearing bridge. Continuous.

Section II.—The two-span, drum-bearing bridge. Continuous for moments; discontinuous for shears.

Section III.—Two center-bearing swing spans with their adjacent ends locked together. Continuous for shears; discontinuous for moments.

SECTION I

THE CENTER-BEARING SWING BRIDGE

2. A center-bearing swing bridge with equal spans.—A center-bearing swing bridge usually has two equal spans; and when it is closed with its ends resting upon the end supports, or when the ends have been lifted, it acts as a girder continuous over three supports for live loads upon any part of either span. The moments at the end supports, 1 and 3, are zero, and the only unknown external moment is that at the center support, 2. It is convenient to consider the stresses due to the concentrated and uniform loads separately.

3. A single concentrated load on one span.—For a single concentrated load, P, on the first span, the bending moment at support 2 is readily found from Eq. (3), Chapter VI.

For this condition $M_1 = 0$, $M_3 = 0$, $P_2 = 0$, and $l_1 = l_2 = l$; hence

$$M_2 = -\frac{Pl}{4}(k - k^3). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Reactions.—From Eq. (5), Chapter VI, S_1 becomes R_1 at support 1. Substituting the value of M_2 from Eq. (1), we have

$$R_1 = \frac{P}{4}(4 - 5k + k^3). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$R_2 = \frac{P}{2}(3k - k^3). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$R_3 = -\frac{P}{4}(k - k^3). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Shears.—The shears are readily found in the ordinary manner when the reactions are known, thus:

$$\left.\begin{array}{ll} \text{when} & x < kl, \quad S_x = R_1, \\ \text{when} & x > kl, \quad S_x = R_1 - P \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

when $x < kl$, S_x is positive;

$x = kl$, the shear passes from positive to negative, indicating the point of maximum moment;

$x > kl$, S_x is negative.

The shear throughout the second, or right, span is equivalent to R_3, but positive when R_3 is negative.

Moments.—The moment for any point x, between support 1 and $x = kl$ is $M = R_1 x$, or

$$M_{x<kl} = \frac{P}{4}(4 - 5k + k^3)x. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This is always positive. M_x is a maximum when $x = kl = 0.43275l$. At this point

$$M = 0.20747Pl.$$

When x is between kl and support 2,

$$M = R_1x - P(x - kl),$$

or

$$M_{x>kl} = \frac{P}{4}(4kl - 5kx + k^3x). \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

This may be either positive or negative, depending upon the value of x. It passes from positive to negative when

$$x_0 = \frac{4l}{5 - k^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

This point of zero moment is known as the **point of contraflexure.** As k becomes larger, x_0 increases and the point of contraflexure lies nearer the second support. As k becomes smaller it is located nearer the first support, the minimum being

$$x_0 = \tfrac{4}{5}l, \text{ when } k = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The maximum negative moment is at support 2 when the load is at

$$k = 0.5773l.$$

$$M_{\max.} = -0.0962Pl.$$

For a single load in the first span, the shears in the second span are all positive and the moments negative.

Typical shears and moments are represented in Fig. 7A. The shears vary as indicated by (*b*) and the moments as in (*c*). The point of contraflexure is at 0.

4. Table of reactions.—Center-bearing swing bridges with two equal spans are built much more frequently than those of any other type. The calculation of the live-load stresses, when such a bridge is acting as a continuous girder, requires that the values of the reactions be found for each panel of a truss, or for several equidistant points in each arm of a girder. Equations (2), (3), and (4) are tedious of repeated application, and considerable time can be saved by evaluating them for various numbers of panels, or subdivisions, for some convenient single panel load, and tabulating these values of the reactions as proportional parts of the assumed panel load.

Table 7A shows the reactions R_1, R_2, and R_3 for a load of 1000 lbs. at any panel, or division point, in the first, or left, span for numbers of panels from 2 to 20, this load having been placed successively at each

panel or division point. The total reactions for any series of joint loads, or concentrations, may be found by summation. It must be remembered that the values of the reactions in the table were calculated on the assumption that E and I were constant throughout the two spans.

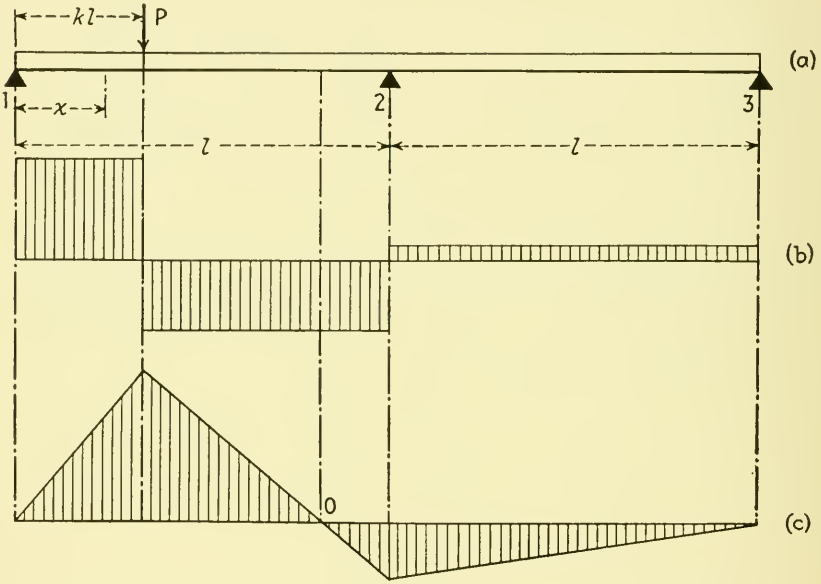


Fig. 7A.

5. A uniform load on one span.—Assume a uniform load of w per unit of length upon the first, or left, span. Refer to Eq. (4), Chapter VI, and note that $M_1 = O$, $M_3 = O$, $w_2 = O$, and $l_1 = l_2 = l$. Then

$$M_2 = -\frac{wl^2}{16}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Reactions.—From Eq. (5), Chapter VI, for a uniform in place of a concentrated load,

$$R_1 = \tfrac{7}{16}wl. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$R_2 = \tfrac{5}{8}wl. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$R_3 = -\tfrac{1}{16}wl. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Shears.—The shear at any point x, in the first span is

$$R_1 - wx, \quad \text{or} \quad S_x = \tfrac{7}{16}wl - wx = \frac{w}{16}(7l - 16x). \quad . \quad . \quad . \quad (14)$$

The shear becomes zero at $x = \tfrac{7}{16}l$.

TABLE 7A

SWING-BRIDGE REACTIONS FOR $P=1000$ POUNDS IN THE LEFT ARM. BEAM CONTINUOUS OVER THREE SUPPORTS, TWO EQUAL ARMS; E AND I ASSUMED CONSTANT THROUGHOUT THE GIRDER

No. of Panels	Values of k	R_1 +	R_2 +	R_3 −
2	1÷2	406.25	687.50	93.75
3	1÷3	592.59	481.48	74.07
	2÷3	240.74	851.85	92.59
		833.33	1333.33	166.66
4	1÷4	691.41	367.18	58.59
	2÷4	406.25	687.50	93.75
	3÷4	167.97	914.06	82.03
		1265.63	1968.74	234.37
5	1÷5	752.00	296.00	48.00
	2÷5	516.00	568.00	84.00
	3÷5	304.00	792.00	96.00
	4÷5	128.00	944.00	72.00
		1700.00	2600.00	300.00
6	1÷6	792.82	247.69	40.51
	2÷6	592.59	481.48	74.07
	3÷6	406.25	687.50	93.75
	4÷6	240.74	851.85	92.59
	5÷6	103.00	960.66	63.66
		2135.40	3229.18	364.58
7	1÷7	822.16	212.83	34.99
	2÷7	648.69	416.93	65.62
	3÷7	483.97	603.49	87.46
	4÷7	332.36	763.85	96.21
	5÷7	198.26	889.20	87.46
	6÷7	86.01	970.84	56.85
		2571.45	3857.14	428.59
8	1÷8	844.24	186.52	30.76
	2÷8	691.41	367.18	58.59
	3÷8	544.43	536.14	80.57
	4÷8	406.25	687.50	93.75
	5÷8	279.78	815.44	95.22
	6÷8	167.97	914.06	82.03
	7÷8	73.73	977.54	51.27
		3007.81	4484.38	492.19
9	1÷9	861.46	165.98	27.44
	2÷9	724.97	327.84	52.81
	3÷9	592.59	481.48	74.07
	4÷9	466.39	622.77	89.16
	5÷9	348.42	747.60	96.02
	6÷9	240.74	851.85	92.59
	7÷9	145.40	931.42	76.82
	8÷9	64.47	982.17	46.64
		3444.44	5111.11	555.55
10	1÷10	875.25	149.50	24.75
	2÷10	752.00	296.00	48.00
	3÷10	631.75	436.50	68.25
	4÷10	516.00	568.00	84.00
	5÷10	406.25	687.50	93.75
	6÷10	304.00	792.00	96.00
	7÷10	210.75	878.50	89.25
	8÷10	128.00	944.00	72.00
	9÷10	57.25	985.50	42.75
		3881.25	5737.50	618.75
11	1÷11	886.55	135.99	22.54
	2÷11	774.23	269.72	43.95
	3÷11	664.16	398.95	63.11
	4÷11	557.47	521.42	78.89
	5÷11	455.29	634.87	90.16
	6÷11	358.74	737.06	95.80
	7÷11	268.96	825.72	94.68
	8÷11	187.06	898.61	85.67
	9÷11	114.17	953.48	67.65
	10÷11	51.42	988.06	39.48
		4313.05	6363.88	681.93
12	1÷12	895.98	124.71	20.69
	2÷12	792.82	247.69	40.51
	3÷12	691.41	367.18	58.59
	4÷12	592.59	481.48	74.07
	5÷12	497.25	588.83	86.08
	6÷12	406.25	687.50	93.75
	7÷12	320.46	775.75	96.21
	8÷12	240.74	851.85	92.59
	9÷12	167.97	914.06	82.03
	10÷12	103.00	960.66	63.66
	11÷12	46.73	989.87	36.60
		4755.20	6989.58	744.78
13	1÷13	903.96	115.16	19.12
	2÷13	808.60	228.95	37.55
	3÷13	714.61	340.01	54.62
	4÷13	622.67	446.97	69.64
	5÷13	533.45	548.48	81.93
	6÷13	447.66	643.15	90.81
	7÷13	365.95	729.63	95.58
	8÷13	289.03	806.56	95.59
	9÷13	217.57	872.55	90.12
	10÷13	152.25	926.26	78.51
	11÷13	93.76	966.32	60.08
	12÷13	42.79	991.35	34.14
		5192.30	7615.39	807.69

TABLE 7A—*Continued*

No. of Panels	Values of k	R_1 +	R_2 +	R_3 −
14	1÷14	910.81	106.96	17.77
	2÷14	822.16	212.83	34.99
	3÷14	734.60	316.51	51.11
	4÷14	648.69	416.91	65.60
	5÷14	564.96	512.94	77.90
	6÷14	483.96	603.50	87.46
	7÷14	406.25	687.50	93.75
	8÷14	332.36	763.85	96.21
	9÷14	262.85	831.45	94.30
	10÷14	198.25	889.21	87.46
	11÷14	139.12	936.04	75.16
	12÷14	86.00	970.85	56.85
	13÷14	39.31	992.81	32.12
		5629.32	8241.36	870.68
15	1÷15	916.74	99.85	16.59
	2÷15	833.93	198.81	32.74
	3÷15	752.00	296.00	48.00
	4÷15	671.41	390.52	61.93
	5÷15	592.59	481.48	74.07
	6÷15	516.00	568.00	84.00
	7÷15	442.07	649.19	91.26
	8÷15	371.26	724.15	95.41
	9÷15	304.00	792.00	96.00
	10÷15	240.74	851.85	92.59
	11÷15	181.93	902.81	84.74
	12÷15	128.00	944.00	72.00
	13÷15	81.67	969.99	51.66
	14÷15	36.59	993.48	30.07
		6068.93	8862.13	931.06
16	1÷16	921.93	93.63	15.56
	2÷16	844.24	186.52	30.76
	3÷16	767.27	277.95	45.22
	4÷16	691.41	367.19	58.60
	5÷16	617.00	453.49	70.49
	6÷16	544.43	536.13	80.56
	7÷16	474.06	614.38	88.44
	8÷16	406.25	687.50	93.75
	9÷16	341.37	754.76	96.13
	10÷16	279.79	815.43	95.22
	11÷16	221.86	868.77	90.63
	12÷16	167.97	914.06	82.03
	13÷16	118.47	950.56	69.03
	14÷16	73.73	977.54	51.27
	15÷16	34.12	994.26	28.38
		6503.90	9492.17	996.07
17	1÷17	926.52	88.13	14.65
	2÷17	853.35	175.65	29.00
	3÷17	780.78	261.96	42.74
	4÷17	709.14	346.43	55.57
	5÷17	638.71	428.46	67.17
	6÷17	569.82	507.42	77.24
	7÷17	502.75	582.74	85.49
	8÷17	437.82	653.78	91.60
	9÷17	375.33	719.93	95.26
	10÷17	315.59	780.58	96.17
	11÷17	258.91	835.13	94.04
	12÷17	205.58	882.96	88.54
	13÷17	155.91	923.47	79.38
	14÷17	110.22	956.03	66.25
	15÷17	68.80	980.05	48.85
	16÷17	31.99	994.86	26.85
		6941.22	10,117.58	1058.80
18	1÷18	930.61	83.23	13.84
	2÷18	861.48	165.95	27.43
	3÷18	792.86	247.64	40.50
	4÷18	725.01	327.79	52.80
	5÷18	658.20	405.88	64.08
	6÷18	592.66	481.40	74.06
	7÷18	528.67	553.84	82.51
	8÷18	466.48	622.67	89.15
	9÷18	406.34	687.40	93.74
	10÷18	348.52	747.50	96.02
	11÷18	293.27	802.45	95.72
	12÷18	240.85	851.75	92.60
	13÷18	191.51	894.88	86.39
	14÷18	145.51	931.33	76.84
	15÷18	103.12	960.58	63.70
	16÷18	64.58	982.11	46.69
	17÷18	30.15	995.43	25.58
		7379.82	10,741.83	1121.65
19	1÷19	934.24	78.87	13.11
	2÷19	868.71	157.31	26.03
	3÷19	803.62	234.86	38.48
	4÷19	739.17	311.13	50.30
	5÷19	675.61	385.62	61.23
	6÷19	613.14	457.93	71.07
	7÷19	551.97	527.63	79.60
	8÷19	492.35	594.24	86.59
	9÷19	434.47	657.38	91.85
	10÷19	378.55	716.58	95.13
	11÷19	324.81	771.42	96.23
	12÷19	273.51	821.39	94.90
	13÷19	224.81	866.17	90.98
	14÷19	178.94	905.27	84.21
	15÷19	136.18	938.16	74.34
	16÷19	96.65	964.58	61.23
	17÷19	60.63	984.00	44.63
	18÷19	28.38	995.88	24.26
		7815.74	11,368.42	1184.16
20	1÷20	937.53	74.94	12.47
	2÷20	875.25	149.50	24.75
	3÷20	813.34	223.31	36.65
	4÷20	752.00	296.00	48.00
	5÷20	691.41	367.19	58.60
	6÷20	631.75	436.50	68.25
	7÷20	573.22	503.56	76.78
	8÷20	516.00	568.00	84.00
	9÷20	460.28	629.44	89.72
	10÷20	406.25	687.50	93.75
	11÷20	354.09	741.81	95.90
	12÷20	304.00	792.00	96.00
	13÷20	256.16	837.69	93.85
	14÷20	210.75	878.50	89.25
	15÷20	167.97	914.06	82.03
	16÷20	128.00	944.00	72.00
	17÷20	91.03	967.94	58.97
	18÷20	57.25	985.50	42.75
	19÷20	26.84	996.31	23.15
		8253.12	11,993.75	1246.87

The shear throughout the second span is

$$S = \tfrac{1}{16}wl. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Moments.—The bending moment at any point in the first span is

$$M_x = \tfrac{7}{16}wlx - \frac{wx^2}{2} = \frac{w}{16}(7l - 8x)x. \quad . \quad . \quad . \quad (16)$$

The maximum positive moment occurs when $x = \tfrac{7}{16}l$ and is

$$M_{\max.} = \tfrac{49}{512}wl^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The moment becomes zero at $x = \tfrac{7}{8}l$. This is the point of contraflexure.

The moment in the second span is:

$$M_{x_2} = -\frac{w}{16}lx_2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

In this expression x_2 is measured from support 3. It is negative throughout span 2.

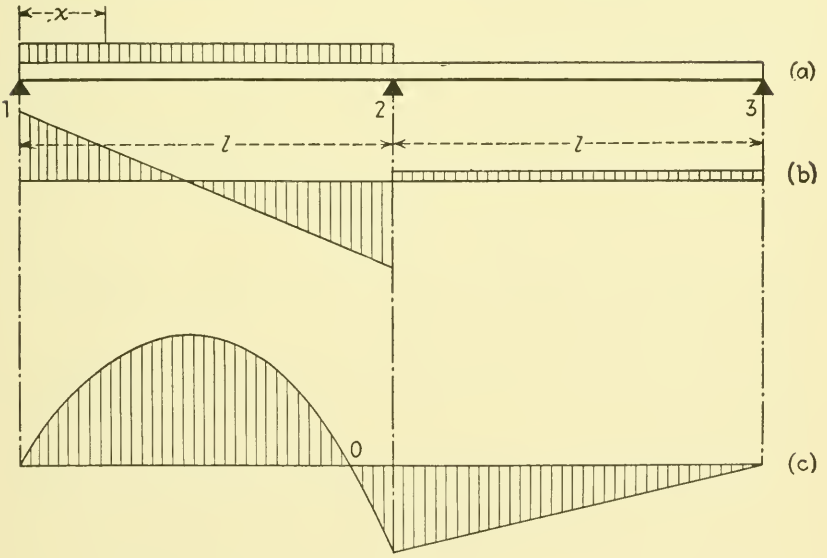


FIG. 7B.

Typical shears and moments are represented in Fig. 7B. The shears vary as indicated by (*b*) and the moments as in (*c*). The point of contraflexure is at 0.

6. A uniform load on both spans.—Assume a uniform load of w per unit of length upon both spans. From Eq. (4), Chapter VI, when

$$M_1 = 0, \quad M_3 = 0, \quad \text{and} \quad l_1 = l_2 = l.$$

$$M_2 = -\frac{wl^2}{8}.$$

Reactions.—From Eq. (5), Chapter VI, for a uniform load in place of a concentrated load.

$$R_1 = \tfrac{3}{8}wl, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

$$R_2 = \tfrac{5}{4}wl. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

$$R_3 = \tfrac{3}{8}wl. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Shears.—The shear at any point, x, in the first span is

$$R_1 - wx, \quad \text{or} \quad S_x = \tfrac{3}{8}wl - wx = \frac{w}{8}(31 - 8x). \quad . \quad . \quad (22)$$

The shear becomes zero at $x = \frac{3}{8}l$.

The shears in the second span are the same as in the first when x_2 is measured from support 3.

Moments.—The bending moment at any point in the first span is

$$M_x = \tfrac{3}{8}wlx - \frac{wx^2}{2} = \frac{w}{8}(3l - 4x)x. \quad . \quad . \quad . \quad . \quad . \quad (23)$$

The maximum positive moment occurs where $x = \frac{3}{8}l$.

$$M_{\text{max.}} = \tfrac{9}{128}wl^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

The moment becomes zero at $x = \frac{3}{4}l$, which is the point of contraflexure.

The expressions for the moments in the second span are the same as in the first when x_2 is measured from support 3.

Typical shears and moments are represented in Fig. 7C. The shears vary as shown by (*b*) and the moments as in (*c*). The point of contraflexure is at 0.

7. End conditions assumed for calculating stresses.—When a center-bearing swing span is closed, with its ends just touching their supports, any live load upon one arm will cause an uplift at the free end of the other arm. A live load passing across the span would thus cause pounding at the ends of the span. This could be prevented by latching down the ends. The latches and the mechanism for operating them would be complicated. It would also be necessary to anchor the end supports to the masonry sufficiently to resist the negative end reactions from the latches. Experience has shown that more satisfactory results

are obtained when the ends are lifted enough to overcome all tendency to pound. The end reactions under all conditions then become positive, and the end supports upon the piers have only to be secured in position.

For ordinary swing spans it has been found that the ends should be lifted until the dead-load positive end reactions are about one and one-half times the maximum live-load negative reactions when one arm is loaded. In some cases the droop of the ends due to temperature changes causes additional end reactions which, when combined with those due to uplift, become too large to be easily provided for. In such cases the end uplift can be reduced without danger of pounding.

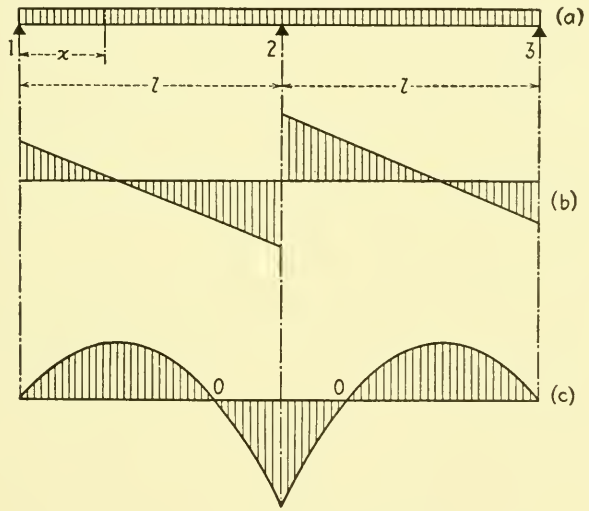


FIG. 7C.

8. Load conditions for calculating stresses.—The maximum stresses in the main girders, or trusses, of swing spans should be calculated for each of the following conditions of loading.

Case I.—Dead load: Bridge open, or closed with no end reactions.

Case II.—Dead load: Bridge closed, with its ends lifted to cause positive end reactions equal to one and one-half times the maximum live-load negative reactions, without impact.

Case III.—Live load: Bridge closed, but with no dead-load end reactions. One arm loaded and considered as a simple span.

Case IV.—Unbroken live load: Bridge closed and considered as a continuous girder.

Case V.—Unbroken live load: Bridge closed and considered as a continuous girder, but with the live load placed so as not to cause negative reactions.

Cases I, II, and III require no comment. In Cases IV and V the stresses in some parts of a continuous girder, or truss, may be greater when the live load is broken and each part placed where it will contribute to the stress in the member. Such an exact division of the live load is not likely to occur during its passage, particularly in the case of railway bridges. Unbroken loads in the position, and of the length, required for maximum stresses in the various members are much more probable. If the bridge is located so that broken loads are likely to occur, they should be used in Cases IV and V, but this will seldom be required.

The placing of the live loads for maximum stresses in one arm for Case V requires special attention. Case V, as stated, requires that there be no negative reactions. The live load, when placed for maximum stress in any member of the left arm, for example, must extend across the center support and upon the right arm until the reaction at the right end is either zero or positive, depending upon the particular member of the left arm under consideration.

The dead-load and live-load stresses from Cases I to V must be combined to find the maximum stresses for which the girders, or trusses, must be designed. The following combinations should be considered.

Case I alone.

Case I with Case III.

Case I with Case V.

Case II with Case IV.

The positive shears near the end of the arm, and some of the negative shears, will be a maximum when the load is on one arm only. The maximum positive bending moments also occur when only one arm is loaded. When the ends are not lifted, a load on one arm will cause the end of the unloaded arm to rise and the loaded arm to act as a simple span for live load; hence Case I should be combined with III. When the ends are neither lifted nor latched down, Case I will also be combined with Case V. When the ends are lifted, Case II should be combined with Case IV.

The girders, or trusses, should be designed for the maximum combined stresses in each member. When stresses of opposite sign occur in a member, due to separate combinations, the member should be designed for the largest combined stress of either kind. When the stress reversal is caused by one passage of the live load, an addition may be made to the stress of each kind to provide for the relatively

sudden reversal of the stress. This is frequently done by adding to each of the alternating stresses one-half of the smaller.

When the bridge is to be designed for a dynamic allowance, or impact, added to the live-load stresses, the addition should be made before making the combinations. This method will result in greater stress reversals than would have occurred had the simple live-load stresses been combined with the dead-load stresses. Reversal may be indicated in some members which would not have been subject to reversal had the impact method not been used.

9. Stresses and design of a center-bearing swing bridge, with two equal arms.—Assume a single-track, through, center-bearing swing span, with two equal arms of the dimensions shown in Fig. 7D (*a*). Let the live load be Cooper's Class E60. The dead loads for such a bridge will be about as follows:

Trusses and bracing 1750 lbs. per linear foot of span, one-half being considered as concentrated at the upper and one-half at the lower panel points.

	Panel Loads	
	Upper, Pounds	Lower, Pounds
Trusses and bracing..... $\frac{1750}{2\times2}\times30$	13,125	13,125
Steel in floor system..... $\frac{750}{2}\times30$	.....	11,250
Track..... $\frac{400}{2}\times30$	.....	6,000
Total at intermediate panel points.....	13,125	30,375
At the end lower panel point assume.....		22,000

Note that, in the truss shown, the center line of the upper chord member *FE*, produced, passes through *a*.

Case I.—Dead load: Bridge open, or closed with no end reactions.—The dead-load stresses may be calculated by the simple analytical or graphical methods, and no explanations of such methods need be given. The stresses are shown in Table 7D, column 2.

Case II.—Dead load: Bridge closed with its ends lifted.—When the live load is placed in span 2 in the position to cause the maximum negative reaction at *a*, this reaction is found to be 33,300 lbs. One

and one-half times this is 49,950 lbs. The stresses may be found by the ordinary methods. They are given in column 3.

Case III.—Live load: Bridge closed, no dead-load end reactions, simple span.—The maximum stresses are easily found by the usual methods. They are shown in column 4, upper series of figures. Below each stress is also given the impact from the formula.[1]

$$I = S\frac{2000 - L}{1600 + 10L}.$$

S is the calculated stress, L the loaded length of track when the stress is a maximum, and I is the addition for dynamic increment, or impact. The sum of S and I is also given in the table for each member.

Case IV.—Unbroken live load: Bridge closed and considered as a continuous girder.—In this case the ends of the span are considered as just touching the supports, but held down so that negative reactions, as well as positive, may be developed. Either or both arms may be loaded in order to produce the greatest stresses, depending upon the member under consideration. The truss in this case is a true continuous girder for live loads, and the ordinary methods of calculation become long and tedious. The direct use of influence lines has been found to be the most expeditious method of calculation, and the results are relatively as accurate as the areas of the members when made up from commercial rolled steel plates and shapes.

Typical calculations of an influence line for a chord and a web member will be given.

Chord BCD.—The center of moments is at c. The end reaction, R_1, for a unit load at any joint is taken from Table 7A.

TABLE 7B

Position of Unit Load	Value of R_1	Influence Moment at c
b	0.752	$-1\times30+0.752\times2\times30=$ 15.12
c	0.516	$0.516\times2\times30=$ 30.96
d	0.304	$0.304\times2\times30=$ 18.24
e	0.128	$0.128\times2\times30=$ 7.68
g	−0.072	$-0.072\times2\times30=-$ 4.32
h	−0.096	$-0.096\times2\times30=-$ 5.76
i	−0.084	$-0.084\times2\times30=-$ 5.04
j	−0.048	$-0.048\times2\times30=-$ 2.88

[1] Specifications for Railway Bridges, *Trans. Am. Soc. C. E.*, Vol. LXXXVI, p. 471, 1923.

The influence line for moments at c is shown in Fig. 7D (b), by the broken line, $ac'fM_ck$. The positive moments are plotted above the line afk and the negative below it. In a similar manner the influence lines for moments at the remaining panel points b, d, e, and f are calculated and drawn. Note that the moments at e for loads on

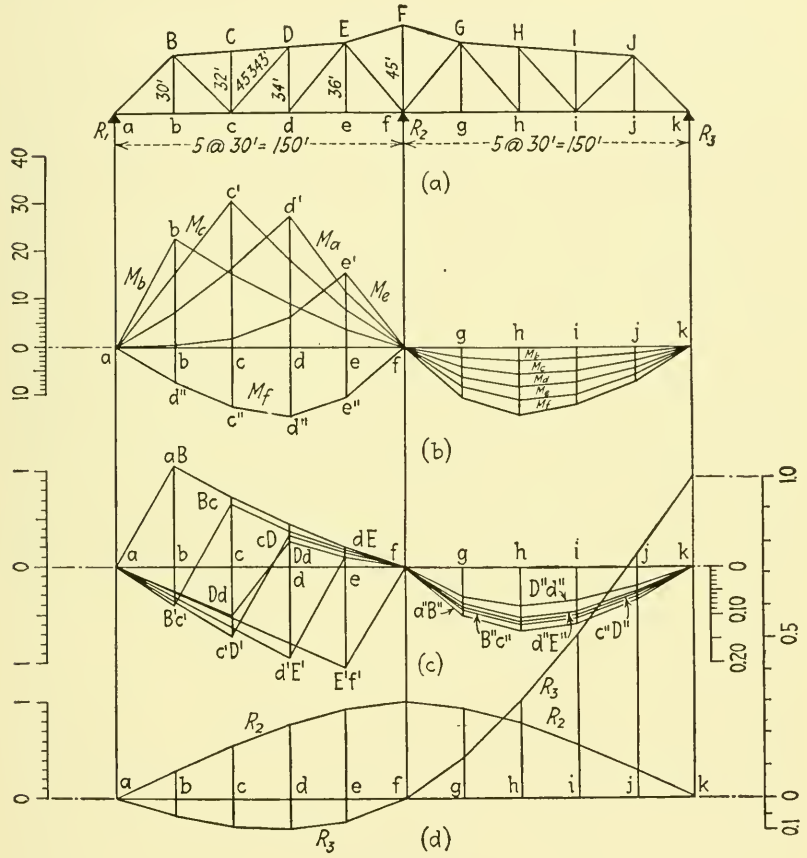

B C D E F G H I J
30' 32' 45 3/4' 34' 36' 45'
R1 a b c d e f R2 g h i j k R3
5@30'=150'
5@30'=150'
(a)
Mb Mc Ma Me Mf
(b)
aB Bc cD dE Dd
B'c' c'D' d'E' E'f'
a"B" B"c" d"E" c"D" D"a"
(c)
R2 R3
(d)
0.10 0.20 0.5 1.0

FIG. 7D.

the left arm are all positive. Had the arm been divided into six panels, in place of five, the moment at e for a load at b would have been negative. The moment at f is negative for loads at any point of either arm.

The influence lines may be drawn on cross-section paper, for convenience in reading their values when the loads are applied. The live-load diagram should be laid out on a strip of paper, to the same horizontal scale as the influence diagram. The load diagram is then

applied to the influence diagram in the position required for the maximum moment at any joint. A little experience will soon indicate which axle load should be placed at any joint and the direction in which the load should be headed. For maximum positive moments in span 1, span 2 should not be loaded. For negative moments in span 1, span 2 should be loaded. For maximum negative moments at the center of the bridge, both spans should be loaded.

When the load diagram is correctly located for the maximum moment at any joint, the ordinates to the influence line, for this joint, are either read directly from the cross-section paper or scaled if plain paper is used. The ordinate for any axle is then multiplied by the axle load and the products summed for all of the axles. It is convenient to scale and sum the ordinates for all duplicate axles before the multiplications are made.

The chord stress for any center of moments is found by dividing the maximum bending moment at the joint by the lever arm of the chord.

Web member cD.—The influence line for web stresses may be calculated either for the shear in the panel or for the actual stress in the web member. When either or both of the chords is inclined, the influence line for stress is the more convenient, and will be used in this example. Chords *EB* and *fa*, produced, intersect 15 panel lengths to the left of *a*. The lever arm of *cD* about this moment center is $17 \times \frac{34}{45.343} = 12.747$ panel lengths.

TABLE 7C

Position of Unit Load	Value of R_1	Moment about Center of Moments M	Influence Stress in $cD = \frac{M}{12.747}$
b	0.752	$0.752 \times 15 - 1 \times 16 = -4.72$	−0.370
c	0.516	$0.516 \times 15 - 1 \times 17 = -9.26$	−0.726
d	0.304	$0.304 \times 15 = 4.56$	0.358
e	0.128	$0.128 \times 15 = 1.92$	0.151
g	−0.072	$-0.072 \times 15 = -1.08$	−0.085
h	−0.096	$-0.096 \times 15 = -1.44$	−0.113
i	−0.084	$-0.084 \times 15 = -1.26$	−0.099
j	−0.048	$-0.048 \times 15 = -0.72$	−0.056

The influence line for the stress in *cD* is shown in Fig. 7D (*c*), by the broken line, *a*, *c'D'*, *cD*, *f*, *c''D''*, *k*. The process for the other webs

is similar. All of the influence lines are plotted in Fig. (*c*). For clearness, the lines for span 2 have been plotted to a much larger scale than was used for those for span 1. It should be noted that when stress influence lines are used, there is a separate line in span 2 for each principal web member of span 1. Had shear influence lines been used, only one line would have appeared for span 2.

Care must be taken to head the live load in the right direction, and put it in the correct position to produce the maximum direct and reverse web stresses. For positive shears near R_1, the load should be headed toward the left, two engines used, and no load placed on span 2. For *cD*, *Dd*, and *dE* the load is headed toward the left, one engine is used, and span 2 is not loaded. For stresses due to negative shear in *Bc*, span 2 is loaded, the load being headed toward the right, and span 1 is not loaded. For *dE* the load heads toward the left and span 2 is loaded. For *Ef*, since *FE*, produced, passes through *a*, the position of the load and the stress in *Ef* is the same as for a simple span, of length *af*. Note that the live load is considered as unbroken for all web stresses in this example.

The direct and reverse stresses for all members of the truss are shown in Table 7D, columns 6 and 7, also the impact additions and the totals to be used when combining Case IV with Case II.

Case V.—Unbroken live load: Bridge closed and considered as a continuous girder. No negative reactions.—The conditions imposed, namely, that the load be unbroken, and placed so that no negative reactions are produced, make a preliminary calculation necessary to determine the portions of the spans to be loaded. Consider the first, or left, span. When the axle loads are in the positions to cause maximum positive moments, such a part of span 2, to the right of R_2, must be loaded, as to neutralize the negative reaction at R_3, due to loads in span 1. When the axle loads are in the positions to cause maximum positive shears in the web members of span 1, a part of span 2, to the right of R_2, must be loaded. The load is headed toward the left for these web members. For the maximum negative shears in span 1, some wheel of the second engine is placed at the critical panel point, and, as all loads in span 2 add to the negative shears, span 2 must be fully loaded.

For the positive moments and shears in span 1, the portion of span 2 that must be loaded to the right of R_2 is most readily found from an influence line for reactions at R_3. Such a line is shown in Fig. 7D (*d*).

As an example, consider the maximum stress in *cD* due to positive shear. The load is headed toward the left with only one engine in

TABLE

1	2	3	4	5	6	7	8	9
Member	I	II	III	III	IV	IV	V	V
			Direct	Reverse	Direct	Reverse	Direct	Reverse
abc	− 22.0	+ 28.0	+245.6 +146.6		+195.5 +116.7	− 33.3 − 19.9	+182.3 + 85.8	
			+392.2		+312.2	− 53.2	+268.1	
cd	−173.4	− 41.1	+299.4 +178.7		+209.6 +125.1	− 88.3 − 52.7	+174.8 + 82.4	− 3.5 − 1.6
			+478.1		+334.7	−141.0	+257.2	− 5.1
def	−290.8	−124.3	+192.0 +114.6		−111.2 − 66.4	+ 79 5 + 47.5	+ 38.1 + 18.1	− 60.8 − 28.7
			+306.6		−177.6	+127.0	+ 56.2	− 89.5
BCD	+ 82.2	− 11.7	−318.9 −190.3		−254.4 −151.8	+ 62.7 + 37.4	−223.6 −105.8	
			−509.2		−406.2	+100.1	−329.4	
DE	+173.8	+ 41.3	−300.1 −179.1		−210.1 −125.4	+ 88.5 + 52.8	−175.2 − 82 5	+ 3.5 + 1.6
			−479.2		−335.5	+141.3	−257.7	+ 5.1
EF	+379.3	+205.5	0		+210.0 + 77.6		+210.0 + 77.6	
					+287.6		+287.6	
aB	+ 31.1	− 39.4	−338.5 −202.0		−276.1 −164.8	+ 47.1 + 28.1	−257.5 −121.3	
			−540.5		−440.9	+ 75.2	−378.8	
Bc	− 85.4	− 23.1	+168.3 +113.1	− 32.5 − 28.7	+133.9 + 89.9	− 42.1 − 25.1	+118.8 + 62.6	+22.4 + 8.3
			+281.4	− 61.2	+223.8	− 67.2	+181.4	+30.7
cD	+138.5	+ 79.7	+ 98.9 + 75.5	− 72.2 − 55.2	+116.6 + 89.1	− 50.2 − 38.4	+116.0 + 42.9	−43.9 −28.3
			+174.4	−127.4	+205.7	− 88.6	+158.9	−72.2
Dd	−111.2	−' 69.6	− 69.9 − 53.4	+ 51.6 + 39.5	− 84.7 − 64.7	+ 36.4 + 27.8	− 80.3 − 29.7	+31 9 +20.6
			−123.3	+ 91.1	−149.4	+ 64.2	−110.0	+52.5
dE	+184.3	+130.1	+182.2 +122.2	− 20.7 − 18.2	+254.6 +112.0	− 7.9 − 8.2	+254.6 +112.0	− 6.8 − 5 5
			+304.4	− 38.9	+366.6	− 16.1	+366.6	−12.3
Ef	−113.3	−113.3	−299.9 −179.0	0	−299.9 −179.0		−299.9 −179.0	
			−478.9		−478.9		−478.9	
Bb *Ee*	+ 30.4	+ 30.4	+129.9 +114.6		+129.9 +114 6		+129.9 +114.6	
			+244.5		+244.5		+244.5	
Ff	−218.0	−118.1	0		−120.7 − 44.6		−120.7 − 44 6	
					−165.3		−165.3	
Cc	− 13.1	− 13.1	0		0		0	

7D

10	11	12	13	14	15	16	17	18		
								SECTIONS		
I and III	I and V	II and IV	Maximum Combinations	$\frac{r}{\frac{l}{r}}$	Unit Stress	Area Reqd.	Area Used	Material	*A* Gr.	*A* Net
		+340.2	I and III +370.2					Same as *def*		
+370.2	+246.1	− 25.2	II and IV − 25.2							
	+ 83.8	+293.6	II and IV +384.7					Same as *def*		
+304.7	−178.5	−182.1	II and IV −273.2							
	−234.6	−301.9	I and V −380.3	6.7	13.2	28.8	28.9	2 webs 18×$\frac{7}{16}$	15.7	13.1
+ 15.8	−380.3	+ 2.7	I and III + 15.8	54	16.0	1.0	24.6	4 ang. 4×4×$\frac{7}{16}$	13 2	11 5
									28.9	24.6
		−417.9	I and IV −462.1	7.3	13.58	34.0	37.8	1 cov. 21×$\frac{7}{16}$	9.2	8.3
								2 webs 18×$\frac{7}{16}$	15 7	13.1
			I and IV					2 ang. $3\frac{1}{2}\times3\frac{1}{2}\times\frac{3}{8}$	4.9	3.4
−427.0	−247.2	+ 88.4	+132.6	49	16.0	8.3	30.8	2 ang. 5×$3\frac{1}{2}\times\frac{1}{2}$	8.0	6.0
					.				37.8	30.8
	+178.9	−294.2	I and IV −385 5	7.3	13.58	28.4	37.8	Same as *BCD*		
−305.4	− 83.9	+182.6	I and IV +273.9	49	16.0	17.1	30.8			
			I and V							
+379.3	+666.9	+493.1	+666.9		16.0	41.7		4 eyebars 7×$1\frac{1}{2}$	42.0	42.0
			I and III					1 cov. 21×$\frac{7}{16}$	9 2	8.3
		−480.3	−509.4	7.0	11.47	44.4	44.6	2 webs 18×$\frac{5}{8}$	22 5	19 0
			II and IV					2 ang. $3\frac{1}{2}\times3\frac{1}{2}\times\frac{3}{8}$	4.9	3.4
−509.4	−347.7	+ 35.8	+ 35.8	73	16.0	2.2	36.7	2 ang. 5×$3\frac{1}{2}\times\frac{1}{2}$	8.0	6 0
									44.6	36.7
+196.0	+ 96.0	+200.7	II and IV +200.7	5.6	16.0	12.6	17.4	2 channels 15″×33 lb.	19.8	17.4
−146.6	− 54.7	− 90.3	I and III −146.6	91	9.91	14.8	19.8			
		+285.4	I and III +312.9	5.4	16.0	19.6	20.4			
+312.9	+297.4	− 8.9	II and IV − 8.9	100	9 19	1 0	23.5	2 channels 15″×40 lb.	23.5	20.4
−234.5	−221.2	−219.0	I and III −234.5	5.6 73	11.47	20.5	20 6	2 channels 15″×35 lb.	20.6	
			I and V					2 webs 18×$\frac{5}{8}$	22.5	17.5
+488.7	+550.9	+496.7	+550.9		16.0	34.4	34.5	4 ang. 6×4×$\frac{1}{2}$	19 0	17 0
									41.5	34 5
			Any	7 9				2 webs 22×$\frac{11}{16}$	30.2	
−592.2	−592.2	−592.2	−592.2	72	11.56	51.2	53.6	4 ang. 6×4×$\frac{5}{8}$	23.4	
									53.6	
			Any					1 web $10\frac{1}{2}\times\frac{1}{2}$	5 2	4.2
+274.9	+274.9	+274.9	+274.9		16.0	17 2	17.4	4 ang. 6×4×$\frac{7}{16}$	16 7	13.2
									21 9	17.4
			I and V	7.8				2 webs 22×$\frac{1}{2}$	22.0	
−218.0	−383.3	−283.4	−383.3	69	11.83	32.4	33.5	4 ang. $3\frac{1}{2}\times3\frac{1}{2}\times\frac{7}{16}$	11.5	
									33.5	
			Any	3.0						
− 13.1	− 13.1	− 13.1	− 13.1	128	7.23	1.8	16.7	4 ang. 6×4×$\frac{7}{16}$, $\frac{3}{4}$″ *b* to *b*.	16.7	

front of the train, and with axle 11 at panel point d. With the unbroken load in this position, it is found, by scaling and summing the reactions at R_3 due to each axle on span 1, that the negative reaction at R_3 is $-$ 10,200 lbs. The length of the train load to the right of R_2, in span 2, to neutralize this negative reaction may then be found from the reaction influence line, (d). The uniform load in panel fg causes a positive reaction at R_3 equal to

$$\frac{0.128}{2} \times 30 \times 3000 = + 5760 \text{ lbs.}$$

The load must extend into panel gh until 10,200 $-$ 5760 $=$ + 4440 lbs. is produced at R_3. The length of uniform load necessary is found from the expression,

$$\left(0.128 + \frac{0.0059x}{2}\right) \times (3000) = 4440,$$

from which $x = 9.5$, very nearly. The uniform load should then extend 9.5 ft. beyond g into panel gh.

The maximum stress in cD with one arm loaded and with wheel 11 at d is

	Pounds
	$-$ 50,240
The positive stress due to loads in span 2 is:	
$\frac{0.058}{2} \times 30 \times 3{,}000$	$= +$ 3825
$\frac{0.825 + \left[0.835 + (.113 - .085)\frac{9.5}{30}\right]}{2} \times 9.5 \times 3000$	$= +$ 2545
	+ 6,370
The resultant stress in cD for Case V then is	$-$ 43,870

These maximum stresses for Case V in all members of the truss are shown in Table 7D, columns 8 and 9. The impact additions and the totals to be used, when combining Case V with Case I, are also given.

Combined stresses.—The combinations of the stresses from Case I with Case III, Case I with Case V, and Case II with Case IV are shown in columns 10, 11, and 12. Column 13 gives the maximum combinations for which the various truss members must be designed.

The "Specifications for Steel Railway Bridge Superstructure," reported by the Special Committee on Specifications for Bridge Design

and Construction of the American Society of Civil Engineers, Art. 112,[2] provides that "if the alternate stresses occur in succession during the passage of one train, as in stiff counters, each stress shall be increased by 50 per cent of the smaller." This provision applies equally well to a swing span, for some members usually are subjected to such alternate stresses. In the example under consideration it is found by inspection that the only members affected by this provision are the chords *cd*, *BCD*, and *DE*, for the combination of Cases II and IV. In column 13, 50 per cent of the smaller stress has been added to each of the alternating stresses in column 12, for this combination.

Design of members.—The unit stresses and methods called for by the Specification referred to above have been used in the design of the various truss members. It is also assumed that ordinary structural steel would be used. Columns 14, 15, 16, and 17 indicate the process, and the resulting sections are shown in column 18.

10. Stresses corrected for actual truss conditions.—The methods of analysis thus far given are based upon the ordinary theory of beams and constant moments of inertia throughout the girder or truss. The common theory of the deflection of beams does not take into account the deflections due to the web distortions, but only those due to changes in length of the chords. In beams with solid webs, the error introduced is not serious; but in the case of trusses with ordinary web members, it may be too large to be neglected. For example: The end deflection due to the chord stresses in the truss treated in Chapter IX, Art. 16, Table 9L, is about 1.74 in., while that due to stresses in the web members is 0.52 in., or about 30 per cent of that due to the chords. Of the total end deflection, about 77 per cent is due to the chords and 23 per cent to the webs. The moment of inertia was also assumed to be constant throughout the length of the truss, while, as a matter of fact, it increased from the ends toward the center pier, owing to the increased depth of the truss and larger sections at and near the center.

The two errors in the common theory are, in some degree, compensating; hence it is better to ignore both than to attempt to make corrections for either alone.

When it is desired to correct a design to take account of the actual truss conditions, the method of redundant members is the most convenient.

In the case of a center-bearing swing bridge, it is convenient to consider the center reaction, R_2, as redundant. The truss is then assumed to be supported only at its ends.

[2] *Trans. Am. Soc. C. E.*, Vol. LXXXVI, p. 478, 1923.

Let P = the stress in any member due to the given loads;
u = the stress in any member due to a unit load applied upward at the center;
l = the length, and A the area of any member, then

$$R_2 = -\frac{\sum \frac{Pul}{EA}}{\sum \frac{u^2 l}{EA}}. \quad \ldots\ldots\ldots \quad (25)$$

By the application of Eq. (25) for a unit load at each of the loaded joints, an influence line can be calculated and drawn, but the labor involved is considerable, and a more convenient method will be given.

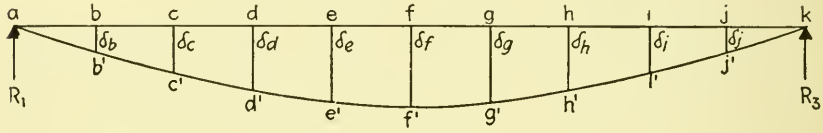


Fig. 7E.

Let Fig. 7E be a deflection diagram of the bridge when supported at its ends and loaded with a unit load at its center. By Maxwell's principle of reciprocal deflections, the deflection at f due to a unit load at any other point, as c, is equal to δ_c, the deflection at c due to a unit load at f; hence the reaction at f due to a unit load at c is $\frac{\delta_c}{\delta_f}$, or in general:

$$R_2 = \frac{\delta}{\delta_f}, \quad \ldots\ldots\ldots \quad (26)$$

in which δ is the deflection at any joint due to a unit load at f, when the bridge is assumed to be supported only at its ends; δ_f is the deflection at f; and R_2 is the center reaction due to a unit load at the given joint. If the deflections be plotted to such a scale that δ_f is unity, the curve $af'k$ becomes an influence line for R_2.

11. Application to the truss shown in Fig. 7D (a).—The method outlined in Art. 10 will be applied to the truss shown in Fig. 7D (a). The lengths, cross-sections, stresses, and values of $\frac{ul}{A}$ for a unit load at f, with the truss supported at R_1 and R_3, are given in Table 7E. The unit load at f is assumed as acting downward.

TABLE 7E

Member	Length l	Cross Section A	u	$\frac{ul}{A}$
abc	720	30.7	+0.500	+11.73
cd	360	30.7	+1.324	+15.53
def	720	30.7	+1.667	+39.10
BCD	721.6	37.8	−0.940	−17.94
DE	360.8	37.8	−1.327	−12.67
EF	375.8	43.7	−1.740	−14.96
aB	509.1	46.8	−0.707	− 7.69
Bc	509.1	19.8	+0.624	+16.05
cD	544.1	26.4	−0.588	−12.12
Dd	408	23.5	+0.417	+ 7.24
dE	561.9	45.2	−0.542	− 6.74
Ef	561.9	53.6	0	0
Ff	540	33.5	+1.000	+16.12

The values of $\frac{ul}{A}$ in the last column, when divided by E, are the elastic deformations of each member. As only comparative results are required, the division by E need not be performed.

Construct a Williot diagram for the values of $\frac{uv}{A}$, beginning at the joint *f*. Fig. 7F is the diagram for one-half of the span. Scale the deflection of each loaded joint from the line *aa'*. Divide each deflection by the deflection of the joint *f*. The results are given in Table 7F.

TABLE 7F

Joint Loaded	Deflection δ	Actual Influence, R_2	Approximate Influence, R_2 from Table 7A	Error of Approximate	Error, Per Cent of Actual
a	0.0	0.0	0.0	0.0	0.0
b	106.0	0.2797	0.2960	+0.0163	+5.83
c	211.5	0.5581	0.5680	+0.0099	+1.77
d	304.0	0.8022	0.7920	−0.0102	−1.27
e	358.2	0.9452	0.9440	−0.0012	−0.13
f	379.0	1.0000	1.0000	0.0	0.0

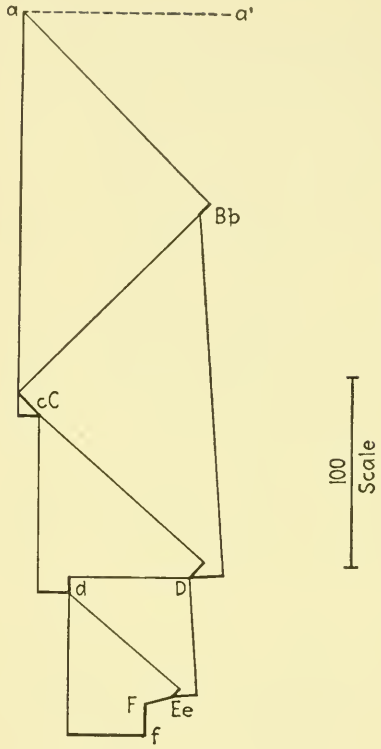


Fig. 7F.

The influence line for R_2 having been determined, reactions R_1 and R_3 may be found by taking moments about R_3 and R_1, and the influence lines for moment and shear at any point may be found and compared with those of Fig. 7D (*b*) and (*c*). The process is the same as outlined in Art. 9. The complete influence lines may then be constructed, the axle loads applied, and the stresses found and compared with those previously determined by the approximate method. The example will not be fully carried out. Closely approximate results may be found by comparing the influence areas, which amounts to assuming that a certain load per linear foot of span determines the stress at each joint, successively, both for the approximate and the more exact method, by the use of the actual reactions. The results are given in Table 7G.

TABLE 7G

Comparison of Bending Moments

Joint	Sum of Influence Line Ordinates		Difference	Per Cent of Error
	Exact	Approximate		
b	+51.225	+51.000	0.225	−0.43
c	+72.450	+72.000	0.450	−0.62
d	+63.675	+63.000	0.675	−1.06
e	+24.900	+24.000	0.900	−3.61
f	−87.750	−90.000	2.25	+2.56

A precisely similar method may be used for the shears, or the web stresses, if preferred.

12. A center-bearing draw bridge with unequal spans.—Let the length of the first span be l_1 and that of the second l_2. Consider a single load in the first span at a distance kl_1 from the left support, R_1. Apply Eq. (3), of Chapter VI, observing that

$$M = 0, \quad M_3 = 0, \quad \text{and} \quad P_2 = 0; \text{ then}$$

$$M_2 = -P_1 \frac{l_1^2(k - k^3)}{2(l_1 + l_2)}. \quad \ldots \quad (27)$$

From Eq. (5) S_1 becomes R_1 at support 1, and, by substituting the value of M_2 from (27),

$$R_1 = P_1\left[(1 - k) - \frac{l_1(k - k^3)}{2(l_1 + l_2)}\right]. \quad \ldots \quad (28)$$

$$R_2 = P_1\left[k + \frac{l_1(k - k^3)}{2(l_1 + l_2)}\left(1 + \frac{l_1}{l_2}\right)\right]. \quad \ldots \quad (29)$$

$$R_3 = -P_1 \frac{l_1^2(k - k^3)}{2(l_1 + l_2)l_2}. \quad \ldots \quad (30)$$

It is sometimes convenient to let nl_1 represent l_2, n being the ratio of l_2 to l_1. The equations then become

$$M_2 = -\frac{P_1 l_1}{2(1 + n)}(k - k^3). \quad \ldots \quad (31)$$

$$R_1 = \frac{P_1}{2(1 + n)}[2(1 + n) - (3 + 2n)k + k^3]. \quad \ldots \quad (32)$$

$$R_2 = \frac{P_1}{2n}[(1 + 2n)k - k^3]. \quad \ldots \quad (33)$$

$$R_3 = -\frac{P_1}{2(1 + n)n}(k - k^3). \quad \ldots \quad (34)$$

If n in Eqs. (32), (33), and (34) is made unity, length l_2 becomes equal to l_1 and the expressions reduce to the forms given in Eqs. (2), (3), and (4). All remaining values are readily found when the reactions are known, and the method of solution of a swing bridge with unequal spans may be the same as for one with equal arms. The number of stresses to be found is greater because of the lack of symmetry.

A swing bridge of this type is often called a bob-tailed draw.

SECTION II

THE RIM-BEARING SWING BRIDGE

13. A rim-bearing swing bridge. Truss partially continuous over four supports.—Figure 7G (*a*) represents the outline of a rim-bearing swing bridge of three spans, continuous over four supports. It is

evident that loads upon span 1 will cause heavy negative reactions at support 3; in fact, they would be larger than the dead-load reactions. Should an attempt be made to resist them, it would be found that special anchorages would be required. Should anchorages adequate to provide for the negative reactions be supplied, this form of truss would be undesirable, for the loads upon the drum would be unequal when the span was unsymmetrically loaded. This is one of the many cases where it is wiser to avoid the difficulty than to meet the theoretical conditions by unusual devices.

Of the various designs that have been used to overcome the large negative reactions due to unsymmetrical loads, probably the best and

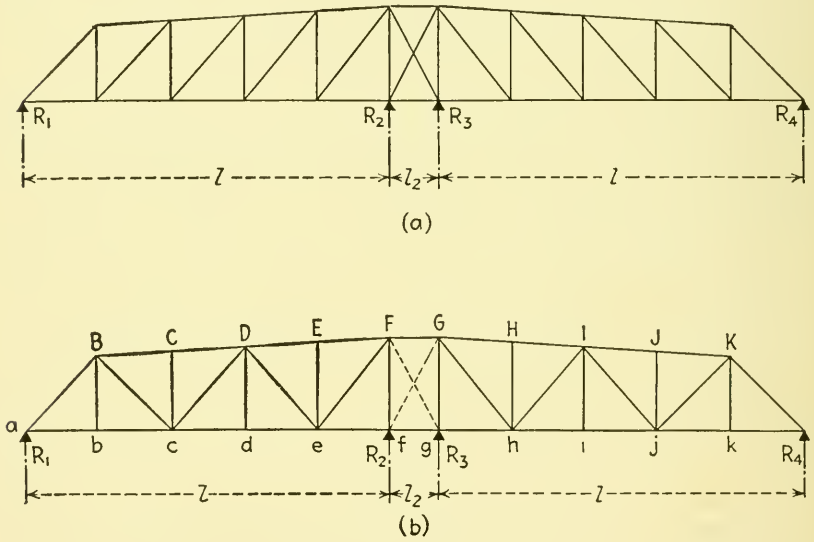


Fig. 7G.

most feasible is to make the diagonals in the middle panel, above the drum, of light sections, strong enough to resist vibrations and end wind forces, when the bridge is open, but weak enough to prevent full continuous-girder action across the drum bearings.

Such a truss is *partially continuous*. The middle panel chords are made strong enough to provide for the full bending moments, due to continuity; but the middle web members are designed so that they act only as braces for the middle posts when the span is open, and cannot carry the shears theoretically due to a continuous condition. Experience has shown that such trusses are permanently satisfactory, and they may be considered as standard designs for this purpose.

Various designs, providing links connected to the tops of the middle posts by pins, and similar devices to avoid the shearing stresses, have been built, but, with the resulting complications in details, they have not proved of enough value to justify their costs.

Figure 7G (*b*) represents one form of a partially continuous swing draw bridge which has proved to be a satisfactory design. Here the

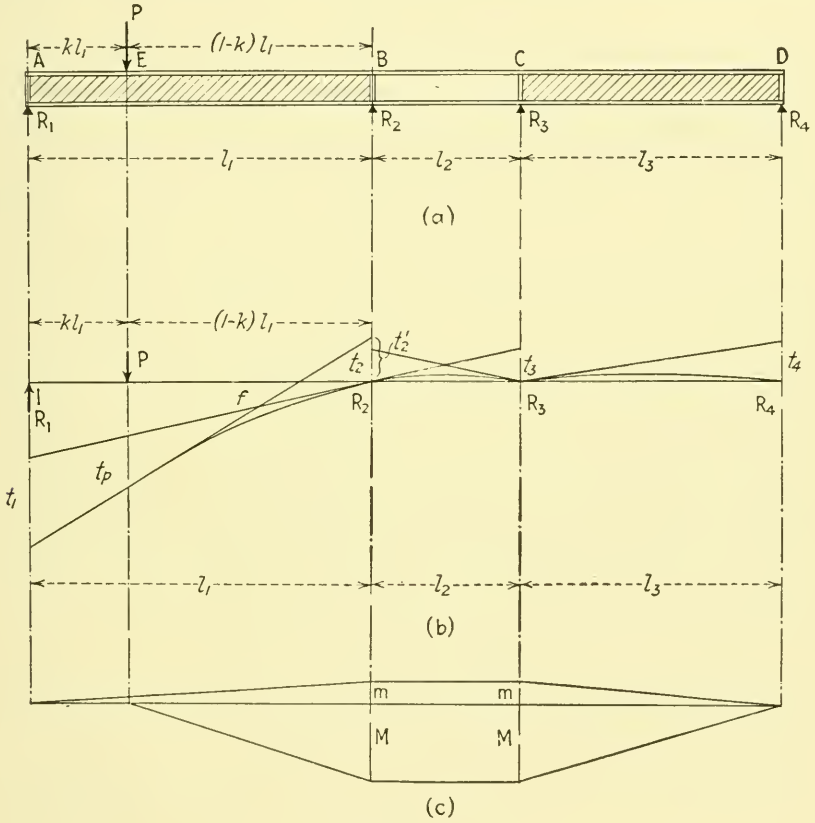


FIG. 7H.

middle panel diagonals, *Fg* and *fG*, are made of light nominal sections, determined from practical considerations. They must be able to resist all stresses due to endwise wind pressure and vibrations when the span is open, but are assumed to carry no shearing stresses due to unsymmetrical live loads.

14. Reactions for a beam partially continuous over four supports. No shear in the middle span.—Let Fig. 7H (*a*) represent a beam, of

uniform moment of inertia, supported at R_1, R_2, R_3, and R_4. The span lengths are l_1, l_2, and l_3. The web is assumed to be omitted in span 2, but the flanges continue to be of full section between B and C. Assume a single load at any point of span 1, distant kl_1 from A. Let it be required to find the four reactions.

The deflection of a loaded beam, Chapter IX, Art. 1, Eq. (1), is:

$$\Delta = \int_A^D \frac{Mmdx}{EI} \quad \ldots\ldots\ldots \quad (35)$$

If reaction R_1 is considered to be omitted, this expression gives the deflection at $A = \Delta$. If now a reaction of one unit is assumed at A, the beam being otherwise unloaded, the deflection is

$$\delta = \int_A^D \frac{m^2dx}{EI} \quad \ldots\ldots\ldots \quad (36)$$

The actual reaction at A due to the load P, then is:

$$R_1 = \frac{\Delta}{\delta} = \frac{\int_A^D \frac{Mmdx}{EI}}{\int_A^D \frac{m^2dx}{EI}} \quad \ldots\ldots\ldots \quad (37)$$

The integration can be made, by the usual methods, separately for the parts AE, EB, BC, and CD, and the division performed as indicated. It is interesting to substitute the method of area-moments of Chapter IX, Art, 6; and this will be done not only to obtain a convenient solution, but to indicate the utility of the method.

Let Fig. (*b*) represent the deflected beam and the tangent-deviations, to greatly exaggerated scale, and Fig. (*c*) the bending moments due to the load, P, and the end reaction, R_1, of unity.

The bending moment at B and C, due to the load P, are each equal to $M = -P(1-k)l_1$. We can then write at once by inspection,

$$EIt_p = -P(1-k)l_1 \times \tfrac{1}{2}(1-k)l_1 \times \tfrac{2}{3}(1-k)l_1 = -\frac{P}{3}(1-k)^3 l_1^3.$$

$$EIt_2' = -P(1-k)l_1 \times \tfrac{1}{2}(1-k)l_1 \times \tfrac{1}{3}(1-k)l_1 = -\frac{P}{6}(1-k)^3 l_1^3.$$

The intersection f is at $\frac{1}{3}(1-k)l_1$ from R_2, and

$$EIt_1 = -\frac{\frac{P}{3}(1-k)^3 l_1^3}{\frac{2}{3}(1-k)l_1}[\tfrac{2}{3}(1-k)l_1 + kl_1]$$

$$= \frac{P}{6}(k - k^3)l_1^3 - \frac{P}{3}(1 - k)l_1^3. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

$$EIt_2 = EIt_3 = -P(1 - k)l_1 \times l_2 \times \frac{l_2}{2} = -\frac{P}{2}(1 - k)l_1l_2^2$$

$$EI(t_2 + t_3) = -P(1 - k)l_1l_2^2$$

The tangent-deviation at R_1, due to the deflection of span 2, then is

$$EIt_1'' = -P(1 - k)l_1l_2^2 \times \frac{l_1}{l_2} = -P(1 - k)l_1^2l_2. \quad . \quad . \quad . \quad . \quad . \quad (39)$$

$$EIt_4 = -P(1 - k)l_1 \times \tfrac{1}{2}l_3 \times \tfrac{2}{3}l_3 = -\frac{P}{3}(1 - k)l_1l_3^2,$$

but the tangent-deviation at R_1 due to t_4 is $t_4 \times \frac{l_1}{l_3}$; hence,

$$EIt_1''' = -\frac{P}{3}(1 - k)l_1l_3^2 \times \frac{l_1}{l_3} = -\frac{P}{3}(1 - k)l_1^2l_3. \quad . \quad . \quad . \quad . \quad . \quad (40)$$

Summing (38), (39), and (40),

$$EI(t_1 + t_1'' + t_1''')$$

$$= \frac{P}{6}(k - k^3)l_1^3 - \frac{P}{6}(1 - k)(2l_1^3 + 6l_1^2l_2 + 2l_1^2l_3). \quad . \quad (41)$$

Consider now the tangent-deviations due to the unit reaction at R_1. The bending moments at B and C are each equal to $m = l_1$; hence,

$$EIt_1 = l_1 \times \tfrac{1}{2}l_1 \times \tfrac{2}{3}l_1 = \tfrac{1}{3}l_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

$$EIt_2 = EIt_3 = l_1 \times l_2 \times \tfrac{1}{2}l_2 = \tfrac{1}{2}l_1l_2^2,$$

and

$$EIt_1'' = l_1l_2^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

$$EIt_4 = l_1 \times \tfrac{1}{2}l_3 \times \tfrac{2}{3}l_3 = \tfrac{1}{3}l_1l_3^2,$$

and

$$EIt_1''' = \tfrac{1}{3}l_1^2l_3. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

Summing (42), (43), and (44):

$$EI(t_1 + t_1'' + t_1''') = \tfrac{1}{6}(2l_1^3 + 6l_1^2l_2 + 2l_1^2l_3). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

Dividing (41) by (45),

$$R_1 = P\left[(1 - k) - \frac{l_1(k - k^3)}{2l_1 + 6l_2 + 2l_3}\right]. \quad . \quad . \quad . \quad (46)$$

The remaining reactions can now be found.

$$R_2 = P - R_1 = P\left[k + \frac{l_1(k - k^3)}{2l_1 + 6l_2 + 2l_3}\right]. \quad . \quad . \quad (47)$$

$$R_3 = -R_4 = P\frac{l_1(k - k^3)}{2l_1 + 6l_2 + 2l_3} \times \frac{l_1}{l_3}. \quad . \quad . \quad . \quad (48)$$

In most swing bridges, spans $l_1 = l_3 = l$, and the reactions become,

$$R_1 = P\left[(1 - k) - \frac{l(k - k^3)}{4l + 6l_2}\right]. \quad . \quad . \quad . \quad . \quad . \quad (49)$$

$$R_2 = P\left[k + \frac{l(k - k^3)}{4l + 6l_2}\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

$$R_3 = -R_4 = P\frac{l(k - k^3)}{4l + 6l_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

When the general expressions for the reactions as given above are known, the actual reactions for any system of loads may be found.

The method of influence lines, used for the center-bearing swing span analysis in Art 9, will be found convenient for calculating the stresses. It must be noted that the influence reactions for a partially continuous swing span cannot be taken from Table 7A, but must be calculated from the equations given above.

The expression for R_1, given in Eq. (46), differs from that in Eq. (28) only by the term $6l_2$ in the denominator.

All of these formulae are for constant moment of inertia.

After preliminary sections are found from the use of these equations, corrections can be made to conform to the actual moments of inertia by a method similar to that outlined in Arts 10 and 11.

SECTION III

DOUBLE SWING BRIDGES

15. Double swing bridge. Ends connected by a shear lock. Partially continuous over four supports.—In some cases where a large clear opening is required for navigation two adjacent swing spans have been used, as indicated by Fig. 7 I, providing a clear opening of BD. When

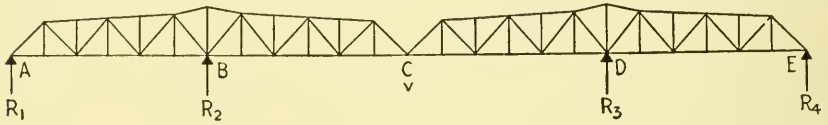


FIG. 7 I.

the bridge is closed the ends are locked together at C, so that shear, but no moment, is transmitted across the joint. The cantilever ends are thus forced to deflect together. If the ends were left free the pier reactions could be found from ordinary static expressions. As they are locked, each cantilever will react against the other by a force V. When this mutual reaction is known, all of the others are readily found.

Let it be assumed that the ends are not connected at C. Find the following deflections: (1) The deflection δ'_n at C, of the structure AC, for a single load placed at any point of AC. (2) The deflection δ'_c at C, of the structure AC, for a unit load at C. (3) The deflection δ''_c at C, of the structure CE for a unit load at C. If now the two structures be considered as connected at C, this point will deflect a distance δ_n under the load P, and δ_n will be less than δ'_n on account of the resistance of structure CE.

Let V be the mutual reaction between the arms at C; then for the structure AC,

$$\delta_n = \delta'_n - V\delta'_c$$

and for the structure CE,

$$\delta_n = V\delta''_c.$$

Solve these expressions for V and

$$V = \frac{\delta'_n}{\delta'_c + \delta''_c}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

When δ'_n is downward V is positive, or upward upon AC. When the two structures are alike and symmetrical about C, $\delta''_c = \delta'_c$ and

$$V = \frac{\delta'_n}{\delta'_c + \delta'_c} = \frac{1}{2} \times \frac{\delta'_n}{\delta'_c}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

The right reaction of an ordinary swing bridge, AC, is

$$R_3 = \frac{\delta'_n}{\delta'_c}, \text{ hence}$$

$$V = \tfrac{1}{2}R_3. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

The reaction V then is one-half of that for the same structure on rigid supports.

16. A single load on the first span.—The two spans are assumed to have the same moment of inertia throughout. Assume a single load, P, in span 1, and let V represent the mutual reaction of the shear-locked cantilevers upon each other. The method of area-moments will afford a simple means of finding the value of V. In Fig. 7J (a), ABC represents the first span and CDE the second. The deflection curves are sketched to greatly exaggerated scale. Figure (b) represents the bending moments due to the load P and the reaction V. They must be taken with their proper algebraic signs. The mutual reactions, V, being equal, the spans symmetrical, and the moments of inertia

constant from end to end of the two structures, the tangent-deviations at C must also be equal, or

$$t_{V_2} = t_{V_3} = t_V.$$

Let t_1 be the tangent-deviation at A and t_4 that at E. Suppose the tangents from A and E to be produced to intersection with the vertical through C; then

$$t_{V_1} = t_1 \frac{l_2}{l_1} \quad \text{and} \quad t_{V_4} = t_4 \frac{l_2}{l_1}.$$

Let the tangent-deviations at C, due to the deflections of the spans l_2, be t_{V_2} and t_{V_3}, respectively. The sum $t_{V_1} + t_{V_2}$ must then be equal to $t_{V_4} + t_{V_3}$. Call these sums t_V.

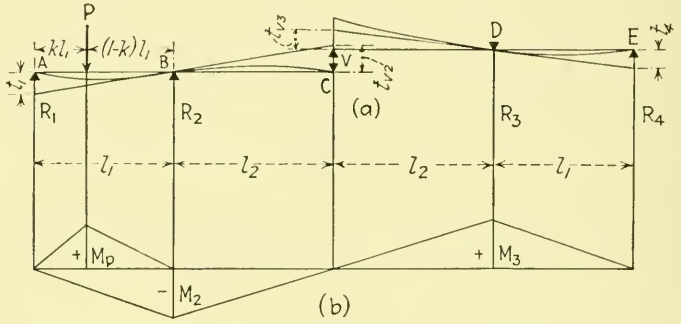


FIG. 7J.

The bending moment, $-M_2$, at $B = -Vl_2$ and M_3 at $D = Vl_2$, also that at $P = M_P = Pk(1-k)l_1$. We can now write, by inspection,

$$EIt_1 = Pk(1-k)l_1(\tfrac{1}{2}l_1)\tfrac{2}{3}[kl_1 + \tfrac{1}{2}(1-k)l_1] - Vl_2(\tfrac{1}{2}l_1)(\tfrac{2}{3}l_1)$$

$$= \frac{Pl_1^3}{6}(k - k^3) - \frac{Vl_1^2 l_2}{3},$$

but $t_{V_1} = t_1 \times \dfrac{l_2}{l_1}$; hence

$$EIt_{V_1} = \frac{Pl_1^2 l_2}{6}(k - k^3) - \frac{Vl_1 l_2^2}{3} \quad \ldots\ldots\ldots\ldots \quad (55)$$

$$EIt_4 = Vl_2(\tfrac{1}{2}l_1)(\tfrac{2}{3}l_1) = \frac{Vl_1^2 l_2}{3},$$

but $t_{V_4} = t_4 \times \dfrac{l_2}{l_1}$, hence

$$EIt_{V_4} = \frac{Vl_1 l_2^2}{3} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (56)$$

$$EIt_{V_2} = -Vl_2(\tfrac{1}{2}l_2)(\tfrac{2}{3}l_2) = -\frac{Vl_2^3}{3}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

Similarly,

$$EIt_{V_3} = \frac{Vl_2^3}{3}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

Since $t_{V_1} + t_{V_2} = t_{V_4} + t_{V_3}$,

$$\frac{Pl_1^2l_2}{6}(k - k^3) - \frac{Vl_1l_2^2}{3} - \frac{Vl_2^3}{3} = \frac{Vl_1l_2^2}{3} + \frac{Vl_2^3}{3};$$

from which

$$V = \frac{l_1^2(k - k^3)}{4(l_1 + l_2)l_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

This value of V is one-half of the reaction, R_3, shown by Eq. (30).

The value of V having been found for a load at any point on span 1, the reactions may readily be calculated from simple moment equations.

17. A single load on the second span.—Let Fig. 7K (*a*) represent

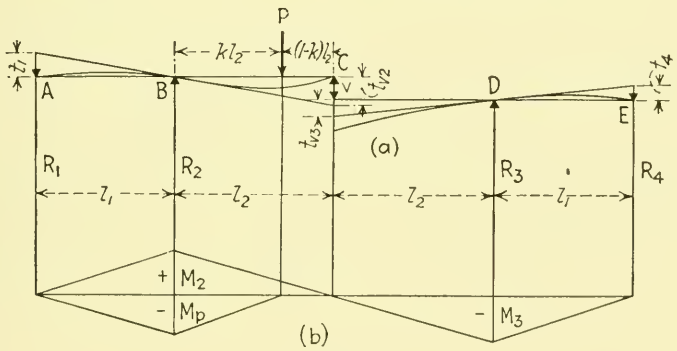


FIG. 7K.

the conditions diagrammatically, and let Fig. (*b*) represent the bending moments, in a precisely similar manner to that described in Art. 16. The bending moment, M_2, at B, due to V, $= Vl_2$, and $-M_3$ at $D = -Vl_2$. The moment at B, due to P, $= -M_p = -Pkl_2$. We can now write by inspection,

$$EIt_1 = Vl_2(\tfrac{1}{2}l_1)(\tfrac{2}{3}l_1) - Pkl_2(\tfrac{1}{2}l_1)(\tfrac{2}{3}l_1) = \frac{Vl_1^2l_2}{3} - \frac{Pkl_1^2l_2}{3},$$

but $t_{V_1} = t_1 \times \dfrac{l_2}{l_1}$; hence

$$EIt_{V_1} = \frac{Vl_1l_2^2}{3} - \frac{Pkl_1l_2^2}{3}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

$$EIt_{V_2} = Vl_2(\tfrac{1}{2}l_2)(\tfrac{2}{3}l_2) - Pkl_2(\tfrac{1}{2}k)l_2(l_2 - \tfrac{1}{3}kl_2),$$

or

$$EIt_{V_2} = \frac{Vl_2^3}{3} - \frac{Pl_2^3}{6}(3k^2 - k^3). \quad \ldots \quad (61)$$

$$EIt_4 = -Vl_2(\tfrac{1}{2}l_1)(\tfrac{2}{3}l_1) = -\frac{Vl_1^2l_2}{3},$$

but $t_{V_4} = t_4 \times \frac{l_2}{l_1}$; hence,

$$EIt_{V_4} = -\frac{Vl_1l_2^2}{3} \quad \ldots \quad (62)$$

$$EIt_{V_3} = -Vl_2(\tfrac{1}{2}l_2)(\tfrac{2}{3}l_2),$$

or

$$EIt_{V_3} = -\frac{Vl_2^3}{3}. \quad \ldots \quad (63)$$

Since $t_{V_1} + t_{V_2} = t_{V_4} + t_{V_3}$,

$$\frac{Vl_1l_2^2}{3} + \frac{Vl_2^3}{3} - \frac{Pkl_1l_2^2}{3} - \frac{Pl_2^3}{6}(3k^2 - k^3) = -\frac{Vl_1l_2^2}{3} - \frac{Vl_2^3}{3},$$

from which

$$V = Pk\frac{l_1}{2(l_1 + l_2)} + P\frac{l_2(3k^2 - k^3)}{4(l_1 + l_2)}. \quad \ldots \quad (64)$$

If, in Eq. (28), k is substituted for $1 - k$, $1 - k$ for k, l_1 for l_2, and l_2 for l_1, the equation becomes,

$$R_1 = Pk\frac{l_1}{l_1 + l_2} + P\frac{l_2(3k^2 - k^3)}{2(l_1 + l_2)}. \quad \ldots \quad (65)$$

Thus the value of V in Eq. (64) is found to be one-half of the reaction R_1 in Eq. (28).

The value of V being known for a load at any point of span 2, the reactions may be found from simple moment equations.

18. Lateral bracing.—In through swing bridges of the type shown in Fig. 7D (*a*), portal frames are placed above the traffic clearance line between the end posts at *B*, and between the center diagonal posts, *Ef*, and the vertical posts, *Ff*. These portal frames serve to transmit the lateral forces on the upper chords to the plane of the lower chords, at *a* and *f*, when the bridge is closed, and to *f* when it is open. On account of the flexibility of the portal frames and posts, there is little need of considering the upper bracing as continuous.

In the plane of the unloaded chords, the lateral truss should be treated as two simple, horizontal spans when the bridge is closed, and as two cantilevers when it is open.

In the plane of the loaded chords, the lateral truss should be con-

sidered as a horizontal, continuous truss when the bridge is closed, and as two cantilever spans when it is open.

The lateral systems and the general stability of the bridge should be investigated for three conditions of loading.

First.—When the bridge is closed and there is no live load on it, the lateral wind pressure should be assumed as 50 lbs. per square foot on one and one-half times the area of the truss and the floor system, as seen in elevation.

Second.—When the bridge is closed and carrying live load the wind pressure should be taken as 30 lbs. per square foot on the same area, and combined with a horizontal lateral force at the loaded chord of 10 per cent of the live load. For railway bridges this is sometimes specified as a horizontal wind pressure of 30 lbs. per square foot on a train 10 ft. high and assumed to act 7 ft. above the rail.

Third.—When the bridge is open the wind pressure is assumed as 15 lbs. per square foot on the same area as above.

The maximum stress selected from these three cases is the proper basis for proportioning each respective member of the lateral bracing.

As a movable bridge is actually a machine, great care must be taken to make sure that the laterals and sway systems are well designed and of ample strength. All light, flimsy sections must be avoided. This is particularly important in any parts of the structure that are subjected to stresses and vibrations from the operating machinery. Many of the defects in the action of the machinery are directly traceable to deflections and vibrations of the structural parts supporting the mechanism.

CHAPTER VIII

ANALYSIS OF BASCULE AND VERTICAL-LIFT BRIDGES

1. General.—While nearly all swing bridges, except the shear-pole and jack-knife types, are continuous or partially continuous, most types of bascule and vertical-lift bridges are simple spans or cantilevers, or a combination of both. The ordinary static relations serve for the calculation of the stresses, without resort to their elastic properties. The calculations are much simpler than those for swing bridges.

Bascule bridges are usually cantilevers for dead loads and simple supported spans for live loads. There are a few conditions that should receive attention. In trunnion bascules it is important that the trunnion reactions be thoroughly analyzed. The maximum dead-load stresses during movements of the leaves must be determined. When two bascule leaves open from the middle of a navigable channel they are locked together when closed, so that stresses due to shear are transmitted across the joint, and the two leaves are forced to deflect together under live loads. The stresses due to this condition must be found. Particular attention must be given to an analysis of the stresses due to wind pressure when the bascule leaves are open.

Vertical-lift bridges are usually simple spans, supported at their ends, for both dead and live loads, and it is seldom that unusual problems are presented for solution. This is equally true of their lateral and wind bracing.

2. Dead-load reactions of a closed bascule bridge.—Let Fig. 8A represent a single-leaf bascule span.

Let W = the weight of the leaf, acting at its center of gravity, c. g.;
m = the distance of c. g. from g;
l = the lever arm of a link FG connecting the leaf to the counterweight device;
θ = the angle which FG makes with the horizontal.

The weight W acting at c. g., distant m from g, causes a shear and a couple at the trunnion end of the truss or girder. The shear is equal to the weight W. The arm of the couple is l, and T and T' the two reactions from the couple tt'; then

$$T = -T' = W\frac{m}{l}.$$

The vertical reaction is

$$R_V = W - T' \sin \theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The proportions of some bascule leaves are such that R_V is negative. The horizontal reaction is

$$R_H = T' \cos \theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The resultant reaction at g is

$$R = \sqrt{R_V^2 + R_H^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The tangent of the angle that the resultant reaction makes with the horizontal is

$$\tan \beta = \frac{R_V}{R_H}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

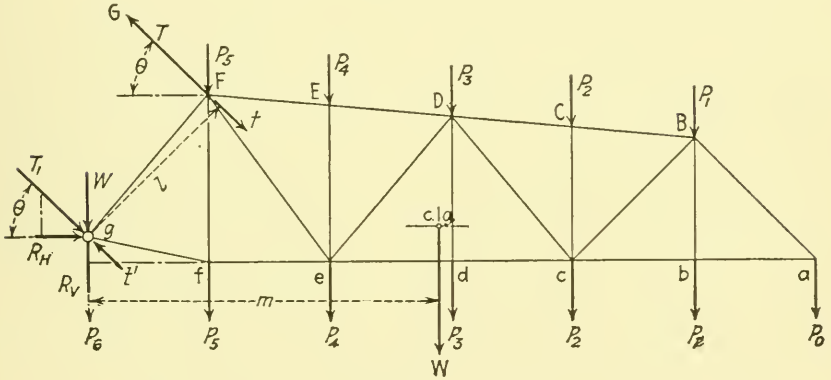


FIG. 8A.

Strict attention should be given to the magnitude and direction of the trunnion reactions and to the design, so as to provide for all stresses developed at the trunnion and its bearings, throughout the movements of the span.

3. Dead-load reactions of an open bascule bridge.—In Fig. 8B let (*a*) represent a bascule bridge opened 90° from its horizontal position. When in this position, with no wind pressure upon the structure, there will be no stress in link FG nor any horizontal reaction at the trunnion g. The vertical reaction then is

$$R'_V = W. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Most designs are made so that the bascule cannot be opened a full quadrant. Let Fig. 8B (*b*) represent such a structure. Equilibrium must be maintained by a shear and a couple at the trunnion end. The

shear is equal to W. Let the arm of the couple be l' and T' and T'' be the two reactions, then

$$T' = -T'' = W\frac{m'}{l'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The vertical reaction is

$$R'_V = W - T'' \sin \theta'. \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

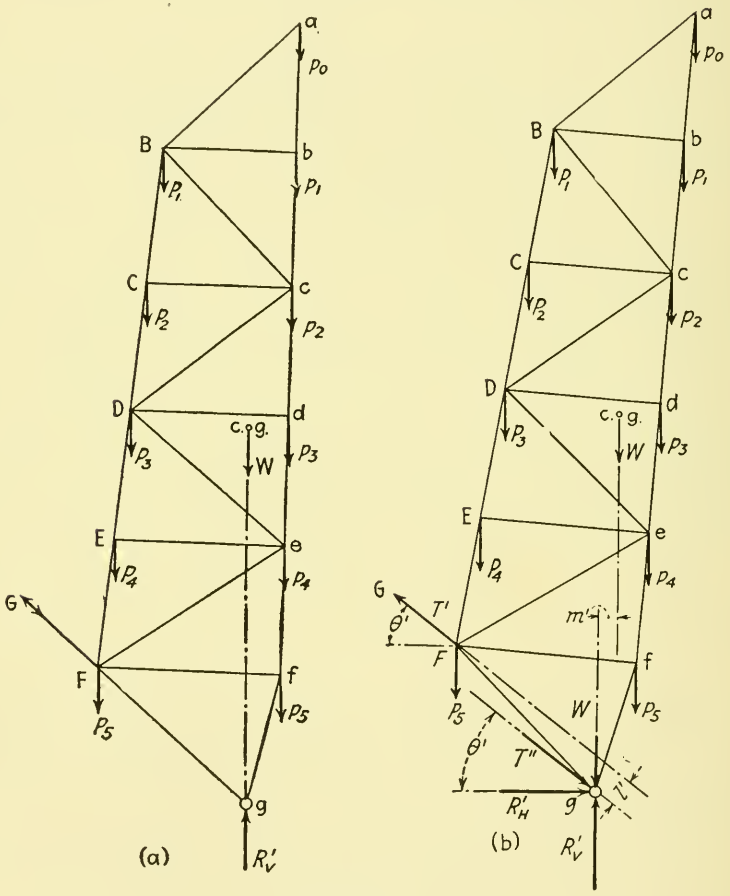


FIG. 8B.

This will always be positive. The horizontal reaction is

$$R'_H = T'' \cos \theta'. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The resultant reaction is

$$R' = \sqrt{R_V'^2 + R_H'^2}. \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The tangent of the angle that the resultant reaction makes with the horizontal is

$$\tan \beta' = \frac{R'_V}{R'_H}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

4. Dead-load stresses.—When the bridge is closed and the reactions are known, the dead-load stresses are easily calculated by the ordinary analytical or graphical methods. After the bascule leaf has been revolved 90°, the stresses again are readily determined. During the movement of the span the dead-load stresses are constantly changing, and in the case of some members they may pass through a maximum greater than those existing when the leaf is either closed or open.[1]

Let S_H = the stress in any member of the truss when the leaf is closed;

S_V = the stress in the same member after it has been revolved 90° from its closed position.

Let Fig. 8C (*a*) represent the leaf in its closed position and Fig. 8C (*b*) show it after it has revolved through an angle α from its closed position.

Find the stress in one of the lower chords, as *cd*.

$$S' = \frac{M}{d} = \frac{p_0 \cos \alpha h_0 + p_1 \cos \alpha h_1 + P_1 \cos \alpha h_1 + P_2 \cos \alpha h_2 + \text{etc.}}{d}$$

$$+ \frac{p_0 \sin \alpha d + p_1 \sin \alpha d + P_1 \sin \alpha_{V1} + \text{etc.}}{d}$$

$$= \frac{\Sigma ph \cos \alpha + \Sigma P_V \sin \alpha}{d} = \frac{M_H \cos \alpha + M_V \sin \alpha}{d},$$

or

$$S' = S_H \cos \alpha + S_V \sin \alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

To find the maximum value of S', differentiate with respect to S' and α.

$$\frac{dS'}{d\alpha} = -S_H \sin \alpha_0 + S_V \cos \alpha_0 = 0, \quad \text{hence} \quad \tan \alpha_0 = \frac{S_V}{S_H},$$

$$\sin \alpha_0 = \frac{S_V}{\sqrt{S_H^2 + S_V^2}}, \quad \text{and} \quad \cos \alpha_0 = \frac{S_H}{\sqrt{S_H^2 + S_V^2}};$$

therefore

$$S = S'_{\max.} = \frac{S_H^2 + S_V^2}{\sqrt{S_H^2 + S_V^2}} = \sqrt{S_H^2 + S_V^2}. \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The same reasoning may be applied to any member of the truss.

[1] The method given is taken from a paper entitled "Maximum Stresses in Bascule Bridges," by Mr. W. W. Pagon, Junior (now Member), Am. Soc. C. E., *Trans. Am. Soc. C. E.*, Vol. LXXVI, p. 73.

To find a criterion for the position of the leaf giving the maximum stress, lay off, as in Fig. 8D, $ab = S_H$ and $bc = S_V$, making abc a right angle. Then $ac = \sqrt{S_H^2 + S_V^2} = S$. Draw a circle through a, b,

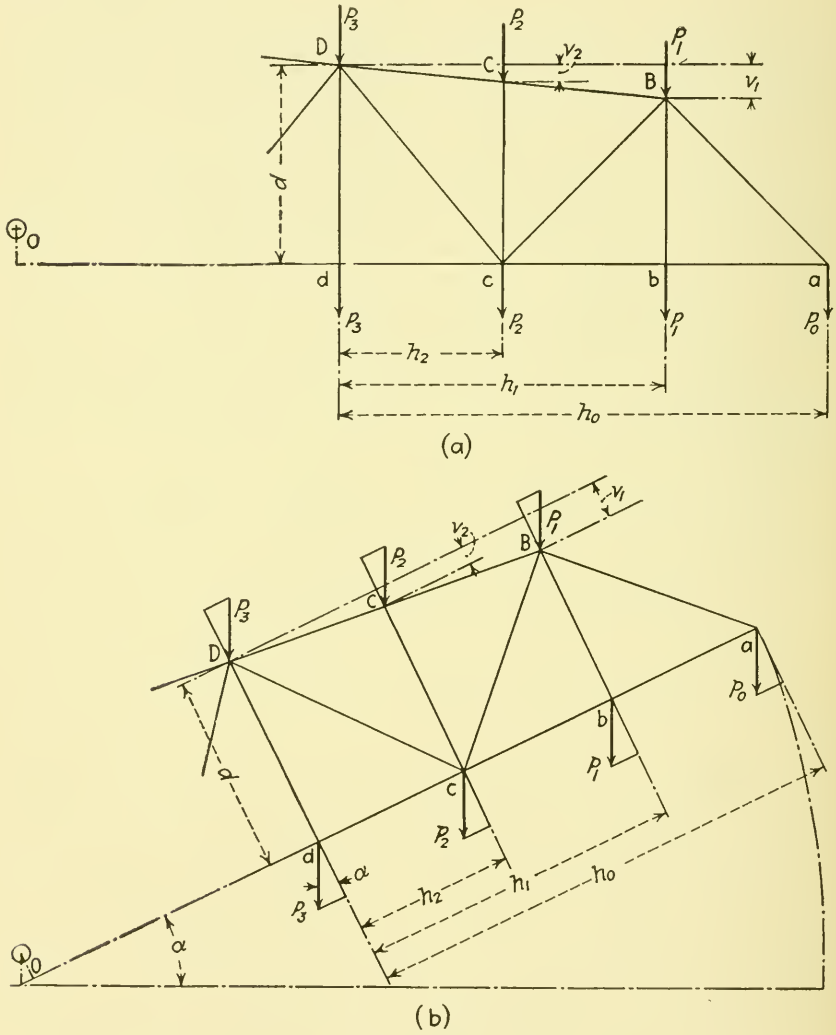


FIG. 8C.

and c. Draw any line, as ad, making any angle α with ab. Project b and c on this line at e and d; then $ae = S_H \cos \alpha$ and $ed = S_V \sin \alpha$; hence $ad = S'$ for the angle α. As abc is a right angle, ac is a diameter

of the circle and is greater than any chord, as ad; hence the maximum stress, $S'_{\max.} = S$, occurs when the diameter ac makes an angle $\tan^{-1} = \frac{S_V}{S_H}$. S_H and S_V are here taken as of the same sign, as compression, and are laid off in the directions ab and bc, and thus represent the stress in any lower chord member. The stress in any other member, when of the same sign, may be laid off and a circle drawn in a similar manner, and a maximum stress is caused by each opening of the leaf.

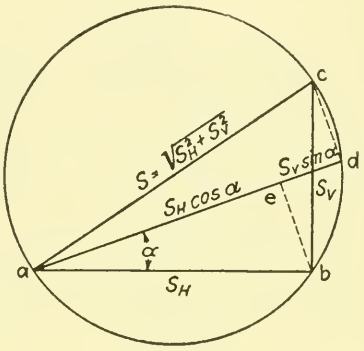


FIG. 8D.

When the stresses are of different signs the circle will be in a different quadrant, a being the origin, and a maximum stress will not be attained.

Criterion for maximum stress.—Whenever S_H and S_V for any member have the same sign, there will be a maximum dead-load stress, greater than either, produced in the member each time the bridge is opened or closed. When this stress occurs the bridge will be open at an angle $\tan^{-1} = \frac{S_V}{S_H}$ with the horizontal. When S_H and S_V are of unlike signs, the maximum stress will be one of the two, whichever is the greater, usually S_H.

Maximum stress.—To find the maximum dead-load stress in any member in a truss revolving about a horizontal axis perpendicular to it: Find S_H for the bridge closed. Note by inspection whether S_H and S_V are of the same sign, and if so find S_V for the bridge open; then the maximum stress is

$$S = \sqrt{S_H^2 + S_V^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

If they are of unlike signs, either S_H or S_V is the maximum stress.

5. Live-load stresses in a double-leaf bascule bridge.—Assume that a double-leaf bascule bridge, as indicated in Fig. 8E, is designed so that when closed it acts as two cantilever arms locked together at g for shears, but also so that no bending moments are transmitted at this point. The stresses in such a structure can be found from the ordinary static relations after the reactions of one arm upon the other at g have been determined. Assume a load P at any point of the first, or left, arm. Equation (64) of Chapter VII gives the mutual reactions of the channel arms of two swing spans with a shear lock at their points of

juncture. In this equation let $l_1 = 0$ and $l_2 = l$. The value of V then becomes

$$V = \frac{P}{4}(3k^2 - k^3). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

From this expression the reactions at g may be found for a load P at any point of the arm ag. Making $l_1 = 0$ in Eq. (64) is equivalent to assuming that the two arms of Fig. 8E are fixed at the trunnion ends a and m. This is not strictly correct, for the leaves act as restrained cantilevers and must be anchored back at A and M. Any elastic distortion of the anchorages will permit small rotary movements at a and m which will modify the deflections at g and the mutual reactions at g. As the anchor ends are usually short and of large sections, the errors introduced by assuming fixed ends are small.

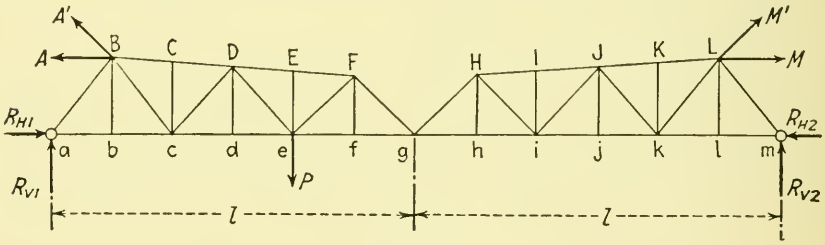


FIG. 8E.

When the construction is such that the elastic movements around a and m are significant, the first design, based upon fixed ends, can be corrected by taking account of the elastic deflections of the anchorages in a manner similar to that outlined in Chapter VII, Arts. 10 and 11, for center-bearing bridges.

6. Wind pressures.—The United States Weather Bureau, after thorough investigation, has adopted the following formula for wind pressure against surfaces normal to the direction of the wind:

$$P = 0.004\frac{B}{30}V^2,$$

in which P = the wind pressure per square foot of exposed surface;
B = the barometric pressure in inches of mercury;
V = the velocity of the wind in miles per hour.

For practical purposes the barometric reading may be ignored, and the formula then becomes,

$$P = 0.004V^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The pressures corresponding to various wind velocities are given in Table 8A.

TABLE 8A

Velocity of Wind, in Miles per Hour	Wind Pressure, in Pounds per Square Foot	Velocity of Wind, in Miles per Hour	Wind Pressure, in Pounds per Square Foot
10	0.4	70	19.6
20	1.6	80	25.6
30	3.6	90	32.4
40	6.4	100	40.0
50	10.0	110	48.4
60	14.4	120	57.6

Probably the most reliable formulae for calculating the effect of wind pressure on surfaces at an angle with its direction are those proposed by Col. Duchemin, a French army officer, and first published in 1842. In 1888 Prof. S. P. Langley made a valuable series of experiments, the results of which varied from those obtained from the Duchemin formulae by less than 3 per cent.

Assume that the wind is blowing horizontally against a surface at right angles to its direction, but inclined at an angle α with the horizontal; them, according to Duchemin:

The pressure normal to the surface is

$$P_n = P\frac{2 \sin \alpha}{1 + \sin^2 \alpha}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

The pressure parallel to the direction of the wind is,

$$P_p = P\frac{2 \sin^2 \alpha}{1 + \sin \alpha}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The lifting or depressing pressure vertically is,

$$P_v = P\frac{2 \sin \alpha \cos \alpha}{1 + \sin^2 \alpha}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

The first of these expressions will most frequently be convenient for use.

7. Bracing of bascule spans.—*Bridge closed. No live load.*—When a bascule bridge is closed and not loaded, the design of the laterals and sway bracing presents no unusual problems. The counterweights are designed to balance fully the dead weight of the bascule leaf; hence, when the bridge is closed there is no dead-load reaction at the free

end. The leaf thus becomes a cantilever for lateral wind pressure as well as for vertical dead load.

A wind pressure of 50 lbs. per square foot corresponds to a velocity of about 112 miles per hour, which is a wind of hurricane, or tornado, intensity. To provide for any contingency the laterals and sway bracing should be designed to withstand a wind pressure of 50 lbs. per square foot, and the general stability of the entire structure, with its towers and counterweights, should be investigated for this pressure.

Bridge closed with live load.—When the bascule is closed the free end reaction from dead load will still be zero, but there will be a positive reaction from the live load, and this will be large enough to cause the leaf to act as a simple span on two supports for lateral wind pressure. The laterals and sway bracing in this case should be designed for a wind pressure of 30 lbs. per square foot. This pressure should be taken as acting on a surface equal to the full projected area of the leaf and its live load in the case of girders with solid webs, and on about one and one-half times the area of the truss, as seen in elevation, plus the area of the floor and the live load. In order to make provision for the dynamic effect of engines and trains on railway bridges, in place of using 30 lbs. per square foot of train surface, it is quite common to use 10 per cent of the vertical train load, combined with 30 lbs. per square foot on one and one-half times the area of the trusses plus the floor system, as seen in elevation, both considered as acting horizontally.

Bridge open.—A wind pressure of 30 lbs. per square foot corresponds to a velocity of about 86.6 miles per hour. This is about enough to overturn unloaded box cars. It is not likely that any attempt would be made to operate a bascule bridge during such a violent wind storm; hence, if the bracing is strong enough for this condition it will be ample. As it would have been proportioned for 50 lbs. per square foot when the span was closed and acting as a cantilever, the bracing so designed would be of ample strength when the span is open.

The general stability of the entire structure should be investigated for a pressure of 30 lbs. per square foot when the leaf is open. In some cases it may be found necessary to increase the distance between the trusses to provide stability. The investigation of the general stability is important, particularly in the case of long, single-track, railway bascule bridges, and should never be neglected.

8. Vertical-lift bridges.—The usual type of vertical-lift bridge is arranged so that the entire span may be lifted vertically to clear traffic in the navigable channel which it crosses. Ropes or chains are attached to the four corners of the span, passed upward and over sheaves on the

tops of the towers, and connected to counterweights, moving vertically which balance the weight of the span.

Dead- and live-load stresses.—As the weight of the span is fully counterweighted, the dead-load reactions are taken by the counterweight ropes. When the span is closed the live-load reactions are the same as for a simple span resting upon the piers, whenever the live load is entirely between the trusses. On some bridges with wide roadways to accommodate dense traffic, some of the roadways, or the sidewalks, or both, are carried by cantilever brackets outside of the trusses. When the brackets carry live load on only one side of the bridge, negative reactions are caused at the ends of the truss on the side of the bridge opposite the loaded cantilevers. As the weight of the span is fully counterweighted, no dead weight is available to resist this uplift. The only resistance to motion is that due to the friction of the counterweight ropes, sheaves, and bearings, and the inertia of the structure. This is only enough to provide for a small unbalanced live load, like that on a narrow sidewalk. In the case of structures with wide sidewalks, or roadways, outside of the main trusses, the designs must provide means to resist the tendency to lift the truss on the side opposite the loads.

It is possible to latch down the four corners of the span by mechanically operated locks, but this results in additional mechanism, and there would also be some danger that the locks might not always be driven when the bridge was closed. It is simpler and safer to provide special supports for the outer ends of the end brackets at the piers. The design should be well considered and adequate provision made for stability.

Bracing.—The lateral bracing of the span should be designed, when closed and unloaded or closed and loaded, for the same lateral forces that were given in Art. 7 for the same cases for bascule bridges.

The sway frames between the trusses should receive special attention. As the whole span moves vertically between a system of longitudinal and transverse guides on the towers, the moving span really becomes a part of a large machine. Bracing that would be adequate for a fixed span might not give sufficient stiffness. It must be strong enough to hold the frame true and out of wind under all conditions. The wind pressure may be applied in any direction, and the reactions at the corners of the span will depend upon the arrangement of the guides on the tower posts.

Figure 8F illustrates one method of guiding a vertical-lift bridge from the vertical legs of the towers. The span must not only be guided, but provision must be made for expansion, and in such a manner that there may be no danger of its becoming cramped against the guides in case it does not always preserve an exactly horizontal position as it

moves upward and downward. The guides must also resist the wind pressures and the effects of vibrations in all directions.

At tower post A two wheels are provided, having small clearances inside of the guides. They turn on trunnions securely anchored to the framework of the lift span. At B a single wheel is placed so that it can resist transverse forces, and at C another is adapted to take those in a longitudinal direction. At D no guide wheels are used. Post A

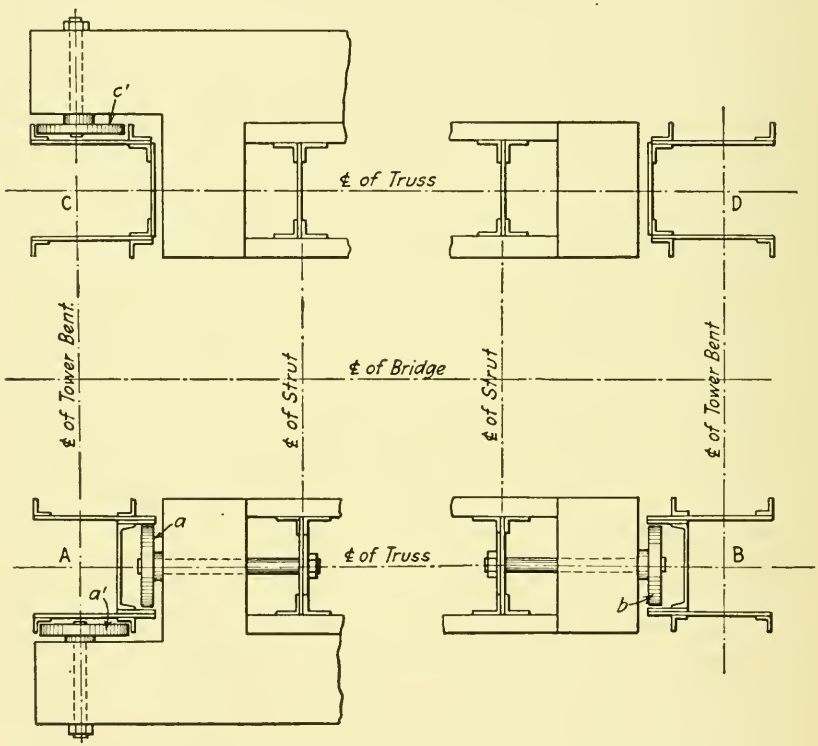


FIG. 8F.

thus becomes an anchorage in both directions. At B transverse motion is prevented but the span is free to expand or contract longitudinally. At C longitudinal motion is prevented but the framework may expand or contract transversely. At D the structure may freely expand or contract in both directions. The actual condition at the corners must be considered when finding the reactions due to wind forces, and provision made to resist them, both in the bracing of the span and in that of the towers.

CHAPTER IX

ELASTIC DEFORMATIONS OF BEAMS, GIRDERS, AND TRUSSES

SECTION I

BEAMS AND GIRDERS

1. General.—The fundamental equation for the deflection of a loaded beam, neglecting the small deflections due to shear, is

$$\Delta = \int \frac{M m dx}{EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In this equation:

M is the bending moment at any section due to the given loads;

m is the bending moment at the same section due to a unit load upon the beam at the point where deflection is required;

dx is a differential distance along the beam;

E is Young's modulus;

I is the moment of inertia of the beam, and

Δ is the deflection.

The development of this equation may be found in any good modern work on Mechanics.

2. Deflection of the end of a cantilever girder, of varying section, with a single load at any point.—Let Fig. 9A be a cantilever girder, AB,

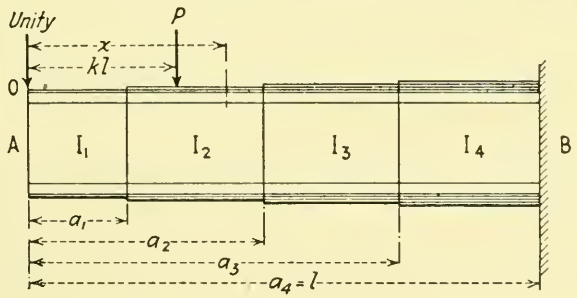


Fig. 9A.

of length l, free at A and fixed in position and direction at B. The girder is assumed to have three cover plates at both top and bottom,

their ends being at distances a_1, a_2, and a_3 from the free end. The moments of inertia of the four sections of the girder are I_1, I_2, I_3, and I_4. The single load, P, is here placed upon the first cover plate at a distance kl from the free end. It is required to find the deflection, Δ, at the free end.

Assume the origin to be at 0 and let x be any distance along the girder from 0. Place a unit load at 0. From 0 to kl the bending moment due to the load P is zero, and Eq. (1) becomes $\Delta = 0$.

Consider the section from kl to a_2. Here $M = P(x - kl)$, $m = x$, and $I = I_2$; hence Eq. (1) becomes:

$$EI_2\Delta_2 = P\int_{kl}^{a_2}(x^2 - klx)dx = P\left[\frac{x^3}{3} - \frac{klx^2}{2}\right]_{kl}^{a_2}$$

$$= P\left[\frac{a_2^3}{3} - \frac{kla_2^2}{2} + \frac{k^3l^3}{6}\right],$$

or

$$EI_2\Delta_2 = \frac{P}{6}[2a_2^3 - 3kla_2^2 + k^3l^3].$$

For section from a_2 to a_3,

$$EI_3\Delta_3 = P\int_{a_2}^{a_3}(x^2 - klx)dx = P\left[\frac{x^3}{3} - \frac{klx^2}{2}\right]_{a_2}^{a_3}$$

$$= P\left[\frac{a_3^3 - a_2^3}{3} - \frac{kl(a_3^2 - a_2^2)}{2}\right],$$

or

$$EI_3\Delta_3 = \frac{P}{6}[2(a_3^3 - a_2^3) - 3kl(a_3^2 - a_2^2)]$$

Similarly,

$$EI_4\Delta_4 = \frac{P}{6}[2(a_4^3 - a_3^3) - 3kl(a_4^2 - a_3^2)]$$

and

$$\Delta = \frac{P}{6E}\left[\frac{2a_2^3 - 3kla_2^2 + k^3l^3}{I_2} + \frac{2(a_3^3 - a_2^3) - 3kl(a_3^2 - a_2^2)}{I_3} + \frac{2(a_4^3 - a_3^3) - 3kl(a_4^2 - a_3^2)}{I_4}\right]. \quad (2)$$

If the load P is at the free end of the cantilever, $k = 0$, and Eq. (2) becomes, by inserting the deflection from 0 to a_1,

$$\Delta = \frac{P}{3E}\left[\frac{a_1^3}{I_1} + \frac{a_2^3 - a_1^3}{I_2} + \frac{a_3^3 - a_2^3}{I_3} + \frac{a_4^3 - a_3^3}{I_4}\right]. \quad (3)$$

If the girder is assumed to have a uniform cross-section

$$I_1 = I_2 = I_3 = I_4 = I,$$

all values of a are equal to l, and Eq. (2) becomes

$$\Delta = \frac{Pl^3}{6EI}(2 - 3k + k^3). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If the load P is at the free end, $k = 0$, and Eq. (4) reduces to

$$\Delta = \frac{Pl^3}{3EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

This is the familiar expression for the deflection of the free end of a cantilever beam, of uniform section, due to a single load at its free end.

When the load P is between 0 and a_1, the first two terms within the brackets of Eq. (2) become

$$\left[\frac{2a_1^3 - 3kla_1^2 + k^3l^3}{I_1} + \frac{2(a_2^3 - a_1^3) - 3kl(a_2^3 - a_4^3)}{I_2} + \text{etc.}\right]$$

When the load is on some section nearer the fixed end than as indicated in Fig. 9A, as for example between a_2 and a_3, all terms containing I_1 and I_2 disappear from the equations, for $M = 0$, hence $\Delta = 0$, for the part of the beam between 0 and the load P.

Care must be taken to use all quantities in Eqs. (2), (3), (4), and (5) in their true denominations when calculating actual deflections. P is in pounds, E is in $\frac{\text{pounds}}{\text{inches}^2}$, a and l are in inches and I in inches^4. Δ will then be in terms of $\frac{\text{pounds} \times \text{inches}^3}{\frac{\text{pounds}}{\text{inches}^2} \times \text{inches}^4}$, or in inches.

The advantage of smaller numbers may be gained by taking the values of a and l in feet. If the deflection Δ is required in inches, the factors outside of the brackets will then become $\frac{288P}{E}$ in Eq. (2), $\frac{576}{E}$ in Eq. (3), $\frac{288Pl^3}{EI}$ in Eq. (4), and Eq. (5) becomes $\Delta = \frac{576Pl^3}{EI}$.

3. Deflection of the end of a cantilever girder, of varying section, with a partial uniform load.—Let Fig. 9B represent the girder. The assumptions are the same as in Art. 2, except that a uniform load of w per unit of length extends from the fixed end to within a distance kl from the free end. From 0 to kl the bending moment, M, due to the load of w per unit of length, is zero; hence Eq. (1) becomes $\Delta = 0$.

Consider the section from kl to a_2. Here $M = w\frac{(x - kl)^2}{2}$, $m = x$, and $I = I_2$, and Eq. (1) becomes

$$EI_2\Delta_2 = w\int_{kl}^{a_2}\frac{(x - kl)^2}{2}xdx = w\int_{kl}^{a_2}\left[\frac{x^2 - 2klx + k^2l^2}{2}\right]xdx$$

$$= w\int_{kl}^{a_2}\left[\frac{x^3}{2} - klx^2 + \frac{k^2l^2x}{2}\right]dx$$

$$= w\left[\frac{x^4}{8} - \frac{klx^3}{3} + \frac{k^2l^2x^2}{4}\right]_{kl}^{a_2}$$

$$= w\left[\frac{a_2^4}{8} - \frac{kla_2^3}{3} + \frac{k^2l^2a_2^2}{4} - \frac{k^4l^4}{24}\right],$$

or

$$EI_2\Delta_2 = \frac{w}{24}[3a_2^4 - 8kla_2^3 + 6k^2l^2a_2^2 - k^4l^4].$$

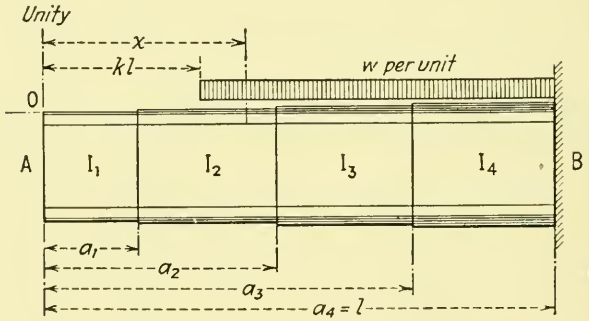


FIG. 9B.

For the section from a_2 to a_3,

$$EI_3\Delta_3 = w\int_{a_2}^{a_3}\frac{(x - kl)^2}{2}xdx = w\left[\frac{x^4}{8} - \frac{klx^3}{3} + \frac{k^2l^2x^2}{4}\right]_{a_2}^{a_3}$$

or

$$EI_3\Delta_3 = \frac{w}{24}[3(a_3^4 - a_2^4) - 8kl(a_3^3 - a_2^3) + 6k^2l^2(a_3^2 - a_2^2)].$$

Similarly,

$$EI_4\Delta_4 = \frac{w}{24}[3(a_4^4 - a_3^4) - 8kl(a_4^3 - a_3^3) + 6k^2l^2(a_4^2 - a_3^2)]$$

and

$$\Delta = \frac{w}{24E}\left[\frac{3a_2^4 - 8kla_2^3 + 6k^2l^2a_2^2 - k^4l^4}{I_2}\right.$$

$$+\frac{3(a_3^4-a_2^4)-8kl(a_3^3-a_2^3)+6k^2l^2(a_3^2-a_2^2)}{I_3}$$

$$+\frac{3(a_4^4-a_3^4)-8kl(a_4^3-a_3^3)+6k^2l^2(a_4^2-a_3^2)}{I_4}\Bigg]. \quad . \quad (6)$$

If the uniform load extends from the fixed to the free end of the cantilever, $k = 0$, and Eq. (6) becomes, by inserting the deflection from 0 to a_1,

$$\Delta = \frac{w}{8E}\left[\frac{a_1^4}{I_1}+\frac{a_2^4-a_1^4}{I_2}+\frac{a_3^4-a_2^4}{I_3}+\frac{a_4^4-a_3^4}{I_4}\right]. \quad . \quad . \quad (7)$$

If the girder is assumed to have a uniform section, $I_1 = I_2 = I_3 = I_4 = I$, all values of a are equal to l, and Eq. (6) becomes,

$$\Delta = \frac{wl^4}{24EI}[3-8k+6k^2+k^4]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

If the uniform load extends over the whole length of a cantilever beam of uniform section, Eqs. (7) and (8) both reduce to

$$\Delta = \frac{wl^4}{8EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

If W be substituted for wl, Eq. (9) becomes

$$\Delta = \frac{Wl^3}{8EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

This is the ordinary expression for the deflection of the free end of a cantilever beam, of uniform section, loaded by the total load W, uniformly distributed over the length of the beam.

When the uniform load ends between 0 and a_1, the first two terms within the brackets in Eq. (6) become

$$\frac{w}{24E}\left[\frac{3a_1^4-8kla_1^3+6k^2l^2a_1^2-k^4l^4}{I_1}\right.$$

$$\left.+\frac{3(a_2^4-a_1^4)-8kl(a_2^3-a_1^3)+6k^2l^2(a_2^2-a_1^2)}{I_2}+\text{etc.}\right].$$

When the end of the load is on some section nearer the fixed end of the beam than as shown in Fig. 9B, as for example between a_2 and a_3, all terms that contain I_1 and I_2 become zero.

If w is taken in pounds per linear foot and the distances a and l are in feet, and the deflection is required in inches, the factors outside of the brackets will become $\frac{864w}{E}$ in Eq. (6), $\frac{2592w}{E}$ in Eq. (7), $\frac{864wl^4}{EI}$ in Eq. (8) and Eq. (9) becomes $\Delta = \frac{2592wl^4}{EI}$, and Eq. (10) is $\Delta = \frac{216Wl^3}{EI}$.

4. Deflection of the end of a cantilever girder, of varying section, with a series of concentrated loads.—The deflection can be found by applying the method of Art. 2 and finding the end deflection due to each load separately. The total deflection will then be the sum of the partial deflections so found. However, it is more convenient to proceed in another way.

Let Fig. 9C represent a cantilever girder, AB, of length l, free at A and fixed in position and direction at B. The loads P_1, P_2 . . . P_5 are at distances $k_1 l$, $k_2 l$ . . . $k_5 l$ from the free end where the moments of inertia are I_1, I_2 . . . I_5. Let x be the distance of any point from the free end. A closely approximate deflection may be found by dividing the girder into any convenient number of parts and finding an average value of I for each part, as indicated in Fig. 9C. For convenience the

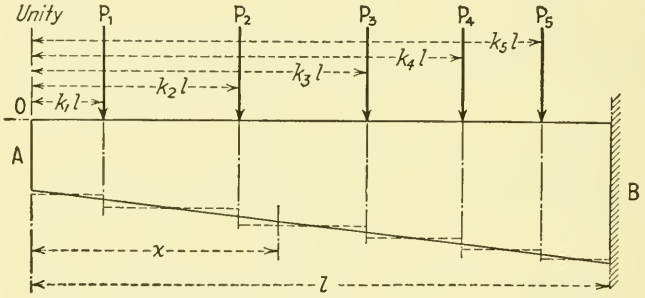


Fig. 9C.

points of division are here made to coincide with the positions of the loads. Let the vertical shear in any segment of the girder be S_a. (Between P_2 and P_3, S_a will equal $P_1 + P_2$.) Let M_a be the bending moment at the left end of any segment of the girder. The bending moment at any point within the segment will then become

$$M_x = M_a + S_a(x - k_a l).$$

Eq. (1) may be written

$$EI\Delta = \int M m dx, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

hence,

$$EI_a\Delta_a = \int_{k_a l}^{k_{a+1} l} [M_a + S_a(x - k_a l)] x dx.$$

Integrating and reducing,

$$\Delta_a = \frac{1}{6EI_a}\left[{}^{x=k_{a+1}l}_{x=k_a l} 3M_a x^2 + 2S_a x^3 - 3S_a k_a l x^2\right]. \quad (12)$$

Δ_a is the end deflection due to the bending moments in the girder segment a. By substituting successively in Eq. (12) for each segment if the girder and finding Δ_a for each, the total deflection, which is the sum of these partial deflections, is found. When considering the segment between loads P_1 and P_2, it must be noted that the moment at P_1 is zero and the term involving M_a in Eq. (12) will become zero.

5. Deflection at the end of a cantilever turntable, by the analytical method.—Figure 9D (*a*) represents one arm of a cantilever turntable

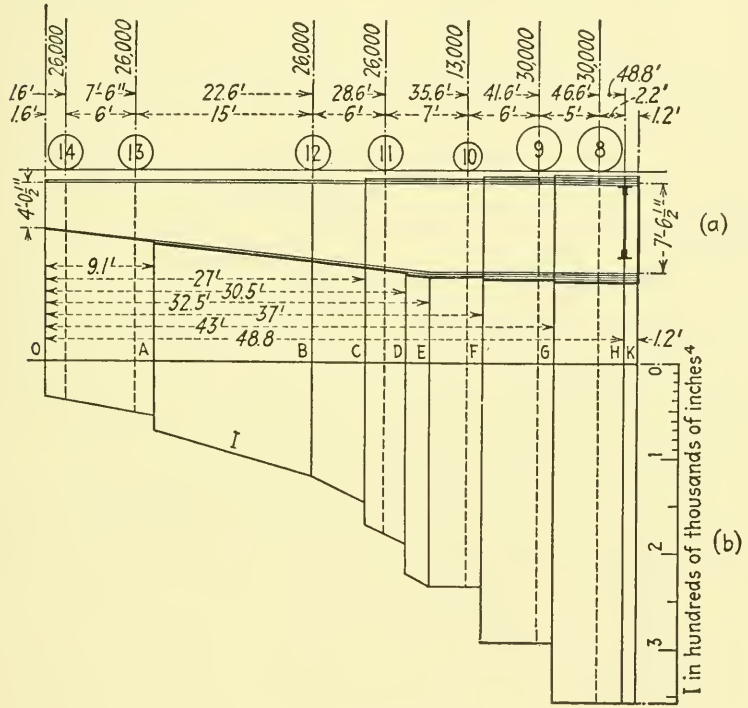


FIG. 9D.

100 ft. long, with a 375-ton Mallet locomotive in balanced position. The maximum deflection occurs at the end of the arm loaded by the tender. It is not necessary to find the deflection due to the dead load, for the table always acts as a balanced cantilever for dead loads, and the live load upon the table does not change the dead-load deflection; moreover, such tables are adjusted in their pits when deflected by their dead loads.

The end deflection due to bending moments in the various segments of the girder from O to H may be found by successive applications of Eq. (12) and summing the partial deflections.

The main girders are supported at H by a cross girder resting upon the center, and a second girder, H_1, is located symmetrically with H beyond the center, K. The bending moment from H to K is a maximum and constant in amount. The end deflection due to moments in this segment of the girder is given by the expression

$$EI_H\Delta_{HK} = \int_H^K M_H m_H dx \quad \text{or} \quad \Delta_{HK} = \frac{1}{EI_H}\int_H^K M_H m_H dx. \qquad (13)$$

$m_H = 1 \times H$, a constant. M_H is also constant; hence,

$$\Delta_{HK} = \frac{H}{EI_H} M_H \left[x \right]_{x=H}^{x=K}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (14)$$

In these expressions, H and K indicate distances from 0.

The turntable arm in Fig. (*a*) is made up as follows:

1 web plate $\frac{1}{2}$ in. thick, 90 in. deep for 17 ft. 6 in. from the center, and 48 in. deep at the end.
4 angles $8 \times 8 \times \frac{3}{4}$ in. full length, placed $90\frac{1}{2}$ in. back to back at the center and $48\frac{1}{2}$ in. back to back at the end.
1 top cover plate $18 \times \frac{13}{16}$ in., full length.
1 bottom cover plate $18 \times \frac{13}{16}$ in. extending to within 9.1 ft. from the end.
1 top cover plate $18 \times \frac{3}{4}$ in. extending to within 27 ft. from the end.
1 bottom cover plate $18 \times \frac{3}{4}$ in. extending to within 30.5 ft. from the end.
1 top and bottom cover plate $18 \times \frac{3}{4}$ in. extending to within 37 ft. from the end.
1 top and bottom plate $18 \times \frac{3}{4}$ in. extending to within 43 ft. from the the end.

Figure 9D (*a*) gives the principal dimensions and the position of the loads. Further data are given in Tables 9A and 9B.

In order to find the average values of I for the calculations it is convenient to plot the critical values of I, given in Table 9A, to scale, below the sketch of the girder as shown in Fig. 9D (*b*). The averages may then be found by careful scaling and simple calculations. One example of the application of Eq. (12) will be given. Consider the end deflection due to the bending moments in the segment of the girder from wheel 12 to wheel 11. Here M_a = 936,000 ft.-lbs., S_a = 78,000

lbs., $k_a l = 22.6$ ft., $k_{a+1} = 28.6$ ft., and $I_a = 144{,}077$ ins.^{4} Substituting these values in the term within the brackets of Eq. (12), we have

$$6EI_a\Delta_a = 3 \times 936{,}000(28.6^2 - 22.6^2) + 2 \times 78{,}000(28.6^3 - 22.6^3)$$
$$- 3 \times 78{,}000 \times 22.6(28.6^2 - 22.6^2) = 1{,}086{,}696{,}000.$$

TABLE 9A

Designation of Girder, Section	Distance from End, Feet	Values of I, Inches^4	Designation of Girder, Section	Distance from End, Feet	Values of I, Inches^4
O	0.0	34,341	E	32.5	234,319
A	9.1	56,115	F	37.0	234,319
A_1	9.1	71,943	F_1	37.0	294,399
B	22.6	119,085	G	43.0	294,399
C	27.0	146,894	G_1	43.0	356,419
C_1	27.0	169,137	H	48.8	356,419
D	30.5	189,767	K	50.0	356,419
D_1	30.5	220,071			

TABLE 9B

Wheel, Number or Point	Distance from End, Feet	Bending Moment M, Feet-Pounds	Shear S, lbs.
14	1.6	0	26,000
13	7.6	156,000	52,000
12	22.6	936,000	72,000
11	28.6	1,404,000	104,000
10	35.6	2,132,000	117,000
9	41.6	2,834,000	147,000
8	46.6	3,569,000	177,000
Cross Girder	48.8	3,958,400	177,000
Center	50.0	3,958,400	177,000

As the value of M_a was used in foot-pounds and all linear dimensions were taken in feet, and the term within the brackets of Eq. (12) is a cubic function, it is necessary to multiply the value given for $6EI_a\Delta_a$ by 12^3 to find the deflection in inches, hence

$$\Delta_a = \frac{1{,}086{,}696{,}000 \times 1728}{6 \times 29{,}000{,}000 \times 144{,}077} = 0.0748 \text{ in.}$$

The end deflection for any number of segments of the girder may be found in a similar manner and the partial deflections summed; but it is more convenient to divide each value of the bracket of Eq. (12) by the number representing the proper value of I_a and then to sum the quotients and finally multiply this sum by 12^3 and divide by $6E$.

A summary of the calculation is given in Table 9C.

TABLE 9C

Segment of Girder	Factors of Terms in Bracket, Eq. (12)				Value of Bracket	I_a	Bracket I_a
	M_a	S_a	$k_a l$	$k_{a+1} l$			
14 to 13	0	26,000	1.6	7.6	15,724,800	48,000	327.6
13 to 12	156,000	52,000	7.6	22.6	829,764,000	91,075	9,110.3
12 to 11	936,000	78,000	22.6	28.6	1,086,696,000	144,077	7,542.5
11 to 10	1,404,000	104,000	28.6	35.6	1,419,964,000	218,863	6,797.0
10 to 9	2,132,000	117,000	35.6	41.6	4,712,572,800	280,380	16,808.0
9 to 8	2,834,000	147,000	41.6	46.6	4,244,772,000	356,419	11,909.0
8 to H	3,569,000	177,000	46.6	48.8	2,370,718,416	356,419	6,650.8
Sum of values of brackets $\div I_a$ from O to H							59,145.2

The end deflection due to bending moments from O to H is:

$$\Delta_{OH} = \frac{59{,}145.2}{6 \times 29{,}000{,}000} \times 1728 = 0.5873 \text{ in.}$$

Substituting values given in the tables and in Fig. (a) in Eq. (14), we find the end deflection due to bending moments in the girder segment from H to K.

$$\Delta_{HK} = \frac{48.8 \times 3{,}958{,}400 \times 1.2}{29{,}000{,}000 \times 356{,}419} \times 1728 = 0.0388 \text{ in.}$$

The total end deflection then is

$$\Delta_{OK} = \Delta_{OH} + \Delta_{HK} = 0.5873 + 0.0388 = 0.6261 \text{ in.}$$

6. The area-moment method for calculating the deflections of beams.—In 1874 the late Prof. Charles E. Greene published a combination of graphical and analytical methods for the calculation of the curvature and deflection of beams, now widely known as the **area-moment method.** It was contained in his "Graphical Method for the Analysis of Bridge Trusses" and later editions of the same work.

Mr. Charles A. Ellis has expanded this method and applied it to many special cases in his "Essentials in the Theory of Frame Structures."

The principles enunciated by Professor Greene and amplified by Mr. Ellis are freely used in the discussion that follows.

In Fig. 9E let AB be a portion of a beam bent by the action of any system of loads. Draw AO and BO normal to the neutral axis and

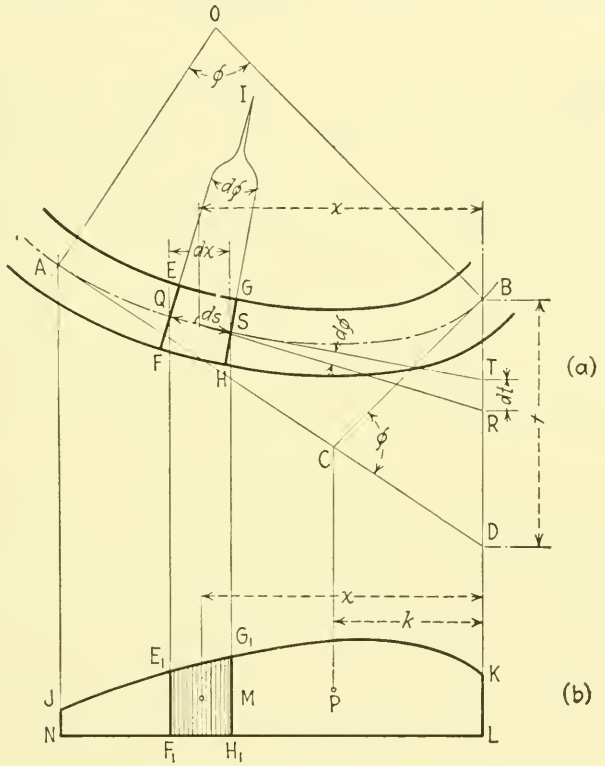


Fig. 9E.

tangents AD and BC intersecting at C. The angle $AOB = BCD = \phi$. Let $EFIIG$ be an elemental segment of the beam of length ds along the neutral axis. Draw FI and HI normal to the neutral axis intersecting at I. The angle $FIH = d\phi$. Draw tangents QR and ST intercepting $TR = dt$ on the line BD. Let $JKLN$, in Fig. (b), represent the moment diagram.

The change of curvature at any point is $\frac{M}{EI}$; hence $d\phi = \frac{M}{EI}ds$, and the total change of curvature from A to B is $\phi = \int_A^B \frac{Mds}{EI}$. For beams

originally straight, or very nearly so, the coordinates may be chosen so that x is measured along the beam, and as the deflections are relatively small dx may be used in place of ds and

$$\phi = \int_A^B \frac{M dx}{EI}. \quad \ldots \quad (15)$$

For a homogeneous beam of uniform section, this becomes

$$\phi = \frac{1}{EI}\int_A^B M dx. \quad \ldots \quad (16)$$

In these expressions Mdx is the elementary area represented by $E_1F_1H_1G_1$ of Fig. (b); hence $\int_A^B Mdx$ is the area of the moment diagram $JKLN$ from A to B.

ϕ is the total angular change due to the curvature of the beam from A to B and is equal to the angle between the tangents at A and B. In Fig. (a), $dt = xd\phi$; hence $t = \int_A^B xd\phi$, or

$$t = \int_A^B \frac{Mxdx}{EI}. \quad \ldots \quad (17)$$

If the beam is homogeneous and of uniform section

$$t = \frac{1}{EI}\int_A^B Mxdx. \quad \ldots \quad (18)$$

The distance $t = BD$ is the *Tangent-deviation at B.*

The expression $Mxdx$ is the moment of the elemental area, M, of the moment diagram, (b), about an ordinate through B; hence the expression $\int_A^B Mxdx$ is the moment of the area $JKLN$ about the ordinate through B and is the *Area-moment* of $JKLN$ about B. If P is the centroid of the moment diagram, the area-moment $= Pk$.

The expressions (15), (16), (17), and (18) state general relations which may be applied to the solution of many problems concerning the curvature and deflection of beams and will be found to simplify the calculations greatly.

When applying these equations, care must be taken to express the various quantities in their proper denominations.

M is in inch-pounds and E is in $\frac{\text{pounds}}{\text{inches}^2}$. I is in inches^4 and should be calculated from gross sections, with no deduction for rivet holes.

$\int_A^B \frac{Mdx}{EI}$ is in $\frac{\text{inches}^2 \times \text{pounds}}{\frac{\text{pounds}}{\text{inches}^2} \times \text{inches}^4}$,

hence is a ratio.

$$\int_A^B Mxdx \text{ is in inches}^3 \times \text{pounds}, \; EI \text{ in inches}^2 \times \text{pounds},$$

and

$$\int_A^B \frac{Mxdx}{EI} \text{ is in } \frac{\text{inches}^3 \times \text{pounds}}{\frac{\text{pounds}}{\text{inches}^2} \times \text{inches}^4},$$

hence is in inches.

7. Deflection at the end of a cantilever turntable, by the area-moment method.—Let Fig. 9F (*a*) represent one arm of a cantilever turntable of the same proportions and with the same loads as for that given in Art. 5. The values of I at critical sections are given in Table 9A. The values of M and the average values of I for the various segments of the girder and the ratios $\frac{M}{I}$ for the same segments are shown in Table 9D. The values of M and I are drawn to scale in Fig. (*b*) and those of $\frac{M}{I}$ in Fig. (*c*). E is assumed to be constant throughout the girder.

TABLE 9D

Section of Girder	Live Load Bending Moment, Feet-Pounds	Moment of Inertia, Inches^4	$\frac{M}{I}$ Pounds / $12 \times$ ins.^{3}
0	0	34,341	0
13	156,000	52,200	2.988
A	240,000	56,115	4.278
A_1	240,000	71,943	3.336
$12 = B$	936,000	119,085	7.860
C	1,280,000	146,894	8.715
C_1	1,280,000	169,137	7.568
D	1,600,000	189,767	8.432
D_1	1,600,000	220,071	7.272
E	1,810,000	234,319	7.724
F	2,290,000	234,319	9.773
F_1	2,290,000	294,399	7.779
G	3,034,000	294,399	10.307
G_1	3,034,000	356,419	8.512
H	3,958,400	356,419	11.105

The portion from O to H is a simple cantilever; hence from O to H, Eq. (17) becomes

$$t_o = \frac{1}{E}\int_o^H \frac{Mxdx}{I} \quad \text{or} \quad t_o = \frac{1}{E}\Sigma \text{ (area-moments)}$$

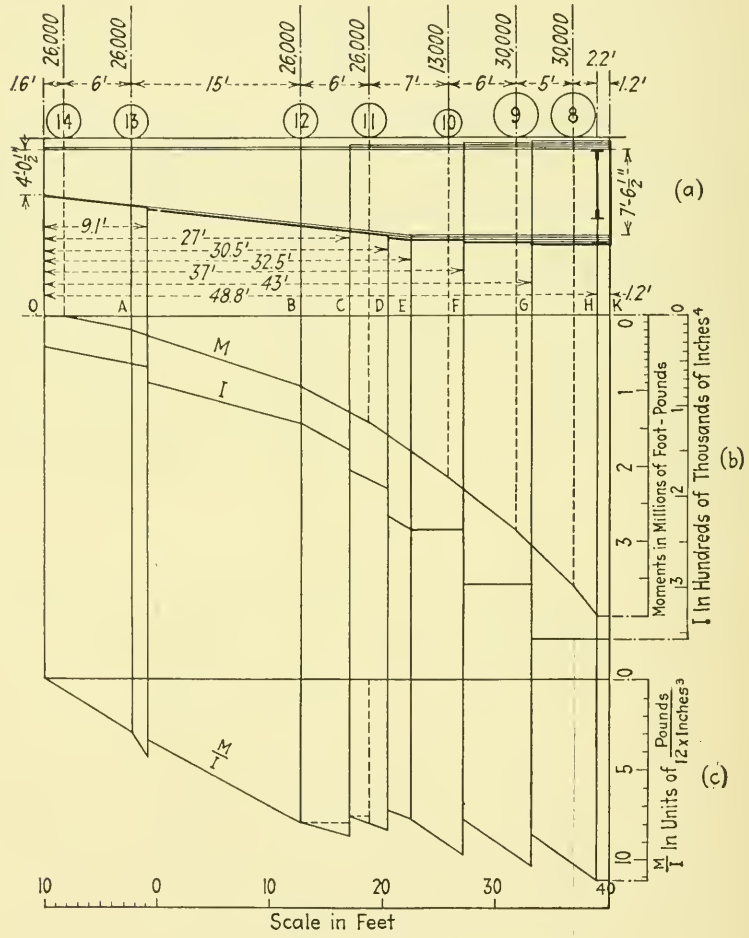


FIG. 9F.

about the ordinate through O. The end deflection due to bending moments from H to H_1 is shown to greatly exaggerated scale in Fig. 9G. Hence

$$t_{H_1} = \frac{1}{E}\int_H^{H_1} \frac{Mxdx}{I} \quad \text{or} \quad t_{H_1} = \frac{1}{E}\text{(area-moment of } HKH_1)$$

about an ordinate through H_1. From similar triangles,

$$t_{HO} = t_{H_1} \times \frac{HO}{HH_1} \text{ and the total end deflection is}$$

$$\Delta_O = t_O + t_{HO}.$$

One example of the method of calculation will be given in full. As in Art. 5, find the end deflection due to the bending moments in the segment of the girder from wheel 12 to wheel 11. The area-moments of $\frac{M}{I}$, Fig. (*c*), are to be found about an ordinate through *O*. Consider first the area from *B* to *C*. It may be taken as a rectangle and a small triangle, as indicated by the dotted line.

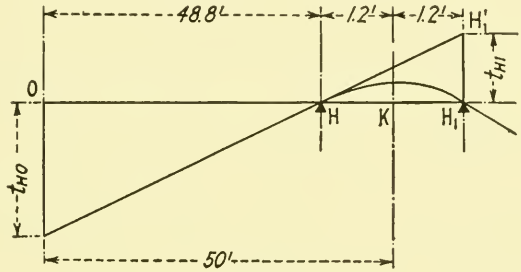


FIG. 9G.

Rectangle from *B* to *C*.

Area $= 7.860 \times 4.4 = 34.584.$

Area-moment $= 34.584 \times 24.8 = 857.683$

Small triangle from *B* to *C*.

Area $= (8.715 - 7.860) \times \frac{4.4}{2} = 1.881.$

Area-moment $= 1.881 \times 25.53 = 48.022$

Rectangle from *C* to 11.

Area $= 7.568 \times 1.6 = 12.109.$

Area-moment $= 12.109 \times 27.8 = 336.630$

Small triangle from *C* to 11.

Area $= (8.432 - 7.568)\frac{1.6}{3.5} \times \frac{1.6}{2} = 0.316.$

Area-moment $= 0.316 \times 28.067 = 8.869$

Total area-moment from *B* to 11 $= 1{,}251{,}204$

$$t_{B-11} = \frac{1}{E}\int_B \frac{Mxdx}{I} = \frac{1{,}251.204}{29{,}000{,}000} \times 1728 = 0.0746 \text{ in.}$$

The analytical method of Art. 5 gave 0.0748 in.

The difference is 0.0002 in., or 0.26 per cent, indicating the accuracy of the area-moment method.

The end deflection may be found in a similar manner for all segments of the girder and the results summed, but it is more convenient first to calculate the area-moments of $\frac{M}{I}$ for each segment, sum these area-moments, and finally divide the sum by E, taking care that the deflection is reduced to inches, unless all values have been taken in inch and pound units.

A summary of the calculations is given in Table 9E. From this table the end deflection due to bending moments from O to II is

$$t_O = \frac{9{,}721.178 \times 1728}{29{,}000{,}000} = 0.5792 \text{ in.}$$

TABLE 9E

Limit of Area	Dimensions for Area	Area, Pounds / $12^2 \times$Ins.^{2}	Distance of Centroid of Area from O, Feet	Area-Moment About O, Pounds / $12^3 \times$Ins.
13	$2.988 \times 7.6 \times \frac{1}{2}$	11.355	5.07	57.560
A	2.988×1.5	4.482	8.35	37.420
A	$1.290 \times 1.5 \times \frac{1}{2}$	0.967	8.60	8.320
$12 = B$	3.336×13.5	45.035	15.85	713.700
$12 = B$	$4.524 \times 13.5 \times \frac{1}{2}$	30.538	18.10	552.700
C	7.860×4.4	34.584	24.80	857.683
C	$0.855 \times 4.4 \times \frac{1}{2}$	1.881	25.53	48.022
D	7.568×3.5	26.486	28.75	761.500
D	$0.864 \times 3.5 \times \frac{1}{2}$	1.519	29.33	44.345
E	7.272×2	14.544	31.50	458.170
E	$0.452 \times 2 \times \frac{1}{2}$	0.452	31.83	14.388
F	7.724×4.5	34.757	34.75	1207.600
F	$2.049 \times 4.5 \times \frac{1}{2}$	4.611	35.50	163.670
G	7.779×6	46.674	40.00	1866.960
G	$2.582 \times 6 \times \frac{1}{2}$	7.584	41.00	310.960
H	8.512×5.8	49.366	45.90	2265.800
H	$2.593 \times 5.8 \times \frac{1}{2}$	7.518	46.867	352.380
Sum to H				9721.178

Referring now to Fig. 9G, the area $III_1 = 11.105 \times 2.4 = 26.652$. The area-moment about II_1 is $26.652 \times 1.2 = 31.982$.

$$t_{H_1} = \frac{31.982 \times 1728}{29{,}000{,}000} = 0.0019058 \text{ in.}$$

$$t_{HO} = 0.0019058 \times \frac{48.8}{2.4} = 0.0388 \text{ in.}$$

The total deflection at O then is:

$$\Delta_O = t_O + t_{HO} = 0.5793 + 0.0388 = 0.6181 \text{ in.}$$

8. Deflection at the end of a cantilever turntable, by the use of deflection influence lines.—Professors Merriman and Jacoby have given an application of influence lines to the calculation of the deflections of beams in "Roofs and Bridges," Part II, edition of 1917, which results in a convenient method of finding the deflection of a beam under any system of loads. It becomes a simple matter to compare the deflections of a point in the beam for various systems of loads.

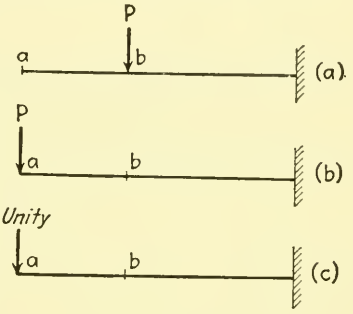


FIG. 9H.

Maxwell's law of reciprocal deflections establishes the fact that the deflection of the point a of the beam in Fig. 9H (a), due to a load P at b, is the same as that at b in Fig. (b) for the load P at a. If, then, the load at a, in Fig. (c), is made one unit and a deflection diagram is constructed for the beam, this diagram is a deflection influence line for a point at the end of the beam. The end deflection due to any system of concentrated loads may then be found by scaling the influence-line ordinates under the loads, multiplying each by the amount of the load, and summing the partial end deflections so found.

Let Fig. 9 I (a) represent one arm of a cantilever turntable girder of the same proportions and with the same loads as given in Art. 5 Fig. (b) shows the critical values of I for various sections of the girder drawn to scale. The values are taken from Table 9D. The girder OH may be divided into any convenient number of equal parts, as 10, as shown in Figs. (b) and (c), and an average value of I found for each segment of the girder. As OH = 48.8 ft., each segment is 4.88 ft., or 58.56 in. long. In a cantilever, m, in Eq. (1), becomes x for the origin at the free end. If δx be the length of a subdivision of the girder, Eq. (1) may be written

$$\Delta_a = \frac{\delta x}{E} \Sigma \frac{m_a x}{I_a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

The summation may be made by constructing an equilibrium polygon, treating the values of $\frac{m}{I}$ as loads. In Fig. (c) draw a moment diagram for m at every point of the girder for load 1 at its end O. As $m\delta x$ is the

area of any segment of the moment diagram, the loads $\frac{m}{I}$ must be applied at the centers of gravity of the segment areas. The pole dis-

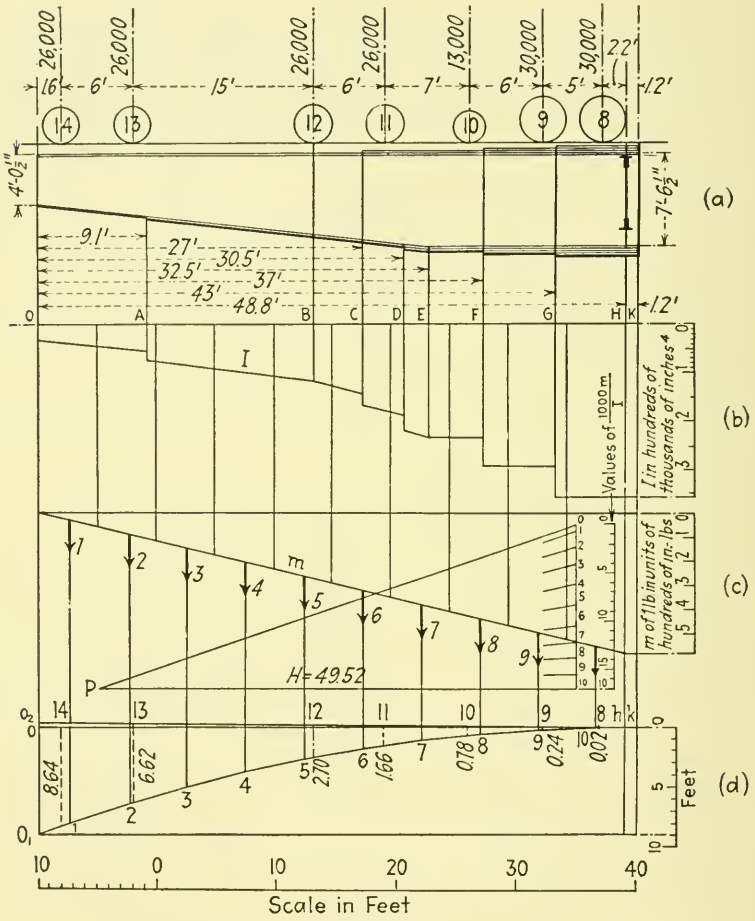


Fig. 9 I.

tance, H, must be laid off to the same scale as the loads; hence it must be expressed in the same units.

$$\Sigma \frac{m}{I} x = Hy,$$

in which y is the ordinate of the equilibrium polygon at the limit to which the summation is to be made. As Δ is very small, the ordinates,

y, should be magnified n times for convenient drafting and scaling, or $y = n\Delta$. Substituting in (19),

$$\Delta = \frac{\delta x}{E} H n \Delta, \quad \text{or} \quad H = \frac{E}{\delta x \cdot n}.$$

Introducing the numerical values from the example.

$$H = \frac{29{,}000{,}000}{58.56 \times n} = \frac{495{,}220}{n}.$$

If n is taken as 10,000, $H = 49.52$.

$\frac{m}{I}$ is expressed in $\frac{\text{pounds}}{\text{inches}^3}$ and H is in the same unit.

Table 9F gives the values of m, I, and $\frac{m}{I}$ in units of the proper denominations and reduced to values convenient for scaling.

TABLE 9F

Designation of Ordinate through Centroid of Segment	Moment of 1 about Ordinate m Inch-Pounds	Moment of Inertia $\div 1000$, or $\frac{I}{1000}$ Inches^4	$\frac{1000m}{I}$ Pounds / Inches^3	Summation of $\frac{1000m}{I}$
1	29.28	39.6	0.74	0.74
2	87.84	53.8	1.63	2.37
3	146.40	84.0	1.74	4.11
4	204.96	100.0	2.05	6.16
5	263.52	116.8	2.25	8.41
6	322.08	155.5	2.07	10.48
7	380.64	220.5	1.72	12.20
8	439.20	260.0	1.69	13.89
9	497.76	306.0	1.63	15.52
10	556.32	356.4	1.56	17.08

In Fig. (*c*), lay off the load line, using the values of $\frac{1000m}{I}$, Table 9F, as loads. For accuracy it is best to lay off the totals given in Column 5 from the origin. Lay off the value of $H = 49.52$. Draw rays to the pole, P. Construct the moment diagram, Fig. (*d*), beginning at the vertical through load 10 and ending at O_1. The line O_1h is the influence line for deflections of the cantilever portion of the girder OH.

The influence line for end deflection due to bending moments in the segment of the girder from H to K requires special consideration.

Remembering Maxwell's law, place a load of unity at O, find the end deflection produced by the bending moments from H to K, due to the load, and reduce this deflection to the scale of Fig. (*d*) and lay it off upward from O to O_2 and draw O_2h. O_2h is the influence line for end deflections due to the bending moments from H to K.

From Eq. (13), the general expression for end deflections due to moments from H to K may be written

$$\Delta_{HK_1} = \frac{1}{E}\int_H^K \frac{M_H m_H dx}{I_H}.$$

In this equation m_H is a constant, for the only moments causing deflection are from H to K. $\int_H^K dx$ is a known distance, equal to HK, and is also constant. M_H is the moment at H due to a load of 1 on the girder at any point between O and K, hence,

$$\Delta_{HK_1} = \frac{m_H \times HK}{E}\sum\frac{H - x}{I_H}. \quad . \quad . \quad . \quad . \quad (20)$$

This is a first-degree equation and may be represented by a straight line, one point of which is at H, the point of no deflection. If a second point is found, the line can be drawn. From Eq. (14)

$$\Delta_{HK_1} = \frac{H}{EI_H} m_H \overset{K}{\underset{H}{[x]}}.$$

Substitute numerical values in this and find Δ_{HK_1} for a load at O.

$$\Delta_{HK_1} = \frac{48.8 \times 48.8 \times 1.2}{29{,}000{,}000 \times 356{,}419} \times 1728 = 0.0000004777 \text{ in.}$$

The scale of Fig. (*d*) is in feet, and the deflection was multiplied by 10,000 in finding H. $\frac{M}{I}$, Table 9F, was multiplied by 1000. The end deflection in inches is

$$\Delta_{HK_1} = \frac{12 \times OO_2}{10{,}000 \times 1000}.$$

But

$$\Delta_{HK_1} = 0.0000004777;$$

hence,

$$OO_2 = \frac{4.777}{12} = 0.398 \text{ ft.}$$

Lay off OO_2 above the base line of Fig. (*d*) and draw O_2h. This is the required influence line for deflections at O due to moments from H to K. The actual deflections at O due to the loads shown in Fig. (*a*)

may now be found by scaling the total ordinates under the respective loads between the influence lines O_2h and O_1 10, for

$$\Delta_a = \frac{y_a \times 12 \times \frac{\text{wheel load}}{1000}}{10{,}000}.$$

Table 9G gives a summary of the calculations.

TABLE 9G

Wheel	Ordinate y, Feet	Factor for Inches	Wheel Load / 1000	Divisor to Reduce to Natural Scale	Deflection at 0, Inches
14	8.64				
13	6.62				
12	2.70				
11	1.66				
	19.62	12	26	10,000	0.6121
10	0.78	12	13	10,000	0.0121
9	0.24				
8	0.02				
	0.26	12	30	10,000	0.0094
Total end deflection in inches					0.6336

9. Deflection at the end of a cantilever turntable, by an approximate method.—A closely approximate deflection under loads can be found by the methods given in Arts. 5, 6, 7, and 8, but each requires more time than is sometimes available. When placing a cantilever table in a pit, after providing for the calculated deflection from live load, it is customary to adjust it about $\frac{1}{4}$ in. higher than indicated, in order to allow for deflections due to shear and to provide swinging clearance above the circle-rail. Great precision in the calculations is unnecessary in such cases. Moreover, while a turntable is designed for definite locomotives of ample weight and length, it is seldom that just such engines are turned, most operations being under smaller engines and with a great variety of wheel spacing.

For a reasonably consistent series of designs, an approximate method of calculating end deflections can be found which will serve in many practical cases.

In Fig. 9J let

$\frac{l}{2}$ = length of one arm of the turntable, in feet;
a = length of a uniform load, in feet;
w = uniform load, in pounds per linear foot;
x = any distance from the end of the uniform load;
E = modulus of elasticity;
I = moment of inertia of the maximum girder section, in inches^4, and
Δ = required end deflection in inches.

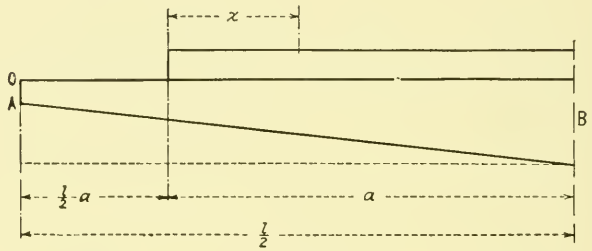


Fig. 9J.

Then

$$\Delta = \int \frac{Mm}{EI} dx = \int_0^a \frac{\frac{wx^2}{2}\left(x + \frac{l}{2} - a\right)dx}{EI}. \quad . \quad . \quad . \quad (21)$$

$$= \frac{1}{EI}\int_0^a \left(\frac{wx^3}{2}dx + \frac{wlx^2}{4}dx - \frac{wax^2}{2}dx\right)$$

$$= \frac{1}{EI}\left(\frac{wa^4}{8} + \frac{wla^3}{12} - \frac{wa^4}{6}\right)$$

$$= \frac{1}{24EI}(3a^4 + 2la^3 - 4a^4),$$

or

$$\Delta = \frac{wa^3}{24EI}(2l - a).$$

But the moment at the fixed end, B, the center of the table, is

$$\frac{wa^2}{2} = M,$$

hence,

$$\Delta = \frac{Ma}{12EI}(2l - a). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

This expression may be written

$$\Delta = \frac{Ma}{KI}(2l - a), \quad \text{or} \quad K = \frac{Ma}{\Delta I}(2l - a). \quad . \quad . \quad . \quad (23)$$

When applied to a turntable loaded with a locomotive in balanced position, the distance, a, for the maximum deflection at the end, will be the distance from the center of gravity of the engine to the center of the end tender axle, in practically all cases.

The end deflections of a series of tables were found, by more exact methods, and the values of M, a, l, Δ, and I substituted in Eq. (23) for each table and the corresponding values of K calculated. The average value of K was about 139,000. Only relatively small variations from this value occurred for tables from 55 to 100 ft. in length.

The approximate formula for end deflection then becomes

$$\Delta = \frac{Ma(2l - a)}{139{,}000\,I}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

In this formula the engine is assumed to be balanced; that is, its center of gravity is at the center of the table, and its tender is full. The value of M is the live-load moment, in foot-pounds, of the wheels at the tender end of the engine about the center of gravity of the engine. I is the moment of inertia, in inches^4, of the maximum section at the center of the main girders of the table. L and a are in feet.

Using data from a series of standard deck cantilever turntables, as built by the American Bridge Company, in Eq. (24), the end deflections shown in Table 9H were found.

TABLE 9H

Length in Feet L	Engine	Length c. to c. of End Axles, Feet	Live Load Moment in Ft.-Lbs. at c.g. of Engine, M	Distance in Feet a	Moment of Inertia in Inches^4 I	$2L-a$	Deflection in Inches Δ
75	213-ton Consolidation, 2–8–0	63.5	1,786,200	33.9	124,700	116.1	0.41
80	250-ton Mikado, 2–8–2	70	2,346,000	36.4	160,000	123.6	0.47
90	325-Mallet, 2–8–8–0	83	3,233,000	42.9	260,800	137.1	0.52
100	375-ton Mallet, 2–8–8–2	92	4,170,800	48.4	356,400	151.6	0.62
110	450-ton Mallet, 2–10–10–2	96	5,200,800	49.9	492,800	170.1	0.65
120	475-ton Mallet, 2–10–10–2	108.5	6,264,400	57.7	647,600	182.3	0.73

Table 9 I gives a comparison of the end deflection of a 100-ft. deck turntable, designed for a 375-ton Mallet, 2–8–8–2 locomotive, calculated by the methods given in Arts. 5, 7, 8, and 9. It will be noted

that the results obtained by the graphical and approximate methods agree very closely with those obtained by the analytical method.

TABLE 9 I

Method of Calculation	Article	End Deflection, Inches	Differences, Inches	Differences, Per Cent
Analytical	5	0.6261	0.0	0.0
Area-moments	7	0.6181	−0.0080	−1.28
Deflection influence lines	8	0.6336	+0.0075	+1.20
Approximate	9	0.6200	−0.0061	−0.99

10. Vertical deflection at the center of a girder, with varying moment of inertia, due to a concentrated load at any point.—Let Fig. 9K represent a plate girder with three cover plates

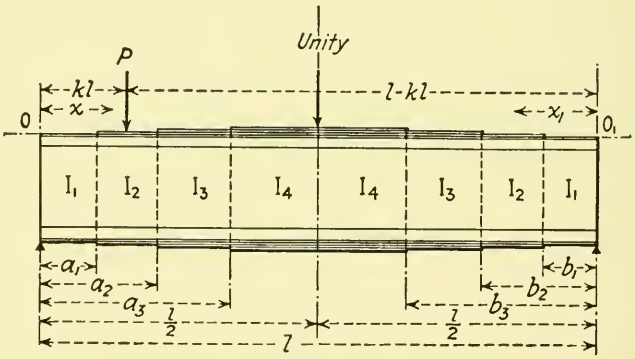


Fig. 9K.

Find the vertical deflection at the center due to a load, P, at any point distant kl from the left support. From Eq. (1)

$$\Delta = \int_0^l \frac{Mmdx}{EI}.$$

Substituting the values given in Fig. 9K,

$$\Delta = \frac{1}{2E}\int_0^{a_1}\frac{P}{I_1}(1-k)x^2dx + \frac{1}{2E}\int_{a_1}^{kl}\frac{P}{I_2}(1-k)x^2dx$$
$$+ \frac{1}{2E}\int_{kl}^{a_2}\frac{Pk}{I_2}(lx-x^2)dx + \frac{1}{2E}\int_{a_2}^{a_3}\frac{Pk}{I_3}(lx-x^2)dx$$

$$+\frac{1}{2E}\int_{a_3}^{\frac{l}{2}}\frac{Pk}{I_4}(lx-x^2)dx+\frac{1}{2E}\int_{0}^{b_1}\frac{Pk}{I_1}x_1^2dx_1$$

$$+\frac{1}{2E}\int_{b_1}^{b_2}\frac{Pk}{I_2}x_1^2dx_1+\frac{1}{2E}\int_{b_2}^{b_3}\frac{Pk}{I_3}x_1^2dx_1$$

$$+\frac{1}{2E}\int_{b_3}^{\frac{l}{2}}\frac{Pk}{I_4}x_1^2dx_1,$$

$$=\frac{P(1-k)}{6E}\left[\frac{a_1^3}{I_1}+\frac{k^3l^3-a_1^3}{I_2}\right]$$

$$+\frac{Pkl}{4E}\left[\frac{a_2^2-k^2l^2}{I_2}+\frac{a_3^2-a_2^2}{I_3}+\frac{\left(\frac{l}{2}\right)^2-a_3^2}{I_4}\right]$$

$$+\frac{Pk}{6E}\left[\frac{b_1^3}{I_1}+\frac{b_2^3-b_1^3}{I_2}+\frac{b_3^3-b_2^3}{I_3}\right.$$

$$+\frac{\left(\frac{l}{2}\right)^3-b_3^3}{I_4}-\frac{a_2^3-k^3l^3}{I_2}$$

$$\left.-\frac{a_3^3-a_2^3}{I_3}-\frac{\left(\frac{l}{2}\right)^3-a_3^3}{I_4}\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

If the cover plates are placed symmetrically with respect to the center of the girder, $b_1 = a_1$, $b_2 = a_2$, and $b_3 = a_3$, and

$$\Delta=\frac{P}{6E}\left[\frac{a_1^3}{I_1}+\frac{k^3l^3-a_1^3}{I_2}\right]$$

$$+\frac{Pkl}{4E}\left[\frac{a_2^2-k^2l^2}{I_2}+\frac{a_3^2-a_3^2}{I_3}+\frac{\left(\frac{l}{2}\right)^2-a_3^2}{I_4}\right]. \quad . \quad (26)$$

If the moment of inertia is constant from end to end of the girder, $I_1 = I_2 = I_3 = I_4 = I$, and

$$\Delta=\frac{Pkl^3}{48EI}[3-4k^2]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

If now the load, P, is placed at the middle of the girder span, $k = \frac{1}{2}$ and

$$\Delta=\frac{Pl^3}{48EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

This is the well-known value of the deflection at the center of a beam of uniform section with a single concentrated load at its center.

While Eqs. (25) and (26) may be further reduced, the forms given will be found convenient for practical application, for all of the terms

in the numerators within the brackets become simple numbers, and the squares and cubes may be taken from tables giving the functions of numbers.

If P is placed on the second cover plate, the expression for deflection becomes

$$\Delta = \frac{P(1-k)}{6E}\left[\frac{a_1^3}{I_1} + \frac{a_2^3 - a_1^3}{I_2} + \frac{k^3l^3 - a_2^3}{I_3}\right] + \frac{Pkl}{4E}\left[\frac{a_3^2 - k^2l^2}{I_3} + \frac{\left(\frac{l}{2}\right)^2 - a_3^2}{I_4}\right] + \frac{Pk}{6E}\left[\frac{b_1^3}{I_1} + \frac{b_2^3 - b_1^3}{I_2} + \frac{b_3^3 - b_2^3}{I_3} + \frac{\left(\frac{l}{2}\right)^3 - b_3^3}{I_4} - \frac{a_3^3 - k^3l^3}{I_3} - \frac{\left(\frac{l}{2}\right)^3 - a_3^3}{I_4}\right] \quad . \quad . \quad . \quad (29)$$

If the cover plates are placed symmetrically with respect to the center of the girder, $b_1 = a_1$, $b_2 = a_2$, $b_3 = a_3$, and

$$\Delta = \frac{P}{6E}\left[\frac{a_1^3}{I_1} + \frac{a_2^3 - a_1^3}{I_2} + \frac{k^3l^3 - a_2^3}{I_3}\right] + \frac{Pkl}{4E}\left[\frac{a_3^2 - k^2l^2}{I_3} + \frac{\left(\frac{l}{2}\right)^2 - a_3^2}{I_4}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

If the moment of inertia is constant, $I_1 = I_2 = I_3 = I_4 = I$, and the expression for Δ is the same as in Eq. (27).

When $k = \frac{1}{2}$ the value of Δ is the same as in Eq. (28).

11. Vertical deflection at the loaded point of a girder with varying moment of inertia.—Let Fig. 9L represent a plate girder with three cover plates.

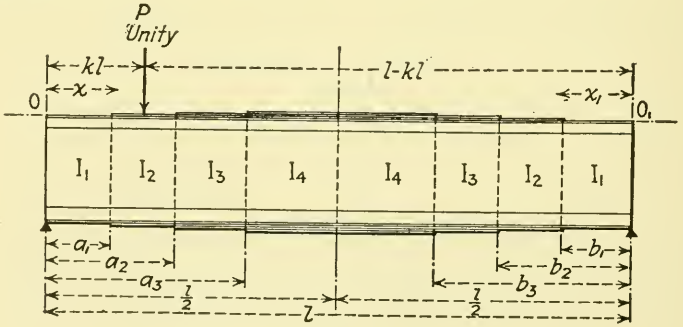


FIG. 9L.

Find the vertical deflection at the point of application of the load, P, distant kl from the left support. Substituting values shown in Fig. 9L in Eq. (1)

$$\Delta = \frac{1}{E}\int_0^{a_1}\frac{P}{I_1}(1-k)^2x^2dx + \frac{1}{E}\int_{a_1}^{kl}\frac{P}{I_2}(1-k)^2x^2dx$$
$$+ \frac{1}{E}\int_{kl}^{a_2}\frac{Pk^2}{I_2}(l-x)^2dx + \frac{1}{E}\int_{a_2}^{a_3}\frac{Pk^2}{I_3}(l-x)^2dx$$
$$+ \frac{1}{E}\int_{a_3}^{\frac{l}{2}}\frac{Pk^2}{I_4}(l-x)^2dx + \frac{1}{E}\int_0^{b_1}\frac{Pk^2}{I_1}x_1^2dx_1$$
$$+ \frac{1}{E}\int_{b_1}^{b_2}\frac{Pk^2}{I_2}x_1^2dx_1 + \frac{1}{E}\int_{b_2}^{b_3}\frac{Pk^2}{I_3}x_1^2dx_1 + \frac{1}{E}\int_{b_3}^{\frac{l}{2}}\frac{Pk^2}{I_4}x_1^2dx_1,$$
$$= \frac{P(1-k)^2}{3E}\left[\frac{a_1^3}{I_1} + \frac{k^3l^3 - a_1^3}{I_2}\right]$$
$$+ \frac{Pk^2l^2}{E}\left[\frac{a_2 - kl}{I_2} + \frac{a_3 - a_2}{I_3} + \frac{\left(\frac{l}{2}\right) - a_3}{I_4}\right]$$
$$- \frac{Pk^2l}{E}\left[\frac{a_2^2 - k^2l^2}{I_2} + \frac{a_3^2 - a_2^2}{I_3} + \frac{\left(\frac{l}{2}\right)^2 - a_3^2}{I_4}\right]$$
$$+ \frac{Pk^2}{3E}\left[\frac{a_2^3 - k^3l^3}{I_2} + \frac{a_3^3 - a_2^3}{I_3} + \frac{\left(\frac{l}{2}\right)^3 - a_3^3}{I_4} + \frac{b_1^3}{I_1}\right.$$
$$\left. + \frac{b_2^3 - b_1^3}{I_2} + \frac{b_3^3 - b_2^3}{I_3} + \frac{\left(\frac{l}{2}\right)^3 - b_3^3}{I_4}\right]. \quad . \quad . \quad . \quad . \quad (31)$$

If the cover plates are placed symmetrically with respect to the center of the girder, $b_1 = a_1$, $b_2 = a_2$, $b_3 = a_3$, and

$$\Delta = \frac{P(1-k)^2}{3E}\left[\frac{a_1^3}{I_1} + \frac{k^3l^3 - a_1^3}{I_2}\right]$$
$$+ \frac{Pk^2l^2}{E}\left[\frac{a_2 - kl}{I_2} + \frac{a_3 - a_2}{I_3} + \frac{\left(\frac{l}{2}\right) - a_3}{I_4}\right]$$
$$- \frac{Pk^2l}{E}\left[\frac{a_2^2 - k^2l^2}{I_2} + \frac{a_3^2 - a_2^2}{I_3} + \frac{\left(\frac{l}{2}\right)^2 - a_3^2}{I_4}\right]$$

$$+\frac{Pk^2}{3E}\left[\frac{a_1^3}{I_1}+\frac{2a_2^3-k^3l^3-a_1^3}{I_2}+\frac{2a_3^3-2a_2^3}{I_3}\right.$$
$$\left.+\frac{2\left(\frac{l}{2}\right)^3-2a_3^3}{I_4}\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

If the moment of inertia is constant from end to end of the girder, $I_1 = I_2 = I_3 = I_4 = I$, and

$$\Delta = \frac{Pk^2l^3(1-k)^2}{3EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

If now P is placed at the middle of the girder span, $k = \frac{1}{2}$, and

$$\Delta = \frac{Pl^3}{48EI}, \text{ as in Eq. (28).} \quad . \quad . \quad . \quad . \quad . \quad (34)$$

12. Vertical deflection at the center of a girder, with varying moment of inertia, due to a partial uniform load.—Let Fig. 9M be a plate girder, with three cover plates, carrying a uniform load of w per unit of length extending from the right support to a point kl from the left support.

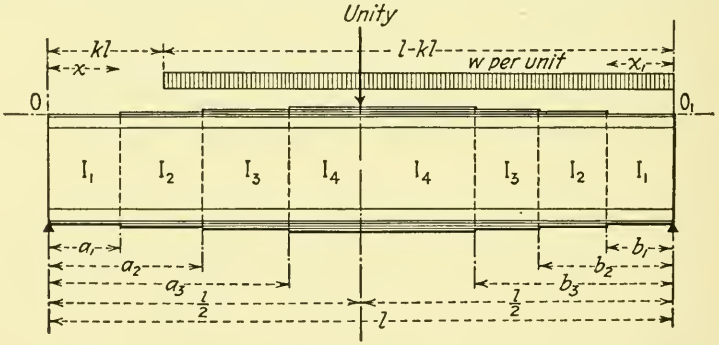


FIG. 9M.

Find the vertical deflection at the middle of the span. Substituting values shown in Fig. 9M in Eq. (1)

$$\Delta = \frac{wl}{4E}\int_0^{a_1}\frac{(1-k)^2x^2dx}{I_1}+\frac{wl}{4E}\int_{a_1}^{kl}\frac{(1-k)^2x^2dx}{I_2}$$
$$+\frac{w}{4E}\int_{kl}^{a_2}\left[\frac{l(1-k)^2x^2dx}{I_2}-\frac{(x-kl)^2xdx}{I_2}\right]$$
$$+\frac{w}{4E}\int_{a_2}^{a_3}\left[\frac{l(1-k)^2x^2dx}{I_3}-\frac{(x-kl)^2xdx}{I_3}\right]$$
$$+\frac{w}{4E}\int_{a_3}^{\frac{l}{2}}\left[\frac{l(1-k)^2x^2dx}{I_4}-\frac{(x-kl)^2xdx}{I_4}\right]$$

$$+\frac{w}{4E}\int_0^{b_1}\left[\frac{l(1-k^2)x_1^2dx_1}{I_1}-\frac{x_1^3dx_1}{I_1}\right]$$

$$+\frac{w}{4E}\int_{b_1}^{b_2}\left[\frac{l(1-k^2)x_1^2dx_1}{I_2}-\frac{x_1^3dx_1}{I_2}\right]$$

$$+\frac{w}{4E}\int_{b_2}^{b_3}\left[\frac{l(1-k^2)x_1^2dx_1}{I_3}-\frac{x_1^3dx_1}{I_3}\right]$$

$$+\frac{w}{4E}\int_{b_3}^{\frac{l}{2}}\left[\frac{l(1-k^2)x_1^2dx_1}{I_4}-\frac{x_1^3dx_1}{I_4}\right],$$

$$=\frac{wl(1-k)^2}{12E}\left[\frac{a_1^3}{I_1}+\frac{k^3l^3-a_1^3}{I_2}+\frac{a_2^3-k^3l^3}{I_2}\right.$$

$$\left.+\frac{a_3^3-a_2^3}{I_3}+\frac{\left(\frac{l}{2}\right)^3-a_3^3}{I_4}\right]$$

$$+\frac{wl(1-k^2)}{12E}\left[\frac{b_1^3}{I_1}+\frac{b_2^3-b_1^3}{I_2}+\frac{b_3^3-b_2^3}{I_3}+\frac{\left(\frac{l}{2}\right)^3-b_3^3}{I_4}\right]$$

$$-\frac{w}{16E}\left[\frac{a_2^4-k^4l^4}{I_2}+\frac{a_3^4-a_2^4}{I_3}+\frac{\left(\frac{l}{2}\right)^4-a_3^4}{I^4}\right.$$

$$\left.+\frac{b_1^4}{I_1}+\frac{b_2^4-b_1^4}{I_2}+\frac{b_3^4-b_2^4}{I_3}+\frac{\left(\frac{l}{2}\right)^4-b_3^4}{I_4}\right]$$

$$+\frac{wkl}{6E}\left[\frac{a_2^3-k^3l^3}{I_2}+\frac{a_3^3-a_2^3}{I_3}+\frac{\left(\frac{l}{2}\right)^3-a_3^3}{I_4}\right]$$

$$-\frac{wk^2l^2}{8E}\left[\frac{a_2^2-k^2l^2}{I_2}+\frac{a_3^2-a_2^2}{I_3}+\frac{\left(\frac{l}{2}\right)^2-a_3^2}{I_4}\right]. \quad . \quad . \quad (35)$$

If the cover plates are placed symmetrically with respect to the center of the girder and the uniform load extends from end to end of the girder, $b_1 = a_1$, $b_2 = a_2$, $b_3 = a_3$, $k = 0$, and

$$\Delta=\frac{wl}{6E}\left[\frac{a_1^3}{I_1}+\frac{a_2^3-a_1^3}{I_2}+\frac{a_3^3-a_2^3}{I_3}+\frac{\left(\frac{l}{2}\right)^3-a_3^3}{I_4}\right]$$

$$-\frac{w}{6E}\left[\frac{a_1^4}{I_1}+\frac{a_2^4-a_1^4}{I_2}+\frac{a_3^4-a_2^4}{I_3}+\frac{\left(\frac{l}{2}\right)^4-a_3^4}{I_4}\right]. \quad . \quad . \quad . \quad (36)$$

If the moment of inertia is constant from end to end of the girder, $I_1 = I_2 = I_3 = I_4 = I$, and

$$\Delta = \frac{5wl^4}{384EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

This is the familiar value for the deflection at the middle of a beam, of uniform section, due to a uniform load extending over its entire length.

13. Horizontal deflection of the upper corner of a simple beam.— When a beam is deflected in a vertical plane by any system of loads, its neutral axis becomes convex downward and a tangent at its end makes an angle ϕ with the horizontal. If the ends of the beam were vertical before deflection, they will be inclined away from the vertical by the angle ϕ after deflection. Thus the upper corner of the beam, which was originally vertically above the support, will be moved a distance Δ in a horizontal direction by the deflection.

Assume a beam, of uniform section, to be deflected in a vertical plane by a uniform load, the conditions being as represented in Fig. 9N.

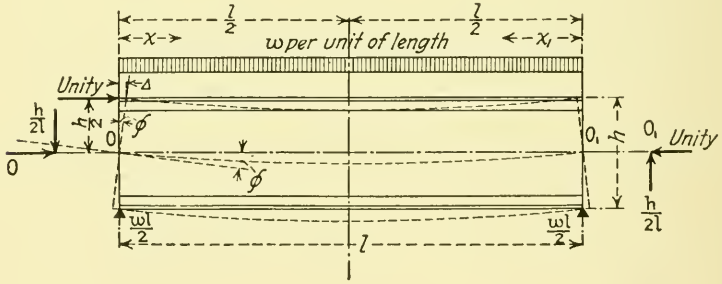


Fig. 9N.

Assume the load of unity to be applied horizontally at the upper left corner, and let Δ be the required horizontal movement, or deflection, of the corner of the beam.

$$\Delta = \int_0^l \frac{Mmdx}{EI} = \int_0^{\frac{l}{2}} \frac{Mmdx}{EI} + \int_0^{\frac{l}{2}} \frac{Mmdx_1}{EI},$$

in which $M = \frac{w}{2}(lx - x^2)$ for both halves of the beam.

$$m = +\frac{h}{2l}x \text{ for the right half and } m = -\frac{h}{2l}x + \frac{h}{2} \text{ for the left}$$

half of the beam; hence,

$$\Delta = \frac{wh}{4EI}\int_0^{\frac{l}{2}}\left[x^2 - \frac{x^3}{l}\right]dx - \frac{wh}{4EI}\int_0^{\frac{l}{2}}\left[x^2 - \frac{x^3}{l}\right]dx$$

$$+ \frac{wh}{4EI}\int_0^{\frac{l}{2}}[lx - x^2]dx;$$

or

$$\Delta = \frac{wh}{4EI}\int_0^{\frac{l}{2}}[lx - x^2]dx = \frac{whl^3}{48EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

If the girder was loaded by a single concentrated load, P, at its center, the horizontal deflection of the same corner would be

$$\Delta = \frac{Ph}{4EIl}\int_0^{\frac{l}{2}}x^2dx - \frac{Ph}{4EIl}\int_0^{\frac{l}{2}}x^2dx + \frac{Ph}{4EI}\int_0^{\frac{l}{2}}xdx,$$

or

$$\Delta = \frac{Phl^2}{32EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

The same expressions may be obtained by another method. The expression $\int_0^l \frac{Mdx}{EI}$ gives the total curvature of the beam throughout its length. If the integration is made from 0 to $\frac{l}{2}$, or from one end to the middle of the beam, the integral will express the angle ϕ through which a tangent to the neutral axis at the end passes, during deflection. By inserting the value of M given above,

$$\phi = \int_0^{\frac{l}{2}}\frac{Mdx}{EI} = \frac{w}{2EI}\int_0^{\frac{l}{2}}(lx - x^2)dx = \frac{wl^3}{24EI}.$$

Now,

$$\Delta = \phi \times \frac{h}{2};$$

hence,

$$\Delta = \frac{whl^3}{48EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38a)$$

Similarly, for a load, P, at the center of the beam,

$$\phi = \int_0^{\frac{l}{2}}\frac{Mdx}{EI} = \frac{P}{2EI}\int_0^{\frac{l}{2}}xdx = \frac{Pl^2}{16EI}.$$

As before, $\Delta = \phi \times \frac{h}{2}$; hence

$$\Delta = \frac{Phl^2}{32EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39a)$$

14. Deflection at the center of a deck-plate girder.—Let it be required to find the vertical deflection at the center of a 70-ft. square deck plate girder span under its dead load and a Cooper E60 engine. The girders are 70 ft. long out to out, and 68 ft. center to center of end supports, and are each of the following proportions:

1 web plate 80 × $\frac{7}{16}$ in.
4 angles 6 × 6 × $\frac{3}{4}$ in., placed 6 ft. 8$\frac{1}{2}$ in. back to back.
1 top cover plate 14 × $\frac{3}{4}$ in., extending the full length of the girder.
1 bottom cover plate 14 × $\frac{3}{4}$ in., 55 ft. effective length.
1 top and bottom cover plate 14 × $\frac{3}{4}$ in., 46 ft. effective length.
1 top and bottom cover plate 14 × $\frac{3}{4}$ in., 32 ft. effective length.

The dead load is as follows:

Wooden deck and track.....	450 lbs. per linear foot.
Steel.....	1040 lbs. per linear foot.
Total.....	1490 lbs. per linear foot.

This is 745 lbs. per linear foot per girder.

Figure 9O (*a*) represents the girder with the live load upon it. The calculated dead, live, and total bending moments at various points along the girder are shown in Table 9J, Columns 2, 3 and 4. They are also shown plotted to scale in Fig. (*b*). The calculated values of I at the same sections are given in Column 5 and drawn to scale in Fig. (*b*). Column 6 gives the values of $\frac{M}{I}$, and these are shown to scale in Fig. (*c*).

As explained in Art. 6, the tangent-deviation under each end reaction is then found by summing the area-moments of $\frac{M}{I}$ from the end to the center of the girder, taking the center of moments at the end support for each half of the girder, and dividing these sums by E, remembering to multiply the quotients by $12^3 = 1728$, since all distances along the girder have been taken in feet and the tangent-deviation is required to be in inches.

Table 9K gives a summation of the area-moment calculations for each half of the girder. The tangent-deviation at R_L is

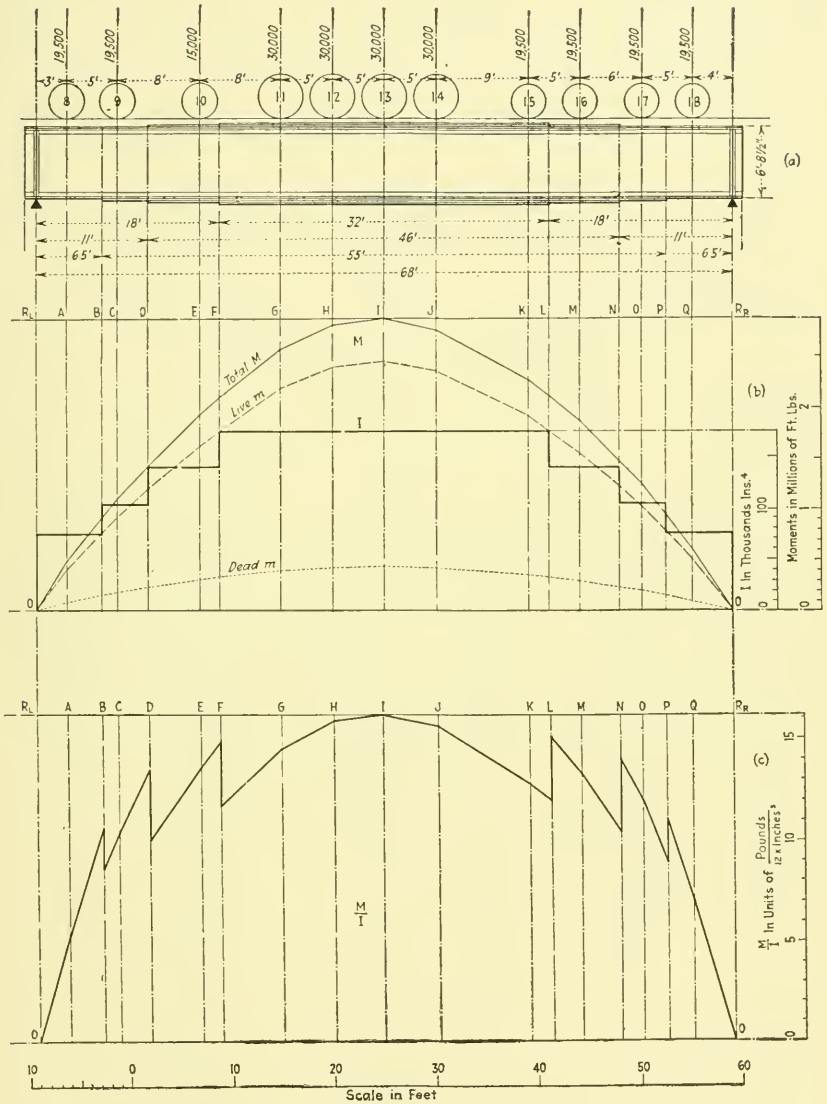
$$t_L = \frac{8104.52}{29{,}000{,}000} \times 1728 = 0.483 \text{ in.}$$

At R_R,

$$t_R = \frac{8093.11}{29{,}000{,}000} \times 1728 = 0.482 \text{ in.}$$

At the center,

$$\Delta = \frac{0.483 + 0.482}{2} = 0.4825 \text{ in.}$$


19,500
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10
11
12
13
14
15
16
17
18
6'-8½"
(a)
18'
32'
18'
11'
46'
11'
65'
55'
65'
68'
R_L A B C D E F G H I J K L M N O P Q R_R
M
Total M
Live m
I
Dead m
(b)
I In Thousands Ins.⁴
Moments in Millions of Ft. Lbs.
(c)
M/I In Units of Pounds / 12 x Inches³
M/I
10 0 10 20 30 40 50 60
Scale in Feet

FIG. 90.

Suppose that the horizontal movement of the left end corners of the girder, with reference to a vertical through the end of the neutral axis at R_L, is required. The angle made by a tangent at the end of the deflected neutral axis with a horizontal plane may be found from Eq.(15).

$$\phi = \int_A^B \frac{M dx}{EI}, \quad \text{or} \quad \phi = \int_A^B \frac{M}{I} dx \div E.$$

$\int_A^B \frac{M}{I} dx$ is the area of the $\frac{M}{I}$ diagram from the center to the end of the girder; hence,

$$\phi = \frac{409.4726}{29{,}000{,}000} \times 144 = 0.002033.$$

TABLE 9J

Girder Section	Dead-Load Moment, Feet-Pounds	Live-Load Moment, Feet-Pounds	Total Moment, Feet-Pounds	I Inches^4	$\frac{M}{I}$ Pounds / 12 Inches^3
A	72,640	376,500	449,140	83,776	5.363
B	148,900	747,500	896,400	83,776	10.702
B	148,900	747,500	896,400	103,402	8.669
C	178,800	906,500	1,085,300	103,402	10.494
D	233,500	1,166,000	1,399,500	103,402	13.537
D	233,500	1,166,000	1,399,500	139,352	10.042
E	309,900	1,598,500	1,908,400	139,352	13.696
F	335,300	1,741,500	2,076,800	139,352	14.906
F	335,300	1,741,500	2,076,800	176,616	11.758
G	393,300	2,170,500	2,563,800	176,616	14.516
H	421,300	2,378,000	2,799,300	176,616	15.849
I	430,600	2,435,500	2,866,100	176,616	16.230
J	421,300	2,342,500	2,763,800	176,616	15.650
K	357,600	1,906,000	2,263,600	176,616	12.816
L	335,300	1,770,000	2,105,300	176,616	11.922
L	335,300	1,770,000	2,105,300	139,352	15.107
M	296,100	1,566,000	1,862,100	139,352	13.360
N	233,500	1,216,000	1,449,500	139,352	10.403
N	233,500	1,216,000	1,449,500	103,402	14.018
O	197,800	1,041,000	1,238,000	103,402	11.980
P	148,900	773,500	922,400	103,402	8.920
P	148,900	773,500	922,400	83,776	11.010
Q	95,400	506,000	601,400	83,776	7.180

TABLE 9K

Limit of Area	Dimensions for Area	Area, Pounds / $12^2 \times$ Inches^2	Distance of Centroid of Area from R_L or R_R, Feet	Area-Moment About R_L or R_R, Pounds / $12^3 \times$ Inches
A	$5.363 \times 3 \times \frac{1}{2}$	8.0445	2.000	16.0890
B	5.363×3.5	18.7705	4.750	89.1599
B	$5.339 \times 3.5 \times \frac{1}{2}$	9.3433	5.333	49.8309
C	8.669×1.5	13.0035	7.250	94.2754
C	$1.825 \times 1.5 \times \frac{1}{2}$	1.3688	7.500	10.2660
D	10.494×3	31.4820	9.500	299.0790
D	$3.043 \times 3 \times \frac{1}{2}$	4.5645	10.000	45.6450
E	10.042×5	50.2100	13.500	677.8350
E	$3.654 \times 5 \times \frac{1}{2}$	9.1350	14.333	130.9350
F	13.696×2	27.3920	17.000	465.6640
F	$1.207 \times 2 \times \frac{1}{2}$	1.2070	17.333	20.8208
G	11.758×6	70.5480	21.000	1481.5080
G	$2.758 \times 6 \times \frac{1}{2}$	8.2740	22.000	182.0280
H	14.516×5	72.5800	26.500	1923.3700
H	$1.333 \times 5 \times \frac{1}{2}$	3.3333	27.333	91.0000
I	15.849×5	79.2450	31.500	2496.2175
I	$0.381 \times 5 \times \frac{1}{2}$	0.9525	32.333	30.7975
Sums about R_L		409.4726		8104.5210
Q	$7.180 \times 4 \times \frac{1}{2}$	14.3600	2.667	38.2933
P	7.180×2.5	17.9500	5.250	95.7333
P	$3.830 \times 2.5 \times \frac{1}{2}$	4.7875	5.667	27.1292
O	8.920×2.5	22.3000	7.750	172.8250
O	$3.060 \times 2.5 \times \frac{1}{2}$	3.8250	8.167	31.2375
N	11.980×2	23.9600	10.000	239.6000
N	$2.038 \times 2 \times \frac{1}{2}$	2.0380	10.333	21.0593
M	10.403×4	41.6120	13.000	540.9560
M	$2.957 \times 4 \times \frac{1}{2}$	5.9140	13.667	80.8246
L	13.360×3	40.0800	16.500	661.3200
L	$1.747 \times 3 \times \frac{1}{2}$	2.6205	17.000	44.5485
K	11.922×2	23.8440	19.000	453.0360
K	$0.894 \times 2 \times \frac{1}{2}$	0.8940	19.333	17.2840
J	12.816×9	115.3440	24.500	2825.9280
J	$2.834 \times 9 \times \frac{1}{2}$	12.7530	26.000	331.5780
I	15.650×5	78.2500	31.500	2464.8750
I	$0.580 \times 5 \times \frac{1}{2}$	1.4500	32.333	46.8833
Sums about R_R.....		411.9820		8093.1110

The condition at the end is shown, greatly exaggerated, in Fig. 9P. The horizontal movement of the top corner is

$$\delta_t = -0.002 \times 42 \qquad = 0.084 \text{ in.}$$

At the bottom corner,

$$\delta_b = +0.002 \times 41.25 = 0.0825 \text{ in.}$$

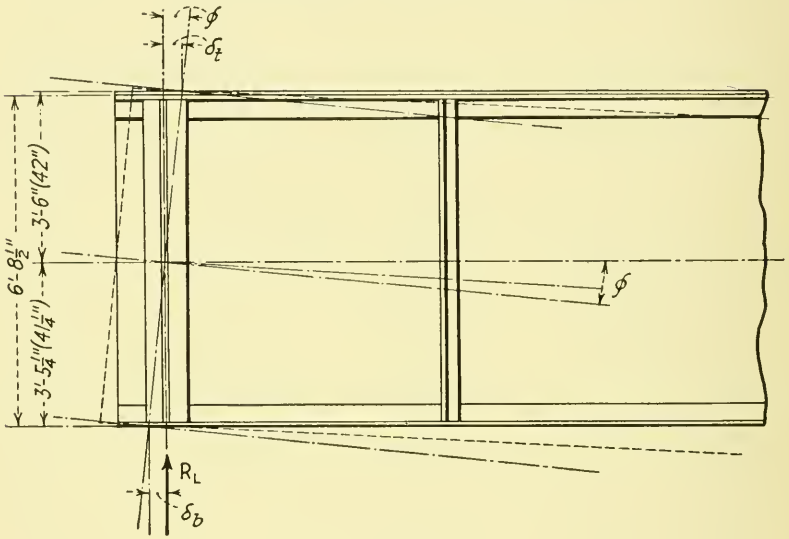

FIG. 9P.

SECTION II

TRUSSES

15. Deflection of a truss.—(1) *Deflection due to stress.*—The deflection of a frame, or truss, in any direction at any point, may be found from the equation

$$\Delta = \Sigma \frac{Sl}{EA} \times u. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

In this expression,

S = the stress in any member;
l = the length of the member;
A = its sectional area;
E = Young's modulus;
u = the stress in the member due to a unit load at the point where, and in the direction in which the deflection is required, and
Σ indicates the summation of the partial deflections in the various members that are subject to stress.

$\frac{S}{A}$ = the unit stress in any member, and may be represented by P, and Eq. (40) may be written,

$$\Delta = \Sigma \frac{Pul}{E}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

$\frac{Pl}{E}$ is the change of length, λ, of any stressed member, and λu is the deflection of the truss at the required point and in the direction of action of the unit load, u, due to the change in the length of the member; hence Eq. (41) may be written,

$$\Delta = \Sigma \lambda u. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

This is in convenient form for use, for the change of length, λ, may be found for each stressed member of the truss and the values of u for all members can be found by any suitable method. In many cases the most convenient method of finding the values of u is to construct a Clerk-Maxwell graphical diagram for the truss when loaded, at the point at which deflection is desired, by the unit load acting in the direction of the required deflection.

Care must be taken to use the values of P and u with their proper algebraic signs. It is convenient to use + for tension and − for compression. The value of δ for any member will be + when P and u have the same sign and − when they are opposite.

(2) *Deflection due to pin clearance.*—When the effective length of any member differs from its calculated length, owing to pin clearance or errors in manufacture, the resulting deflections may be found from Eq. (42) by substituting for λ the actual variations in length of the various members affected, noting that the variation from calculated length is + for elongation and − for shortening. In pin-connected trusses the pin measurements are commonly made between the surfaces of contact of the members and the pins, thus correcting for pin clearances in the shop. In such cases the clearances do not affect deflection.

(3) *Deflection due to temperature.*—When members undergo relative changes of length due to differences in temperature, the deflections are modified.

Let

c = coefficient of expansion;

t = change of temperature of any member, + for an increase and − for a decrease; then

$\lambda_t = ctl$, and the resulting deflection is,

$\delta_t = ctul$ for any member, and for all members affected

$$\Delta_t = \Sigma ctul. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

If t is the same for all members considered as subject to the temperature changes,

$$\Delta_t = ct\Sigma ul. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

In swing-span design the usual assumptions are that the upper chords and web members have a temperature 20° F. higher than the lower chords; or that the upper chords have a temperature 20° F., and the web members 10° F., higher than the lower chords. The first condition is more frequently used.

For steel, c is 0.0000067 per degree Fahrenheit.

(4) *Deflection due to uplift.*—In swing spans the ends of the arms are usually lifted an arbitrary amount to make sure that there shall be no lifting of an end from its support due to the action of a partial live load upon the continuous truss forming the span.

The deflection, Δ_F, due to an uplift F at the end of the truss, is

$$\Delta_F = \sum \frac{P_F ul}{E}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

Here P_F is the unit stress in any member due to the uplift F.

16. Deflections of one arm of a swing span.—The outline of one arm of a double-track, electric railway and highway swing span, with three elevated railway tracks on an upper deck, is shown in Fig. 9 Q. The trusses are 265 ft. 6 in. long from center to center of end bearings.

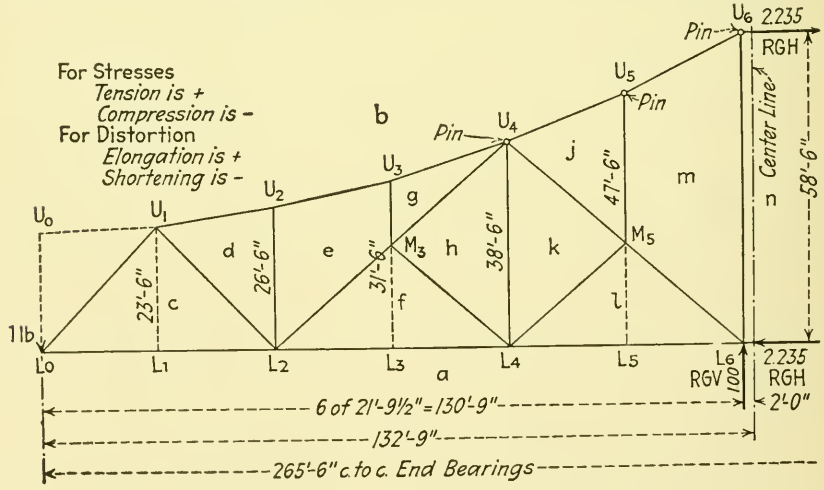

FIG. 9 Q.

(1) *Deflection due to dead load.*—The vertical end deflection, Δ_{L0}, under dead load only, will be found first. It is convenient to determine

the values of u, in Eq. (41), graphically. Place a 1-lb. load, acting vertically downward, at L_0. Consider one arm only and assume point L_6 to be fixed in vertical position. The horizontal reactions at L_6 and U_6, due to the 1-lb. load at L_0, are

$$R_H = 1 \times \frac{130.75}{58.5} = 2.235 \text{ lb.}$$

Beginning at L_0 with the 1-lb. vertical load acting downward, the values of u are shown in Fig. 9R. The values of R_H by scale are found to be $\pm$ 2.24 lbs., checking the diagram very closely.

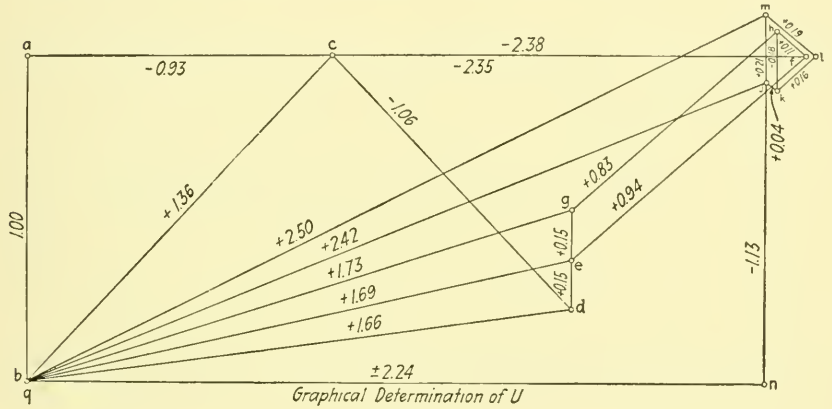

FIG. 9R.

In Table 9L the values of u are shown in column 2, having been scaled from Fig. 9R. Only one arm of the span is considered. The chord members, $U_6 - U_6$ and $L_6 - L_6$, in the center panel, are taken for only one-half of their length. When this panel is omitted and only a single center post is used in a truss, for deflection calculations, this post should be considered as divided by a vertical plane through its axis, and only one-half of the stress and area used in the calculations for one arm.

The values of ul, in column 5, add up to zero, as they should, since L_0, the joint at which the 1-lb. load is applied, is at the same original level as L_6, the joint at which the truss is supported.

The dead-load stresses, column 6, and the gross sectional areas of the members, column 7, are obtained from the stress sheet.

Since E, in Eq. (41), is the same for all members, the values of Pul, column 10, may be added first and their sum then divided by E.

TABLE 9L

1	2	3	4	5	6	7	8	9	10	11		12	13	14	15	16
Member	Stress from 1 Pound at Lo u	Length of Member in Inches l	Product ul	Σul	Dead-Load Stress Bridge Swinging D	Gross Area of Section A	Unit Stress $P=\frac{D}{A}$	$\lambda=\frac{Pl}{E}$	Product Pul	Pin-Play λ_p Fraction	Pin-Play λ_p Decimal	Product $u\lambda_p$	Stress from 76,500 Pounds Uplift W	Unit Stress from Uplift $P_1=\frac{W}{A}$	$\lambda_1=\frac{P_1 l}{E}$	Product P_1ul
$U1$-2	+1.660	264.0	+ 438		+207,000	37.7	+ 5,490	+0.050	+ 2,404,000	.....	.....	.....	−127,000	−3370	−0.031	−1,475,000
$U2$-3	+1.687	268.3	+ 453		+213,000	37.7	+ 5,650	+0.052	+ 2,559,000	.....	.....	.....	−129,000	−3420	−0.032	−1,550,000
$U3$-4	+1.727	274.6	+ 474		+219,000	37.7	+ 5,810	+0.055	+ 2,754,000	.....	.....	.....	−132,100	−3500	−0.033	−1,661,000
$U4$-5	+2.418	282.9	+ 684	+2836	+750,000	67.5	+11,110	+0.108	+ 7,600,000	$+\frac{3}{64}$	+.047	+.114	−185,000	−2740	−0.026	−1,875,000
$U5$-6	+2.504	292.9	+ 733		+774,000	67.5	+11,470	+0.116	+ 8,407,000	$+\frac{1}{32}$	+.031	+.078	−190,500	−2820	−0.028	−2,069,000
$U6$-6	+2.235	24	+ 54		+691,000	154.0	+ 4,490	+0.004	+ 242,000	$+\frac{1}{64}$	+.016	+.036	−171,000	−1110	−0.001	− 60,000
$L0$-2	−0.927	523	− 485		− 77,000	34.0	− 2,260	−0.041	+ 1,096,000	.....	.....	.....	+ 70,900	+2090	+0.038	−1,011,000
$L2$-4	−2.349	523	−1229	−3013	−445,000	46.0	− 9,670	−0.175	+11,873,000	.....	.....	.....	+179,700	+3910	+0.071	−4,798,000
$L4$-6	−2.380	523	−1245		−475,000	46.0	−10,330	−0.186	+12,862,000	.....	.....	.....	+182,100	+3960	+0.072	−4,929,000
$L6$-6	−2.235	24	− 54		−691,000	64.0	−10,800	−0.000	+ 583,000	.....	.....	.....	+171,000	+2670	+0.002	− 144,000
$L0$-$U1$	+1.364	384.6	+ 525		+113,000	45.2	+ 2,500	+0.033	+ 1,313,000	.....	.....	.....	−104,400	−2310	−0.031	−1,213,000
$U1$-$L2$	−1.055	384.6	− 406		−185,000	32.4	− 5,710	−0.075	+ 2,318,000	.....	.....	.....	+ 80,700	+2490	+0.033	−1,011,000
$L2$-$M3$	+0.940	348.9	+ 328		+319,000	46.0	+ 6,930	+0.084	+ 2,273,000	.....	.....	.....	− 71,900	−1560	−0.019	− 513,000
$M3$-$U4$	+0.826	348.9	+ 288		+376,000	57.3	+ 6,560	+0.079	+ 1,889,000	.....	.....	.....	− 63,200	−1100	−0.013	− 318,000
$M3$-$L4$	+0.114	348.9	+ 40		− 57,000	26.5	− 2,150	−0.026	− 86,000	.....	.....	.....	− 8,700	− 330	−0.004	− 13,000
$U4$-$M5$	+0.039	348.9	+ 14		−274,000	61.7	− 4,440	−0.053	− 62,000	.....	.....	.....	− 3,000	− 50	−0.001	− 1,000
$L4$-$M5$	+0.155	348.9	+ 54	+ 177	− 20,000	20.6	− 970	−0.012	− 52,000	.....	.....	.....	− 11,900	− 580	−0.007	− 31,000
$M5$-$L6$	+0.194	348.9	+ 68		−294,000	73.0	− 4,030	−0.048	− 274,000	.....	.....	.....	− 14,800	− 200	−0.002	− 14,000
$U2$-$L2$	+0.151	318	+ 48		− 24,000	19.4	− 1,240	−0.014	− 60,000	.....	.....	.....	− 11,600	− 600	−0.007	− 29,000
$U3$-$M3$	+0.151	147	+ 22		+ 5,000	19.4	+ 260	+0.001	+ 6,000	.....	.....	.....	− 11,600	− 600	−0.003	− 13,000
$U4$-$L4$	−0.178	462	− 82		+131,000	40.7	+ 3,220	+0.051	− 264,000	.....	.....	.....	+ 13,600	+ 330	+0.005	− 27,000
$U5$-$M5$	+0.205	339	+ 70		+ 55,000	19.4	+ 2,830	+0.033	+ 198,000	$+\frac{1}{64}$	+.016	+.003	− 15,700	− 810	−0.009	− 57,000
$U6$-$L6$	−1.128	702	− 792		−371,000	36.4	−10,190	−0.278	+ 8,071,000	$+\frac{1}{64}$	−.016	+.018	+ 86,300	+2370	+0.058	−1,878,000
				$\Sigma ul=0$					$\Sigma Pul=+65,650,000$							$\Sigma Pul=24,690,000$

From the table, $\Sigma Pul = +\ 65{,}650{,}000$; hence,

$$\Delta_D = \Sigma\frac{Pul}{E} = \frac{65{,}650{,}000}{29{,}000{,}000} = 2.26 \text{ in.}$$

(2) *Deflection due to pin clearance.*—The change in length due to pin clearance, column 11, is obtained as follows: The pin holes are $\frac{1}{32}$ in. larger than the pins, and each pin can therefore move $\frac{1}{64}$ in. from the center of the pin hole. Members $U_5 - M_5$ and $U_6 - L_6$ and the half member $U_6 - U_6$ have a pin at one end only; therefore their change in length is $\frac{1}{64}$ in. Member $U_5 - U_6$ has a pin at each end and the change in length is $\frac{1}{32}$ in. At U_4, all of the members are riveted together, except $U_4 - U_5$, which is connected to the gusset plates by the pin. $U_4 - U_5$, therefore, has $\frac{1}{64}$ in. pin play at each end and an additional $\frac{1}{64}$ in. due to the clearance of the pin in the gusset at U_4, making a total of $\frac{3}{64}$ in. These changes in length have the same sign as the dead-load stresses.

The effect of the pin clearances is found by summing the values of $u\lambda_P$ in column 12, and is

$$\Delta_P = 0.249 \text{ in.}$$

The sum $\Delta_D + \Delta_P = 2.26 + 0.249 = 2.509$ in.

(3) *Deflection due to temperature.*—The quantities Σul are given separately in column 5 for the upper and lower chords and the web members, so that the deflection due to a change in temperature of any of these groups may be found from Eq. (44).

Case I.—Assume that the temperature of the upper chords and web members is 20° higher than that of the lower chords.

$c = 0.0000067$, $t = 20$. From column 5, $\Sigma ul = 2836 + 177 = 3013$ and

$$\Delta_T = ct\Sigma ul = 0.0000067 \times 20 \times 3013 = 0.404 \text{ in.}$$

Case II.—Assume that the temperature of the upper chords is 20°, and that of the web members 10°, higher than that of the lower chords.

For the chords $\Delta_{TC} = 0.0000067 \times 20 \times 2836 = 0.380$ in.
For the webs $\Delta_{TW} = 0.0000067 \times 10 \times 177 = 0.012$ in.

Hence, $\Delta_T = 0.392$ in.

(4) *Deflection due to uplift.*—The stresses, W, due to an uplift of 76,500 lbs. at L_0 are found by multiplying the coefficients, u, by 76,500 for each member. As F acts upward while the unit load acts downward, W and u will have opposite signs. The products, $P_1 ul$, will then be

negative, indicating that the deflection, Δ_F, is upward. From the table, $\Sigma P_1 ul = 24{,}690{,}000$, hence

$$\Delta_F = -\frac{\Sigma P_1 ul}{E} = -\frac{24{,}690{,}000}{29{,}000{,}000} = -0.85 \text{ in.}$$

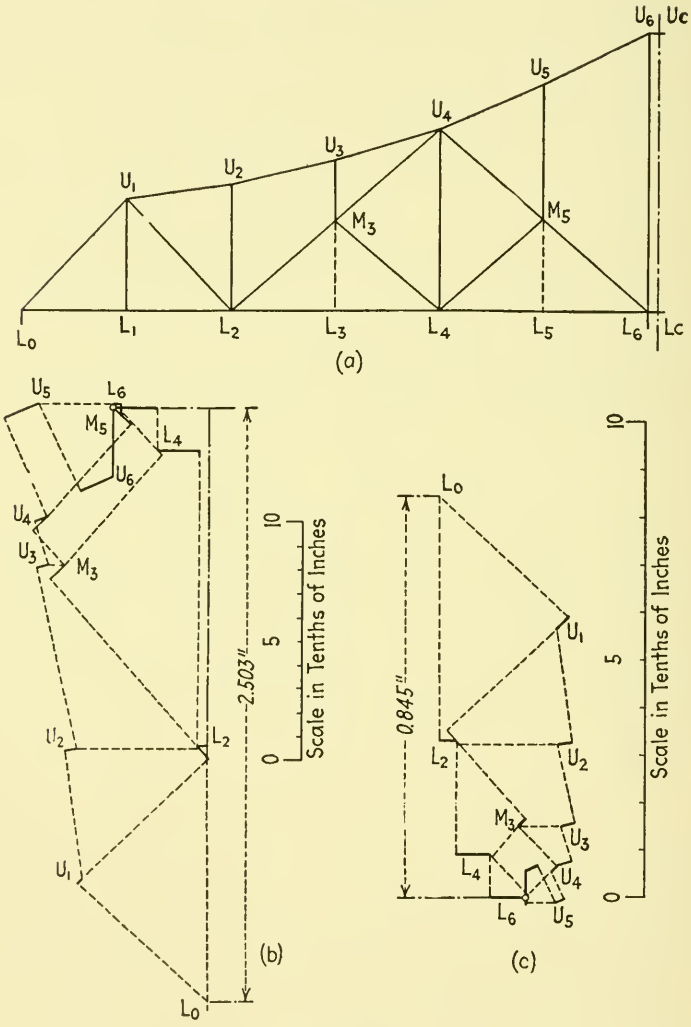

FIG. 9S.

(5) *Williot diagrams.*—When it is desired to find the movements of all joints, due to any condition of stress, this is done by means of the well-known Williot diagrams. In Table 9L, column 9, is given the

change in length, $\lambda = \frac{Pl}{E}$, for each member when subject to dead-load stress only.

Fig. 9S (*b*) gives the diagram for the dead load combined with the pin-clearance deflections. The analytical calculations gave $\Delta_D + \Delta_P = 2.509$ in., while Fig. (*b*) shows 2.503 in. from scaling.

Table 9L, Column 15, gives $\lambda_1 = \frac{P_1 l}{E}$ for each member, due to an uplift of 76,500 lb. The Williot diagram is shown in Fig. 9S(*c*). In this case $\Delta_F = 0.85$ in. by calculation and 0.845 in. by scaling.

The graphical method is thus seen to be very accurate when the work is carefully done.

17. Deflection of frames containing members subjected to bending moments.—The deflection of a member subjected to bending moments, from Eq. (1), is

$$\Delta_B = \int_0^l \frac{M m dx}{EI}.$$

The deflection due to direct stress in a member, from Eq. (40), is

$$\Delta_S = \frac{Sl}{EA} u.$$

The total deflection due to bending moments and direct stress in the various members of the frame then is

$$\Delta = \Delta_B + \Delta_S = \Sigma \int_0^l \frac{M m dx}{EI} + \Sigma \frac{Sl}{EA} u. \quad . \quad . \quad . \quad (46)$$

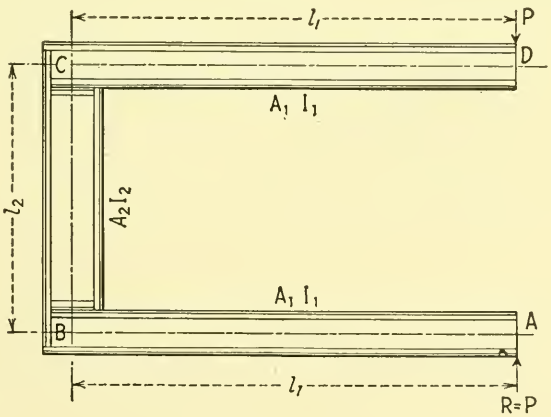

FIG. 9T.

Let Fig. 9T represent a frame, $ABCD$, loaded at D by P and supported at A by a reaction $R = P$. AB and CD have a length l_1, an

area A_1, and a moment of inertia I_1. BC has a length l_2, an area A_2, and a moment of inertia I_2. AB and CD act as beams, while BC is both a beam and a strut. Assume origins at A and C. For beams AB and CD, $M = Px$ and $m = x$, hence

$$\Delta_1 = 2\int_0^l \frac{Mm\,dx}{EI} = 2\frac{P}{EI_1}\int_0^{l_1} x^2 dx = 2\frac{Pl_1^3}{3EI_1}.$$

For BC as a beam $M = Pl_1$, and $m = l_1$; hence,

$$\Delta_2 = \int_0^l \frac{Mm\,dx}{EI} = \frac{Pl_1^2}{EI_2}\int_0^{l_2} dx = \frac{Pl_1^2 l_2}{EI_2}.$$

For BC as a strut $S = P$ and $u = 1$; hence,

$$\Delta_3 = \frac{Sl}{EA}u = \frac{Pl_2}{EA_2}.$$

The total deflection is

$$\Delta = \Delta_1 + \Delta_2 + \Delta_3 = 2\frac{Pl_1^3}{3EI_1} + \frac{Pl_1^2 l_2}{EI_2} + \frac{Pl_2}{EA_2}. \quad . \quad . \quad (47)$$

Example.—Assume AB and CD to be each 25 ft. long and to consist of 1 web 30 by $\frac{3}{8}$ in., 4 angles 6 by 4 by $\frac{3}{8}$ in., spaced $30\frac{1}{4}$ in. back to back, with 6-in. legs outstanding. $A_1 = 25.69$ sq. in. $I_1 = 3767$ in.^{4} Assume BC to be 15 ft. long and to consist of 1 web 40 by $\frac{3}{8}$ in., 4 angles 6 by 4 by $\frac{3}{8}$ in., spaced $40\frac{1}{4}$ in. back to back, with 6-in. legs outstanding. $A_2 = 29.44$ sq. in. $I_2 = 7332$ in.^{4} Let $P = R = 8000$ lbs.

Substituting these values in Eq. (47) and taking $E = 29{,}000{,}000$,

$$\Delta = \frac{2 \times 8000 \times 300^3}{3 \times 29{,}000{,}000 \times 3767} + \frac{8000 \times 300^2 \times 180}{29{,}000{,}000 \times 7332} + \frac{8000 \times 180}{29{,}000{,}000 \times 29.44}$$

or

$$\Delta = 2 \times 0.659 + 0.609 + 0.002 = 1.929 \text{ in.}$$

CHAPTER X

DETAILS OF DESIGN

1. Center-bearing swing spans.—The advantages of center-bearing swing bridges have been stated in Chapter III, Art. 2. Examples, with illustrations, were also given. These represent the simplest and best general arrangements and the most direct methods of delivering the weights of the bridge and its live load to the center pivot. It has been found best to use one or more simple, transverse plate girders, riveted to the main trusses at the center and resting upon the top of the center pivot, with provision for vertical adjustment by means of shims between the bottom of the girders and the top of the pivot. This construction has proved satisfactory in all cases and should be used whenever it is feasible. In order to make the bridge as satisfactory as possible in service, several vital details should be considered.

The bridge must be balanced on the center pier, when turning. Four trailing, or balance, wheels should be used on light spans, and six or more on long and heavy spans which present large areas to wind pressure. These balance wheels should be arranged as indicated in Fig. 10A.

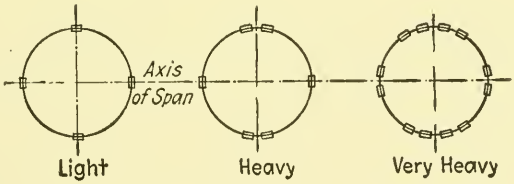

FIG. 10A.

The trusses should be cambered, so that when the ends are lifted the conditions assumed in the calculation of the stresses in the trusses will be closely approximated.

The center must be set level and securely anchored to the pier.

The circular rack on the center pier must be accurately set and thoroughly anchored against the tangential rack force. When this is done, radial struts joining the rack and the center casting are not needed.

The main pinion shaft bearings and supports must be well designed and rigidly attached to the superstructure.

When bevel gears are used in the operating mechanism, the shafts to which they are keyed should turn in union bearings, so that they are held in permanent alignment and the gears in proper mesh.

The end-lifting apparatus must be of ample capacity and of simple construction with adequate provisions for adjustment.

The end latches must be of sufficient strength, and adjustable for proper operating conditions. Their catches must be securely anchored to the rest piers.

The rail joints at the ends of the draw span should be simple and effective, holding the rails to proper line and level, when the bridge is closed. Their operating mechanism should be simple and positive in operation.

The operating, or control, mechanism of the bridge should be adequately housed, and the arrangement within the house should be convenient for operation and maintenance. The operator should have an unobstructed view of the ends of the bridge at the floor level and also of the navigable channel near the bridge.

Other important details will appear as the subject is developed.

2. End lifts and camber.—Article 7, of Chapter VII, directs attention to the fact that when the live load covers only one arm of a swing bridge it causes a negative reaction at the end of the unloaded arm. If the end of the bridge simply touched the pier pedestals, this negative reaction would cause the end of the span to rise from its supports. In order to prevent this, as soon as the span is swung into closed position, the ends are lifted by wedges, of other mechanism, so as to cause a positive reaction between the bridge and its pedestals. The effect of the negative reaction from the live load is then to decrease the positive reaction; and if the latter is equal to, or greater than, the former, the end of the span will not rise from its supports.

Positive end-lift reaction.—The main girders, or trusses, are designed for a positive end reaction, R, equal to 1.5 times the maximum live-load negative reaction, without impact. This positive reaction is shown on the stress sheet. The end-lift operating mechanism must be designed for this reaction and also for the resistances due to operating the bridge latch and the rail lifts, or locks. In some cases the end-lifting mechanism is driven by the same engine, or motor, that turns the span, and its capacity is greater than is required to lift the ends. The resistance is, however, a definite amount due to the stiffness of the structure, and, with properly designed end lifts, the motors cannot exert their full torque and must be operated with less than the normal steam, gas, or electric current.

Normal uplift.—The end-lifting machinery should be adjusted so that at normal temperatures a positive reaction, or uplift, F_1, will be produced, which should equal about 110 per cent of the maximum negative reaction from live loads for single-track bridges, and the negative reaction from live load for double-track bridges.

Camber.—When the bridge is swinging, the ends will deflect under the dead-load stresses and, if there are any pin-connected joints, also because of the pin clearances. The end-lifting apparatus will remove a part of this deflection and the bridge must be cambered to remove the remainder, so that when the ends are raised they will be in their theoretical positions. The camber, C, is given by the formula:

$$C = \Delta_D + \Delta_P - \Delta_{F1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which Δ_D = deflection from the dead-load stresses;
Δ_P = deflection from pin clearances;
Δ_{F1} = upward deflection from the uplift.

Figure 10B indicates the elements that must be considered when determining the required camber.

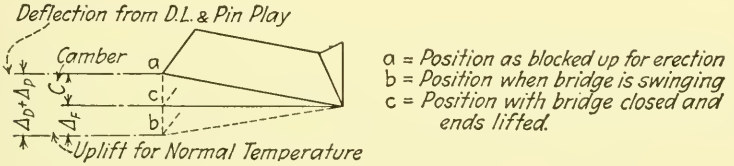

FIG. 10B.

Effect of unequal temperatures.—Since the upper chords and the web members are more exposed to the rays of the sun than the lower chords, they frequently have a higher temperature. This results in a downward deflection of the ends when the bridge is swinging, and in an increased positive reaction when the bridge is closed. In order to provide for this condition, the end lifts are designed to exert the increased uplift, F, equal to 1.5 times the maximum live-load reaction, and to raise the ends through a distance equal to their deflection under this uplift, F.

Swinging clearance.—When the ends of the bridge are lowered, the mechanism must be arranged to provide additional motion at the end of the stroke to allow the necessary clearance for swinging the span clear of its supports. This should be such that if the bridge tilts, so that one end is lowered and the other raised, the balance wheels at the center pier will come to a bearing while there is still clearance at the ends over the pier pedestals.

Let L = the length of one arm of the span;
B = the distance from the center of the span to the balance wheel nearest the end, measured in a direction parallel to the trusses;
K = the clearance between the balance wheels and the track when the bridge is closed and level.

When the ends are deflected downward, owing to the higher temperatures of the top chord and the web members, the clearance at the ends should never be less than $K\frac{L}{B} + \frac{1}{4}$ in. The clearance should be somewhat greater for long spans. For proper adjustment, the clearance, K, should never exceed $\frac{1}{16}$ in.

Method of cambering a truss.—The camber is usually provided by shortening the upper chord members in the panels at the center of the span. The necessary shortening is found from Eq. (42), Art. 15, of Chapter IX, by letting Δ equal the camber, C, considering only the center panel members, and solving for λ, or,

$$\lambda = \frac{C}{u}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Application.—Consider now the bridge for which the deflections were calculated in Art. 15, of Chapter IX. As it is a double-track structure, the end-lifting mechanism will be adjusted to produce, at normal temperatures, an uplift, F_1 equal to the maximum live-load negative reaction, or 51,000 lbs. The deflection corresponding to the stress sheet reaction of 76,500 lbs. was found to be 0.85 in. The deflection due to an uplift of 51,000 lbs. then is

$$0.85 \times 51{,}000 \div 76{,}500 = 0.57 \text{ in.}$$

The camber is found from Eq. (1),

$$C = 2.26 \text{ in.} + 0.249 \text{ in.} - 0.57 \text{ in.} = 1.939 \text{ in.}$$

One-half of this will be provided by shortening $U4 - U5$, and one-half by shortening $U5 - U6$. From Eq. (2) the shortening of $U4 - U5$ will be

$$\tfrac{1}{2} \times 1.939 \div 2.418 = 0.401 \text{ in., or } \tfrac{13}{32} \text{ in.}$$

For $U5 - U6$ it will be

$$\tfrac{1}{2} \times 1.939 \div 2.504 = 0.388 \text{ in., or } \tfrac{3}{8} \text{ in.}$$

The significance of these results is indicated in Fig. 10C.

Camber calculations should be made for every swing span, and definite instructions should be given to the erector as to the method of providing for the camber. In ordinary falsework erection this is readily arranged by setting the blocking under the bottom chord to the exact height required at each panel point.

Care must be taken not to swing the bridge until it has been fully connected and riveted. Whenever, for any reason, it becomes necessary to swing the bridge before the joints are riveted, great care must be taken or the camber will be lost in the operation. In such a case the joints should be thoroughly connected by traffic pins driven in the rivet holes. Their middle diameter should be the same as that of the holes. Enough ordinary bolts should also be used to hold the parts tightly together.

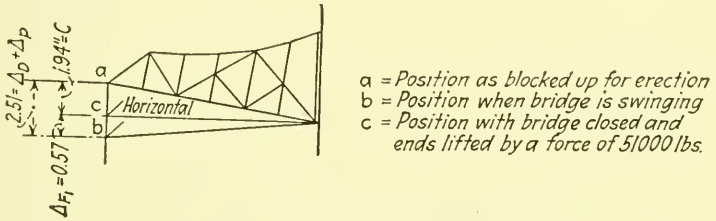

FIG. 10C.

When the conditions are such that it is known, in advance of erection, that the span must be swung before it is finished, it would be wise to set the camber blocking higher than its theoretical position. If some of the camber is then lost, there may still be enough for adjusting the end lift to good working condition. For such cases it is suggested that the ends of the trusses should be adjusted above their theoretical unstressed positions as follows:

Spans up to 150 ft. between end bearings, $\frac{1}{4}$ in.
Spans from 200 to 300 ft. between end bearings, $\frac{1}{2}$ in.
Spans from 300 to 400 ft. between end bearings, $\frac{3}{4}$ in.
Spans from 400 to 500 ft. between end bearings, 1 in.

3. Theory of wedge end lifts.—Experience with several types of end lifts has shown that wedges are the simplest and most satisfactory. When the bridge is closed the loads are carried to the pier pedestals by the simplest of flat, solid bearings. When the wedge-operating mechanism is properly designed the wedges are locked in closed position by the toggle-joint action of the linkage, and the mechanical parts are practically without stress.

The general arrangement of wedge end lifts with respect to the piers and superstructure is illustrated in Chapter III. Reference should be made to Figs. 3B, 3D, 3F, 3K, and 3L.

The design of end wedges and their operating mechanism is a good illustration of the general type of the problems involved in the detailed design of draw bridges. The theory of one type of such mechanism will be given, and in the next article an actual example will be worked out.

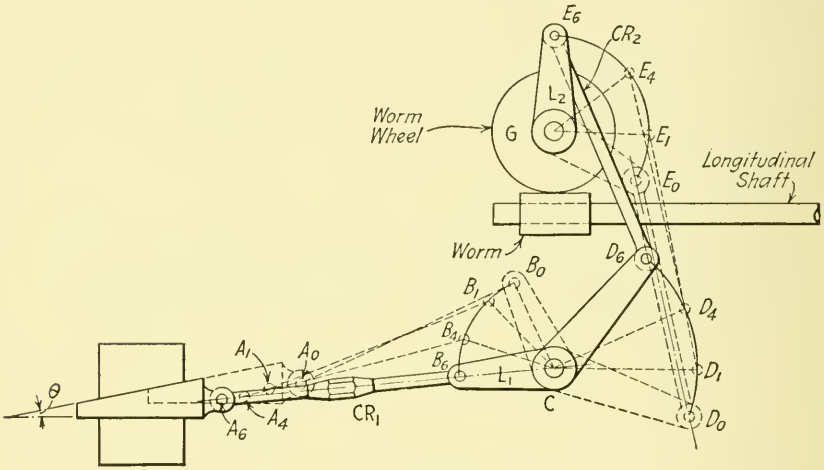

FIG. 10D.

Figure 10D shows the usual arrangement of an end wedge and toggle. The bell-crank lever, L_1, mounted on shaft C, is driven by the connecting rod, CR_2, articulated to lever L_2, which is mounted on the worm-wheel shaft, G. Lever L_1 communicates its motion to the wedge through the connecting rod, CR_1. When the bridge is swinging, the mechanism occupies the position $A_0B_0CD_0E_0G$, as shown by the dotted outlines. The position when the wedges are fully driven is $A_6B_6CD_6E_6G$. The points A_6, B_6, and C should be in a straight line, first, because in that position the toggle is most efficient, and second, since any further motion of L_1 will withdraw the wedge, thus preventing an over-drive.

When driving the wedge there is no resistance, except the friction of the machinery, until it reaches the point A_1, where it begins to bear on the lower wedge seat. The stress in CR_1 then increases continually, from zero at A_1 to a maximum when the wedge reaches A_6.

Refer to Fig 10E and

Let F = reaction required at the end of the uplift;
Δ_F = uplift in inches;

L = effective wedge stroke;
θ = wedge angle;
f = coefficient of friction, here taken as 0.15 on each of the two faces of the wedge;
α = angle of friction for the wedge = $\tan^{-1} f$;
R_C = direction of the resultant load on the bearing, C;
W = pitch line of the worm.

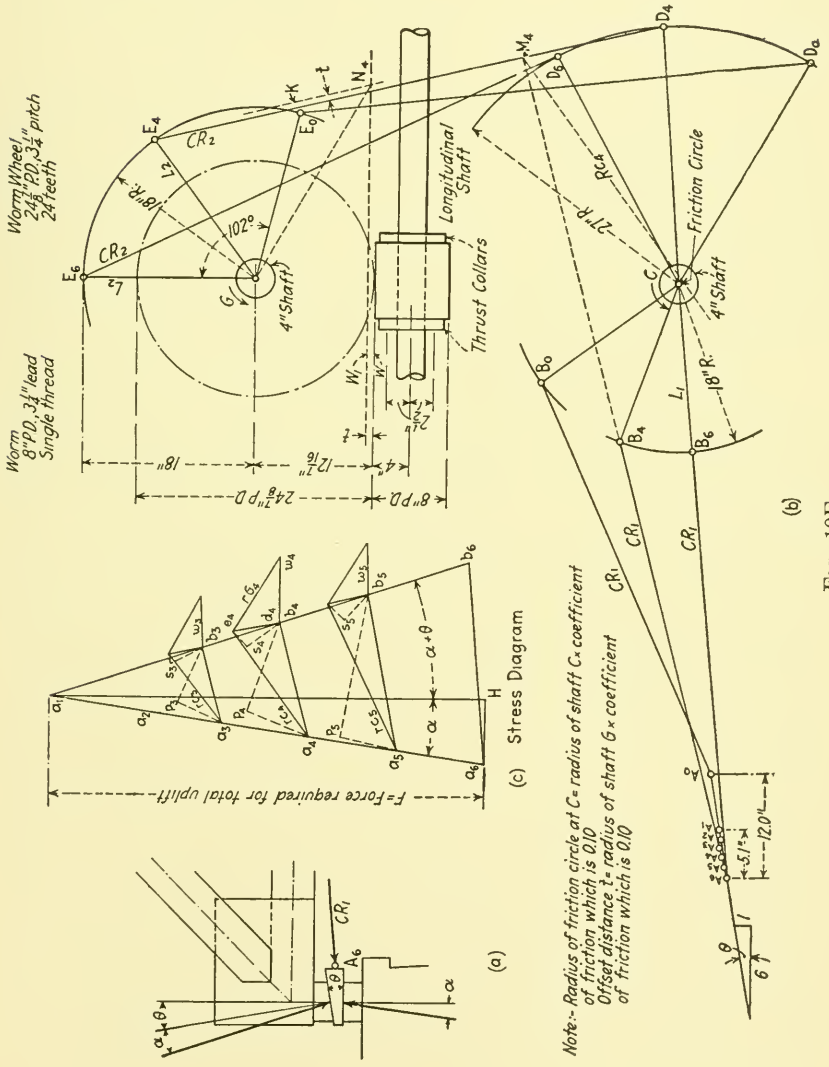

FIG. 10E.

Forces acting on the wedge.—Three forces act on the wedge while it is being driven. These are shown in Fig. (*a*) and are: first, the force in the connecting rod, CR_1; second, the reaction of the lower pedestal, acting at an angle α with the normal to the bottom of the wedge; and third, the pressure of the upper wedge seat, acting at an angle α with the normal to the upper face of the wedge. Note that the last two forces are inclined away from the normals to the wedge faces in a direction to offer further resistance to the driving of the wedge. The vertical component of the force acting on the bottom of the fully driven wedge is equal to F, the reaction required at the end of the uplift.

Stress diagram.—In Fig. (*c*) lay off on a vertical line the distance $a_1 H$, equal to F. Through the point a_1 draw a line parallel to the force acting on the bottom face of the wedge. This line intersects a horizontal line through H at the point a_6. Through a_1 draw a line parallel to the force acting on the upper face of the wedge, and through a_6 draw another parallel to CR_1 of Fig. (*a*). These two lines intersect at b_6. Then $a_1 H$ is the required reaction F, $a_1 a_6$ is the force acting on the bottom of the wedge, $a_1 b_6$ is the force acting on the top of the wedge, and $a_6 b_6$ is the force required in the connecting rod, CR_1.

In Fig. (*b*) the effective stroke of the wedge is divided into five equal parts, and the points A_1, A_2, A_3, A_4, A_5, and A_6 indicate the successive positions of the wedge pin as the wedge is being driven. Points B, D, and E—followed by numerical subscripts—indicate corresponding positions of the other moving points of the mechanism.

In Fig. (*c*) divide $a_1 a_6$ into five equal parts, thus locating points a_2, a_3, a_4, and a_5. In position 4 the force acting on the bottom of the wedge is $a_1 a_4$. The line $a_4 b_4$, drawn parallel to $A_4 B_4$, is the stress in the connecting rod. Produce $A_4 B_4$ to intersect $D_4 E_4$ at M_4. Assume the diameter of shaft C, and draw a friction circle at C with its radius equal to the radius of the shaft multiplied by the coefficient of friction, assumed as 0.10. Through M_4 draw Rc_4 tangent to the friction circle at C. Through a_4 in Fig. (*c*) draw rc_4, parallel to Rc_4, in Fig. (*b*), and through b_4 draw $d_4 e_4$ parallel to $D_4 E_4$. Then $d_4 e_4$ is the stress in CR_2.

Assume the diameter of the worm-wheel shaft, G, and calculate the offset distance t, which is equal to the radius of G multiplied by the coefficient of friction, here assumed as 0.10. Draw the line K parallel to $D_4 E_4$ at a distance t further away from the worm-wheel shaft, and the line W_1 parallel to W, and at a distance nearer to the worm-wheel shaft. The line K intersects W_1 at the point N_4. Draw the line $N_4 G$. In Fig. (*c*) draw rG_4 parallel to GN_4, and w_4 parallel to W_1. The line w_4 gives the tooth load on the worm wheel for driving one

wedge. For two wedges, one on each side of the bridge, the tooth load will be twice as much.

In a similar manner, worm-wheel tooth loads for other positions of the wedge may be found.

4. Example.—The stress diagram in Fig. 10E (*c*) was drawn for the same bridge that served as the example of the method of calculating the deflection of a truss, in Chapter IX, Art. 15, and of camber in Art. 2 of this chapter. The required uplift reaction was found to be 76,500 lbs. and the corresponding vertical uplift was 0.85 in. An outline of the method of designing the various parts of the end-lifting apparatus will now be given.

Wedge stroke.—The wedge angle, θ, is assumed to correspond to a slope of 1 in 6. In order to lift the end of the bridge 0.85 in. the wedge must move $0.85 \times 6 = 5.10$ in., which is the effective stroke.

The distance from the center of the span to the balance wheel nearest the end is 18 ft. 0 in. The length of one arm of the bridge is 132 ft. 9 in., and the clearance between the balance wheels and the track is taken as $\frac{1}{16}$ in. The minimum swinging clearance at the wedges will then be

$$\frac{\frac{1}{16} \times 132.75}{18} + \frac{1}{4} = 0.71 \text{ in.}$$

The temperature deflection is 0.40 in. and Δ_F is 0.85 in. The total vertical motion must then be $0.71 + 0.40 + 0.85 = 1.96$ in., or, say, 2.00 in. The full stroke of the wedge then is $2.00 \times 6 = 12$ in. Laying off this distance on the diagram in Fig. 10E (*b*) will locate the point A_0, from which the position of the mechanism $A_0B_0CD_0E_0G$ at the beginning of the stroke can be determined.

Wedge and wedge pedestals.—The reaction on the wedge is 76,500 lbs. If an allowable bearing pressure of 1500 lbs. per square inch be assumed, the bearing area required between the wedge and its upper and lower pedestals is $76{,}500 \div 1500 = 51$ sq. in. When the bridge is closed and its ends are lifted, the end reaction from live load may be as much as 440,000 lbs., and an impact of, say, 110,000 lbs., a total of 550,000 lbs. This, added to the uplift of 76,500 lbs., produces a total vertical reaction of 626,500 lbs. If the contact unit stress is limited to 16,000 lbs. per square inch, which is the maximum permitted in compression of the wedges and pedestals, the contact area required between the wedge and its seats becomes $626{,}500 \div 16{,}000 = 39.15$ sq. in. The contact area required during motion is the larger, and the wedge and its pedestals must be designed to provide at least 51 sq. in.

Connecting rod CR_1.—The maximum compressive stress in the con-

necting rod CR_1 is represented by the line a_6b_6 in Fig. (*c*), and is 35,600 lbs. The rod and its end connections should be designed for this stress.

Bell crank L_1.—The stress in the connecting rod CR_1, acting upon the crank L_1 at B_4, can be resolved into two components, one normal and one tangent to the arc $B_0B_4B_6$. The normal component causes a direct compression in the arm B_4C. The tangential component produces a bending moment in B_4C.

In Fig. (*c*) a_4b_4 is the stress in the connecting rod CR_1. The dotted line a_4p_4, drawn normal to the direction of B_4C, of Fig. (*b*), is the tangential component of a_4b_4. The dotted line p_4b_4, drawn parallel to B_4C, is the normal component of a_4b_4. In a similar manner the normal and tangential components are found for the other positions of the mechanism. Position 4 gives the largest tangential component, which is 11,000 lbs. The greatest bending moment on the arm B_4C is $11{,}000 \times 18 = 198{,}000$ in.-lbs. at C. The arm B_4C should be designed for a bending moment at C of 198,000 in.-lbs., combined with a compression of 17,800 lbs., the normal component of a_4b_4.

In a similar manner the arm CD_4 is found to be subject to a bending moment slightly larger than B_4C, but the direct stress is small and the section determined for B_4C will also serve for CD_4 in this case. The bending moment and the direct stress should always be found in CD_4, for in some cases the combination might require CD_4 to be stronger than B_4C.

Shaft C.—The maximum resultant force acting on shaft C occurs when the wedge is fully driven and is equal to the stress in the connecting rod CR_1. In Fig. (*c*) this resultant is given by the line a_6b_6 and is 35,600 lbs. This force is the basis of the design of the shaft and its bearings. The shaft forms a pivotal support for the bell crank L_1 and is subjected to bending and frictional stresses only.

Connecting rod CR_2.—The maximum tensile stress in the rod CR_2 is given by the line d_4e_4, in Fig. (*c*), and is 7550 lbs. The rod and its connections must be designed for this stress, but must also be proportioned to withstand a direct compression of a small amount which occurs when the wedge is being withdrawn.

Lever L_2.—The stress in the connecting rod CR_2, acting upon the lever L_2 at E_4, can be resolved into two components, one tangent and one normal to the arc $E_0E_4E_6$. The tangential component causes a bending moment and the normal a compression in L_2. In Fig. (*c*) e_4d_4 is the stress in the rod CR_2. The dotted line s_4d_4, drawn normal to E_4G, is the tangential component of e_4d_4. The dotted line e_4s_4, drawn parallel to E_4G, is the normal component of e_4d_4. In a similar manner the tangential and normal components may be found for the other

positions of the mechanism. Position 4 gives the greatest tangential component, which is 6900 lbs. The maximum bending moment on the lever is 6900 × 18 = 124,000 in.-lbs. Lever L_2 should be designed for a bending moment at G of 124,000 in.-lbs., combined with a compressive stress of 3000 lbs., which is the normal component of e_4d_4.

Worm wheel.—The stress diagrams drawn for positions 3, 4, and 5 give tooth loads on the worm wheel of w_3 = 8600, w_4 = 10,300, and w_5 = 9200 lbs., respectively. Position 4 is the maximum and is 10,300 lbs. for one wedge, or 20,600 for two. The worm wheel may be designed as a gear with 15° involute teeth, the tooth load being 20,600 lbs., assumed to be taken by a single tooth.

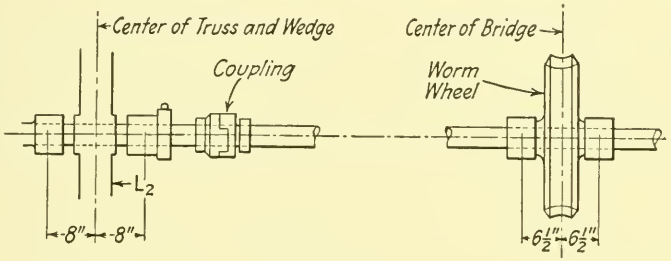

FIG. 10F.

A sketch of the general arrangement of the transverse worm-wheel shaft is shown in Fig. 10F. This shaft G usually extends transversely across the bridge and carries a lever, L_2, at each end. A worm wheel is keyed to it midway between the two levers, so that one worm wheel actuates both end wedges. The shaft is subjected to both torsional and bending moments. The torsional moment is one-half of the worm-wheel tooth load times the radius of the pitch circle, in this case $T = 10{,}300 \times 12\frac{7}{16} = 128{,}100$ in.-lbs. The bearings on each side of the worm wheel carry a load of 10,300 lbs. and these loads cause bending moments in the shaft in a horizontal plane. In this example it is $M = 10{,}300 \times 6\frac{1}{2} = 66{,}950$ in.-lbs. The theory of the design of shafts will be developed in Volume II. Graphical charts will be given which will show the diameters of steel shafts required for combinations of torsional and bending moments. If the proper chart is entered for a torsional moment of 128,100 in.-lbs., and a bending moment of 66,950 in.-lbs., it will be found that the diameter of the shaft must be $4\frac{1}{2}$ in. The worm wheel is keyed to the shaft at the point of maximum stress, hence must be increased in size to allow for the weakening due to the key seat. The final diameter will be $4\frac{1}{2} \times 1.06 = 4\frac{3}{4}$ in.

Similarly, at lever L_2, $T = 10{,}300 \times 12\frac{7}{16} = 128{,}000$ in.-lbs.

The resultant load on the bearings, adjacent to L_2, acts parallel to CR_2, and is 7550 lbs. This lever is midway between the two adjacent bearings; hence, $M = \frac{1}{2} \times 7550 \times 8 = 30{,}200$ in.-lbs. For a combination of these moments a $4\frac{3}{8}$-in. shaft is indicated. Here again, a key seat must be cut to secure lever L_2, and the diameter of the shaft becomes $4\frac{3}{8} \times 1.06$, or $4\frac{5}{8}$ in. The entire shaft should then be made $4\frac{3}{4}$ in. in diameter.

Note that the jaw coupling near L_2 prevents the shaft from acting as a continuous girder for bending moments.

Worm and worm shaft.—The circumference of the worm at the pitch diameter is $8 \times \pi = 25.13$ in. and the lead is $3\frac{1}{4}$ in. A tangential force at the pitch circle of the worm will then move 25.13 in. while advancing the tooth load of the worm wheel through $3\frac{1}{4}$ in. Equating the work done by these forces, we find that the tangential force is $20{,}600 \times 3.25 \div 25.12 = 2660$ lbs., and the torsional moment in the shaft would be $2660 \times 4 = 10{,}640$ in.-lbs., if the worm were 100 per cent efficient. The efficiency of the worm is given by the formula

$$E = \frac{L}{L + 2f\pi R'}$$

in which L is the lead, R the pitch radius of the worm, and f is the coefficient of friction, taken here as 0.10 for cut worms and wheels; then

$$E = \frac{3.25}{3.25 + 2 \times 0.10 \times \pi \times 4} = 0.564.$$

The torsional moment on the worm shaft is $10{,}640 \div 0.564 = 18{,}860$ in.-lbs. To this must be added the collar friction. The thrust collar, against which the worm bears, has an average radius of $2\frac{1}{2}$ in. The friction is $20{,}600 \times 0.10 = 2060$ lbs., and its moment is $2060 \times 2\frac{1}{2} = 5150$ in.-lbs. The total torsional moment on the worm shaft from driving the wedges then is $18{,}860 + 5150 = 24{,}010$ in.-lbs.

A bending moment is also caused by the tendency of the worm to slide away from the worm-wheel teeth, due to the obliquity of the teeth, which may be taken as 15°. This force is $20{,}600 \times \tan 15°$ or $20{,}600 \times 0.268 = 5520$ lbs., less the friction of the contact surfaces, which may be $20{,}600 \times 0.10 = 2060$ lbs., producing a net force of 3460 lbs.

The reaction on each bearing is $\frac{1}{2}$ of $3460 = 1730$ lbs. The worm and shaft are usually forged as a single piece; hence the critical section is at the end of the worm. Assume that the worm is 8 in. long with a bronze thrust washer $\frac{3}{4}$ in. thick at each end, and that the shaft bearings are each $6\frac{1}{2}$ in. long. The lever arm from the center of the bearing to

the end of the worm, then is $\frac{6\frac{1}{2}}{2} + \frac{3}{4} = 4$ in. The bending moment due to the obliquity of the teeth is

$$M_1 = 1730 \times 4 = 6920 \text{ in.-lbs.}$$

The worm-wheel tooth load of 20,600 lbs. acts against the thread of the worm at its pitch circle and parallel to its axis. This causes reactions on the shaft bearings of

$$\frac{20{,}600 \times 4}{8 + 2 \times \frac{3}{4} + 6\frac{1}{2}} = \frac{20{,}600 \times 4}{16} = 5150 \text{ lbs.}$$

The bending moment at the end of the worm is

$$M_2 = 5150 \times 4 = 20{,}600 \text{ in.-lbs.}$$

The total bending moment at the end of the worm then is

$$M = M_1 + M_2 = 6920 + 20{,}600 = 27{,}520 \text{ in.-lbs.}$$

For $T = 24{,}010$ in.-lbs. and $M = 1.5 \times 27{,}520 = 41{,}280$ in.-lbs., a shaft $3\frac{1}{8}$ in. in diameter is required.

When the worm shaft is driven directly by a longitudinal shaft from the center of the span, the longitudinal shaft connections should be made by jaw couplings, to reduce the bending stresses which would otherwise be produced in it by the general deflection of the span.

Rail lifts and latches.—The weights and resistances of the rail lifts and latches were omitted in the example. In an actual design they must be included, and in some arrangements they materially increase the loads on the end-lifting mechanism.

Power required to drive the end wedges.—Suppose it is desired to drive the wedges in 10 seconds. The worm wheel then turns through the angle E_0GE_6, equal to 102°, in 10 seconds. Since the worm wheel has 24 teeth and the worm has a single thread, the worm will turn through $102° \times 24 = 2448°$ in 10 seconds, and in one minute it will turn through $2448 \times 60 \div 10 = 14{,}688°$, or $14{,}688 \div 360 = 40.8$ revolutions. The maximum torsional moment was found to be 24,010 in.-lbs., or 2001 ft.-lbs. Assume an efficiency of the mechanism, from the worm back to the motor, of 80 per cent. The maximum horsepower required to drive the two wedges at one end of the span is

$$\frac{2\pi \times 2001 \times 40.8}{33{,}000 \times 0.80} = 19.44 \text{ h.p.}$$

When one motor drives the wedges at both ends of the bridge the maximum horse-power must be $19.44 \times 2 = 38.88$. When the motor also drives the center wedges, additional power must be provided for this purpose, depending upon the arrangement of, and the resistance in, the center-wedging mechanism. The driving mechanism of these wedges should be adjusted so that they just come to a bearing at the end of the stroke, but do not exert an uplift, hence the resistances are only those due to friction.

5. End and center wedges.—A typical wedge end lift is shown in Fig. 10G (*a*). This detail was used at the ends of a single-track, through, riveted, center-bearing swing span, 247 ft. 1 in. between centers of end bearings, designed for about E60 loading.

The lower pedestal seats on a $\frac{3}{4}$-in. steel plate on the pier. The slope of the top of the wedge is 1 in 6. The wedge *C*11 is operated by the link *F*6, articulated to the wedge and to the bell crank *C*15, which rotates in bearings *B*4. *C*15 is actuated by link *F*2, which in turn is driven by the crank *C*14. This crank is forged integrally with shaft *S*6. The proportions are such that when shaft *S*6 is over-driven, the wedge is withdrawn; in fact, the mechanism can be driven continuously and only raise and lower the end of the truss without damage to any part. The stroke of the wedge is 1 ft. 2 in. A simple crank, *C*17, actuates the end rail-lift shaft by means of the link *F*4. This shaft has cranks attached to the rail-lifting links, *F*7. A bell crank actuating the end latch is also keyed to the same shaft. The bearings of shaft *S*6 and the lower bell-crank shaft are attached to structural steel members, riveted to the sides of the end bottom chords of the truss.

Shims are provided between the upper wedge seat and the bottom of the lower chord of the truss, thus permitting adjustment in the distance from the pier to the lower chord. Links *F*6 and *F*4 are also adjustable. Jaw couplings are used in shaft *S*6, to prevent binding, due to errors in alignment. When the wedges are fully driven, the wedge pin, link pin, and short bell-crank shaft are in a straight line, thus locking the wedge by toggle-joint action.

Figure 10G (*b*) shows one of the center wedges. The arrangement, operation, and adjustment are similar to those of the end wedges; but when the wedge is fully driven, it comes to a bearing without exerting an uplift. When in this position it also is locked by toggle joint action.

These arrangements of wedge end lift and center wedges are more commonly used than any other type. In many of the older designs, cranks offset in the transverse shaft were not used and an over-drive was possible, with consequent damage to the mechanism. In this old

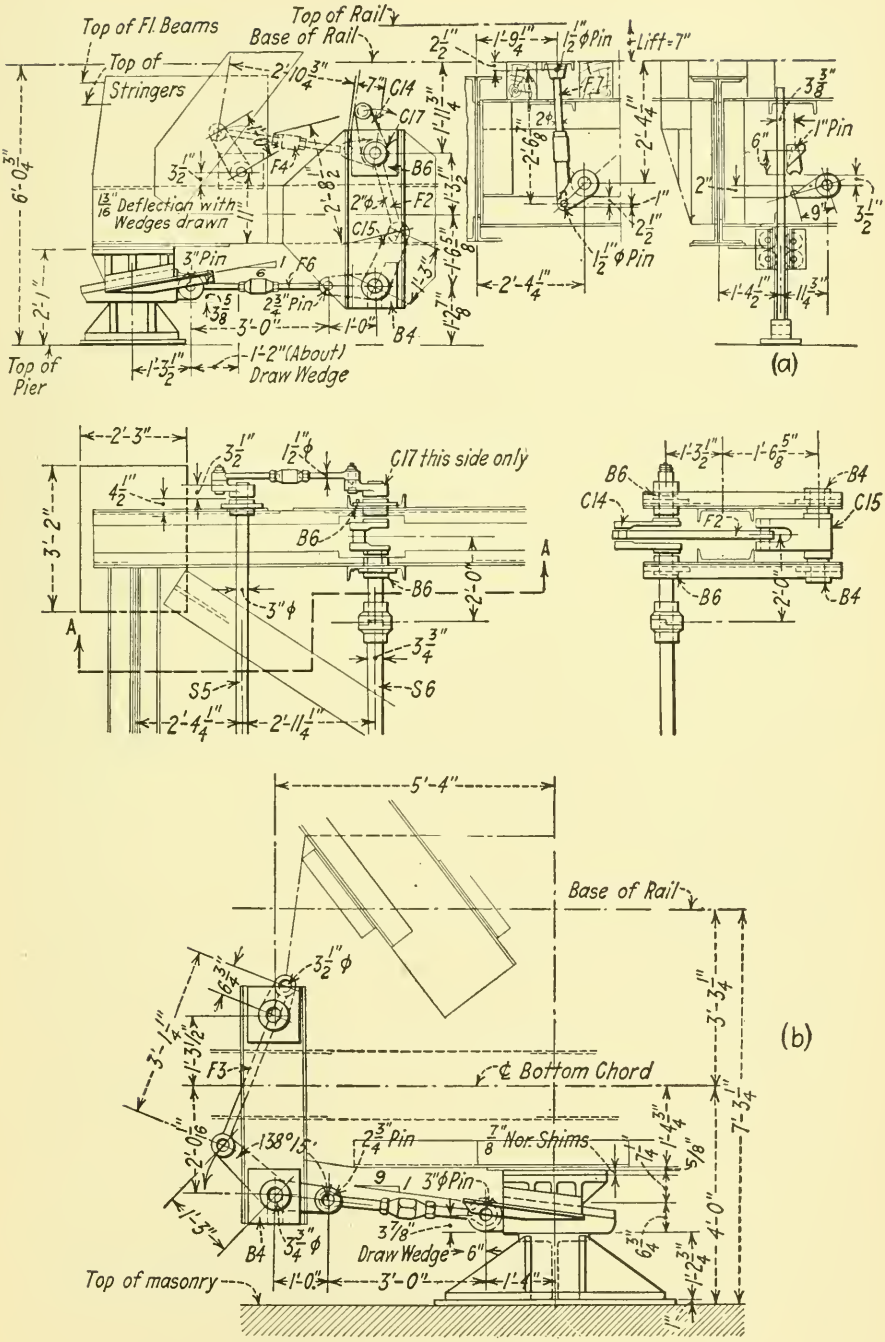
Top of Fl Beams
Top of Stringers
Top of Rail
Base of Rail
Lift=7"
Deflection with Wedges drawn
3" Pin
Top of Pier
1'-2" (About) Draw Wedge
(a)
C17 this side only
B6
B4
C15
C14
F2
S5
S6
5'-4"
Base of Rail
℄ Bottom Chord
F3
138°15'
2¾" Pin
7/8" Nor. Shims
3" ϕ Pin
Draw Wedge
Top of masonry
(b)

FIG. 10G.

construction, link $F2$ was the part that was injured by an over-drive.

6. Link-and-roller end lift.—Figure 10H shows a form of end lift that was extensively used up to about 1900. The type was developed by the late Willard S. Pope, who was the proprietor of the Detroit Bridge and Iron Works. It was improved by the late George S. Morison and used on most of the swing spans which he designed. It was also used by others. The particular design shown was used on the bridge across the Missouri River at Leavenworth, Kansas, which is a single-tract structure 440 ft. between centers of end bearings.

A short link is articulated to a pedestal attached to the lower chord of the truss. A roller turns on a pin in the bottom of the link, and an operating strut articulates to the same pin. The other end of the strut engages trunnions on a moving nut, operated by a horizontal screw driven by gearing. The end lift is symmetrical about the center line of the bridge, the screws being cut with right and left threads. The two pin pedestals are connected by a structural member, and their tops are curved to fit the rollers at the end of the lifting stroke. Thus, the span is accurately centered at both ends when the ends are lifted, and end latches are not required.

The end rail lifts are actuated by the thrust links by means of bell cranks and adjustable vertical struts. In the design shown, the rails move slowly as the mechanism is operated. In a later form, the struts were cut and, when the rails were fully raised, a pawl engaged the upper part of the strut, holding the rails in a fixed position. When the ends of the span were lifted, the lower part of the strut moved downward until, when near the end of the stroke, a cam disengaged the pawl, allowing the rails to fall into place by gravity. The purpose of this was to make sure that the rails were fully seated and not held partly raised by the friction of the rail seats. This device operated well and was used on several bridges, the first being that across the Mississippi River at Alton, Illinois. This is a double-track span of 450 ft. and was built in 1893.

This type of end lift is simple and certain in action and easily maintained. It requires less power than a wedge lift. Over-driving is not probable, as the rollers are practically blocked at the end of the stroke. In large bridges, like those at Alton and Rock Island, two links and rollers, coupled together, are used at each corner of the span. The principal objection to this type is that the dead- and live-load reactions are carried on rollers and pins that form a part of the operating mechanism, and not on large flat surfaces, as when wedges are used.

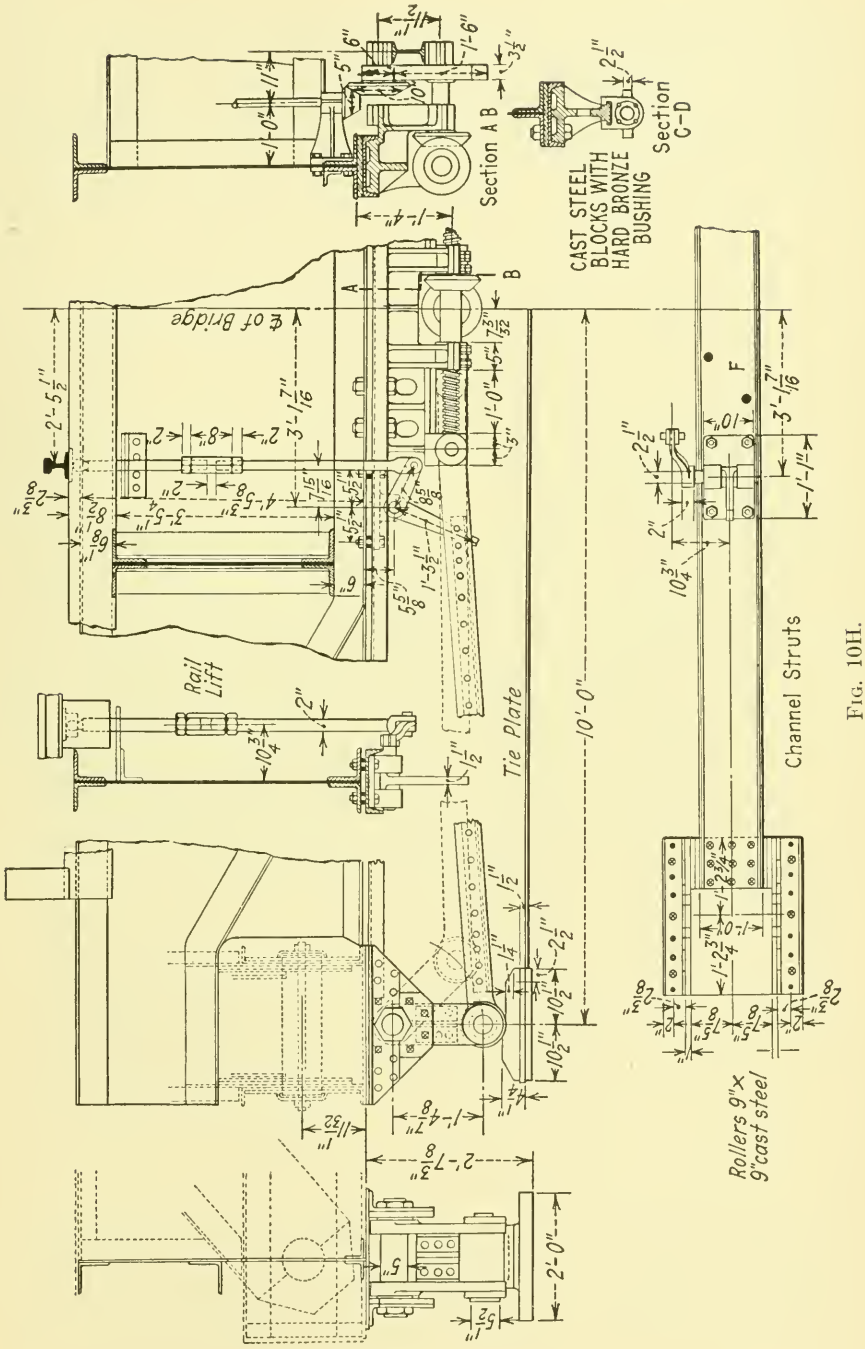
Section A B
Section C-D
CAST STEEL BLOCKS WITH HARD BRONZE BUSHING
℄ of Bridge
Rail Lift
Tie Plate
Channel Struts
Rollers 9" × 9" cast steel

FIG. 10H.

7. Gardner's end lift.—An ingenious type of end lift for swing spans has been developed by Mr. Randall D. Gardner, M. Am. Soc. C. E., of Boston. United States Patent No. 1,091,829 was granted to him on March 31, 1914. The purpose of the design is to combine the mechanical efficiency of the link-and-roller type with the stability of the wedge type, when under load.

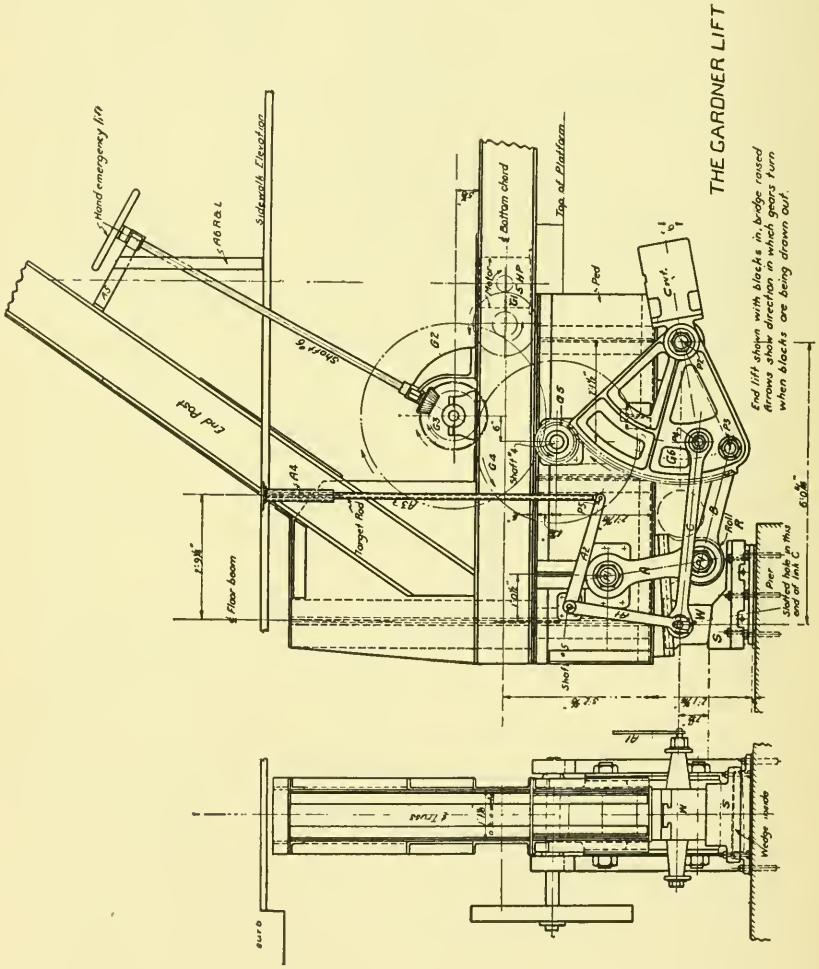

FIG. 10 I.

Figure 10 I shows the general arrangement of the mechanism. The link A carries a roller R, turning on pin $P1$ and is operated by the link B and the segmental gear $G6$, which is driven by a motor, or manually, through an ordinary gear train. A wedge block, W, slides

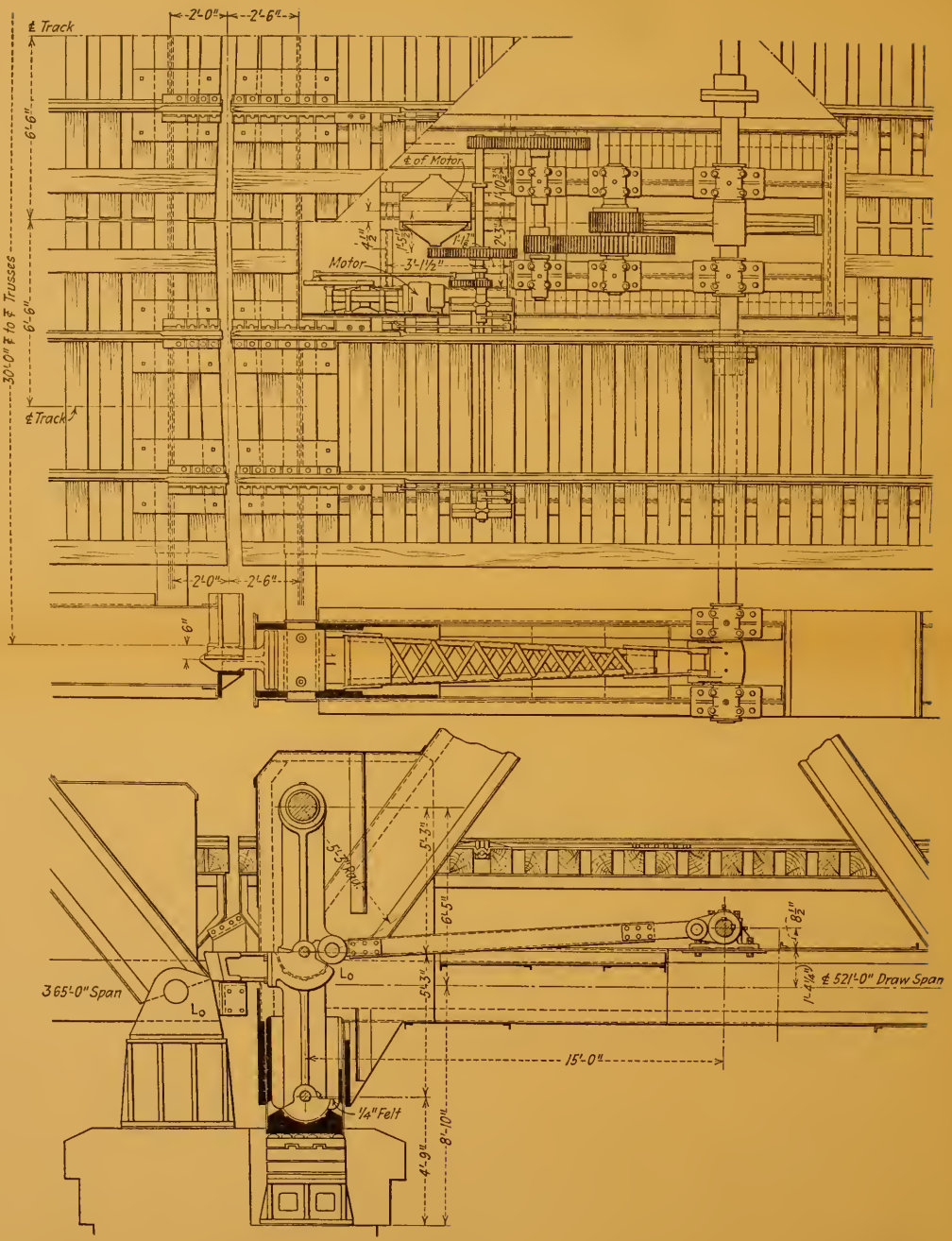

Fig. 10 J.

(To face page 289.)

along a seat on the bottom of the lower chord and is connected to gear *G6* by the link *C* at *P4*. The mechanism is shown in closed position with the end of the span lifted. Link *C* is slotted at its attachment to wedge *W*. The lengths of the links and their points of articulation are selected so that when the mechanism is operated to lower the end of the span, the first movement of link *A* and roller *R* slightly lifts the truss, relieving the wedge of its load. As the movement continues, the slack in the slot in *C* is taken up and the wedge begins to move, the end reaction being taken by the roller. This continues until the end is fully lowered and the roller clears the pier pedestal. When closing the bridge, the movements are reversed, the wedge takes its position and the roller continues to move, lowering the end of the bridge slightly, so that at the end of the lifting stroke the roller clears the pier pedestal, and both dead- and live-load reactions are taken by the wedge block, which is locked in place by the toggle-joint action of the link and the segmental gear.

The wedge never moves under load, both lifting and lowering being performed by the roller-link. This materially reduces friction, increases efficiency, and reduces the power required.

This type of end lift is in use on the Broadway, Chelsea North, and Meridian power-operated, and the Chelsea South and Chelsea Street manually operated bridges in the city of Boston. The operation is satisfactory. An article in *Engineering News*, 1916, I, page 1216, shows half-tone illustrations of the device.

8. Toggle end lift.—During the period from November, 1905, to November 5, 1908, three large bridges were built for the Spokane, Portland, and Seattle Railway Company, under the direction of Mr. Ralph Modjeski, M. Am. Soc. C. E., chief engineer. They cross the Willamette River, near Portland, Oregon, the Oregon Slough, and the Washington Channel of the Columbia River, near Vancouver, Washington. There is a double-track, drum-bearing swing span in each bridge of lengths 521 ft. 0 in., 332 ft. $10\frac{3}{8}$ in., and 466 ft. 10 in., respectively. A special form of toggle end lift was designed by Mr. Byron B. Carter, who was mechanical engineer for Mr. Modjeski. The general arrangement of these end lifts is indicated in Fig. 10J.[1] The following description is quoted from Mr. Carter's paper.

The end reactions at both the Vancouver and Willamette bridges were estimated to be about the same, so it was decided to make the

[1] The Vancouver-Portland Bridges. A Report by Ralph Modjeski, Chicago, 1910. The Operating Machinery of the Willamette River Draw-bridge, near Portland, Oregon, by Byron B. Carter. *Journal of the Western Society of Engineers*, Vol. XXI, p. 529, 1916.

end-lifting mechanisms the same for both bridges. The requirements at the Willamette were that at each corner, or lift, a load of 225,000 lbs. was to be lifted $1\frac{1}{2}$ ins. in 45 seconds and the devices were to carry a total load of 720,000 lbs. Deformation calculations disclosed considerable endwise movements under the action of the live loads provided for. Considerations of all the known forms of end lifts disclosed defects in each, either lack of bearing values under the above loads, lack of efficiency, or tendencies of producing thrusts on the masonry from end movements, due to the live loads, that could not be permitted, especially at Willamette, with its high and slender piers.

To overcome all the above defects, a design was developed that is supposed to be a new form of end lift. To adapt the truss to the lift, the end pin was omitted. The connection is riveted into two main gussets at each side of the chord. In these gussets are placed 12-in. pins, near the top of the connection and outside of the end posts, that carry the end-lift reactions only. These pins, which are removable, are held in plates bolted to the gussets and are prevented from rotating by bolts, through the plates, tapped into the ends of the pins.

Each end lift consists of two cast-steel toggle struts, 11 ft. long from end to end pins. The top joint is bushed with Lumen bushings to the pins in the gussets. The other joints are finished cylindrical surfaces of the castings, 11 in. rad., $20\frac{1}{2}$ in. face, on babbitted surfaces, one in the upper end of the lower strut and one in the lower bearing shoe. Five-inch pins are used here to join the parts and carry the weights only when the jacks are lifted, but they carry none of the loads when the ends are raised. The lower bearing shoe slides vertically about 7 in. in guides, bolted to the lower extensions of the gussets. The lower bearing shoe rests on six segmental rollers 12 in. diameter, of the usual expansion-joint form, except that they are gear-tooth connected to the pedestals and are arranged with pendulum weights to keep the rollers in the vertical position when the end lifts are released. In this manner the great efficiency of a toggle action is combined with a construction of large bearing surfaces and perfect freedom of end motion allowed on a free-moving roller expansion bearing.

The centering and latching of the bridge is produced by a plug with tapering ends, attached to the lower member of the toggles, working in a socket which is attached to the chord of the approach span.

The relations of the plug and socket to the end lift are such that the parallel portion of the plug has entered the socket and aligned the bridge before the lower shoe takes bearing. The taper centers from about 5 in. each side of the bridge center line.

In the upper toggle links are independent pins with bronze bushings for the strut connections. The toggles are actuated by cranks and cross-connecting shafts between the trusses. The bearings for the shafts are attached to the top of the lower chords. Connecting the cranks to the toggles are struts made of structural steel and steel forgings. Mounted on the cross shafts are heavy segmental gears, which are driven by 35-h.p. motor through a single gear train for each end of the bridge. It is thus clear that both sets of toggles for one end of the bridge must act simultaneously while the ends are independent of each

other. The synchronous speed of the motors is depended upon to get practically simultaneous action at the ends, and does so within one or two seconds.

The drawings show all parts in locked position, that is, with the ends lifted, and it is seen that the unbalanced weight of the segmental gear holds all parts in position. The gear train is mounted on a structural framework attached to the two middle stringers at the end panel of the bridge. The cross shaft pierces, but is not supported by, all stringers at their central and neutral axes.

The shafts of the gear train are not in line, but are zigzag with each other, so that shifting, either horizontal or in height, could be done to accommodate the actual diameters of the gear castings.

9. Rocker-and-eccentric end lift.—When a swing span is too long, the ordinary type of wedge end lift is not feasible. When one arm of the span is loaded, the deflection of the trusses causes an angular rotation of the end joint of the bottom chord. If a wedge seat is rigidly attached to the chord, the change in angularity greatly increases the pressure between the wedge and its seat, and that of the lower pedestal and the masonry, at the side toward the center of the span. This action has been known to cause seizing and abrasion of the wedge and seats.

The same condition of loading causes outward movement of the end joint of the bottom chord, due to the elongation of the chord during each passage of the live load. In one case this amounted to more than half an inch. Temperature changes add to this difficulty. The end reactions of a long span are large, particularly in the case of a double-track bridge. The result of the horizontal movement, combined with the heavy reaction, is to apply a large horizontal shear at the top of the pier, tending to overturn it. Most piers are so rigid that they will not deflect enough to absorb the horizontal movement, and the continued application of the shear tends to disintegrate them.

The toggle end lift described in Art. 8, in which the lower end of the ram rests on a nest of rockers, overcomes the deleterious effect of the horizontal movement, but does not so directly correct the effect of the rotation of the joint. Moreover, the device is complicated and expensive.

The rocker-and-eccentric end lift was probably first used by the late C. Shaler Smith, on the C. M. & St. P. Ry. bridge across the Mississippi River, at Sabula, Iowa, which was built by the Edge Moor Iron Co., about 1883. This bridge was in satisfactory use for over twenty-three years, when it was replaced by a modern heavy bridge on account of overload.

Figure 10K shows a side elevation and a section of an adaptation of this design to a very heavy, modern swing span. The angular rota-

tion of the joint is taken by the eccentric-shaft, which is near the bottom chord. The horizontal movements due to the elastic distortions and temperature merely cause slight rotation of the rocker around the shaft. The lower face of the rocker can be made to almost any radius without causing material lifting of the end joint, for the horizontal movements are small. A large radius assists in distributing the load from the theoretical line bearing of the rocker to the pedestal on which it rests. The rocker hangs on the eccentrics like a pendulum, and hangs in central position when raised.

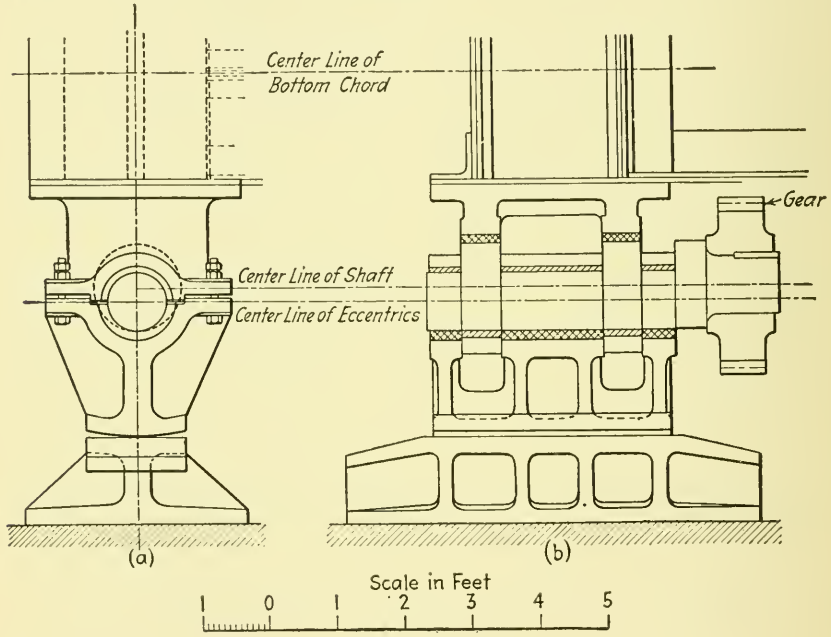

Fig. 10K.

Mechanically, the efficiency is greater than that of an end wedge, but less than that of a toggle with long vertical arms.

As in the toggle, the end reaction from live loads must be taken by the mechanism, but more directly. The whole combination is simpler and more economical than a toggle and there are fewer working parts to maintain, and less probability of its getting out of order.

The linings of the bearings that carry load should be of Class B bronze and liberally proportioned. Provision should be made for thorough lubrication of the shaft in all of its bearings.

10. Reehl's end rail lock.—When sliding locks are used to close the joints in the track rails at the ends of swing spans, the conditions

require a peculiar mechanical motion. The rails must be unlocked before the ends of the span are lowered, and, in closing and lifting, the locks cannot be thrown until the ends are centered and lifted. This is usually accomplished by the use of two motors, one for the end lift and the other for the rail lock, at each end of the span. It is more economical and desirable to operate the center and end wedges and the rail locks by one motor, placed near the center of the span.

Figure 10L shows the general arrangement of a device which successfully accomplishes this result. It was invented by Mr. D. L. Reehl, an assistant engineer with the American Bridge Company, and is protected by United States Patent No. 1,456,266, issued July 6, 1915.

The device is shown with the ends of the span lifted and the rails locked. A gear box is supported between the end stringers at each end of the span. The motor is near the center, and the end gears are driven by longitudinal shafts from the center to the gear boxes. A worm and wheel and a pair of gears reduce the rotational velocity as desired. Gear X is keyed to a short transverse shaft, housed in the sides of the box, and, to unlock, moves in the direction of the arrow. Segmental gear Y is in mesh with X and moves with it and unlocks the rails. When this has been accomplished, the smooth, concave portion of Y contacts with the smooth, convex surface of X, and Y ceases to rotate. The teeth on X then engage those on Z, rotating the transverse shaft that actuates the end wedges and the bridge latch. As the motion continues, the latch is raised and the wedges are withdrawn. All movements are reversed when closing the bridge, the rail locks being driven after the latch has fallen and the wedges have been fully driven.

11. Balanced end latch.—The end latches of swing spans should accurately align the span without shock, and be automatic in action. Figure 10M illustrates a type that has been satisfactorily used on many bridges and may be considered as practically a standard design.

The latch bar L moves vertically between roller guides framed to the end floorbeam. Its lower end carries a roller which engages a catch on the pier. The weight of the latch is nearly counterpoised by the weight E, which can be adjusted to any point on lever l. The latch-roller clears the catch-guide by a small fraction of an inch. While the end wedges are being withdrawn, link m, driven by a crank on a transverse shaft, actuates bell crank n and lever r. After movement begins, lever r engages pawl q and the latch is lifted as movement continues. When fully raised, r and q are in a straight line, and the latch-roller clears the vertical portion of the guide in the catch. The bridge is then turned. The latch-roller moves up the sloped surface u, disingaging pawl q. As turning continues, the roller passes down

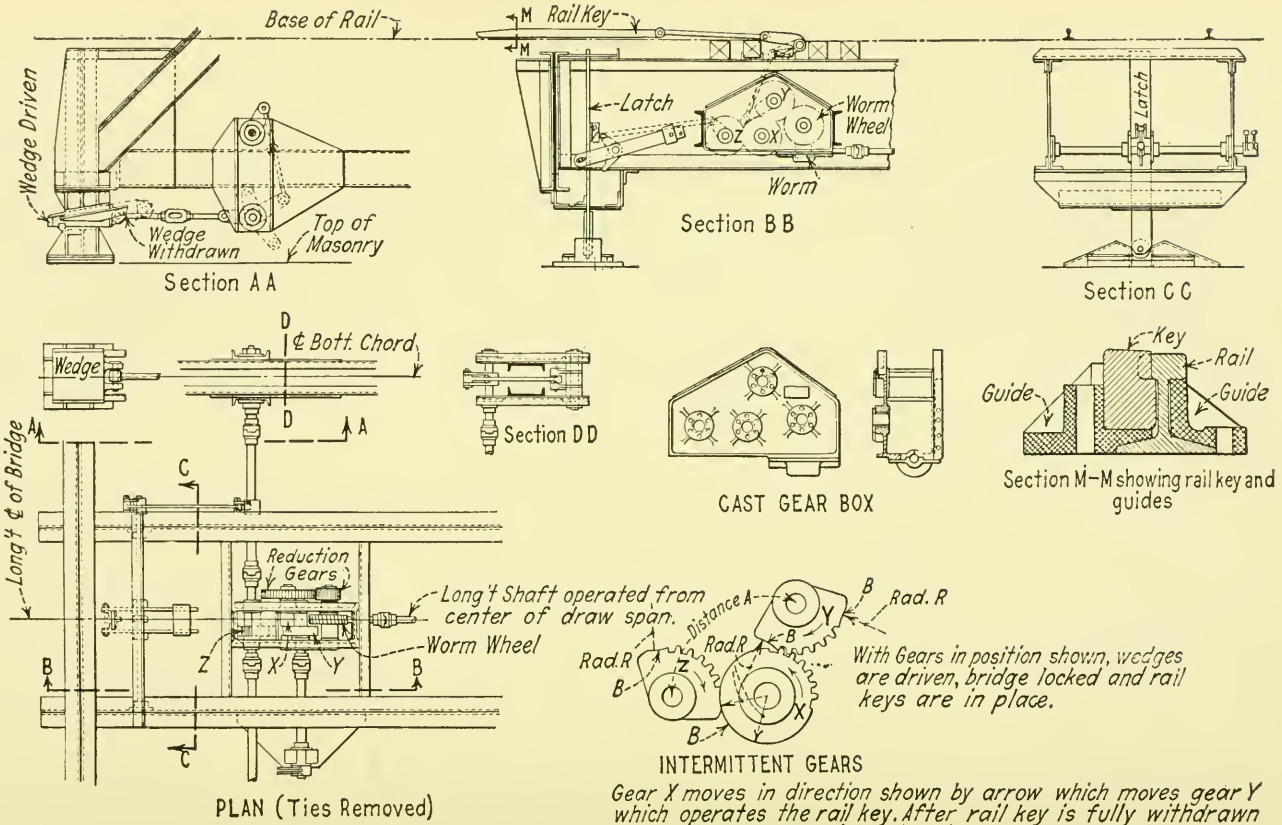
Base of Rail
Wedge Driven
Wedge Withdrawn
Top of Masonry
Section A A
M Rail Key
M
Latch
Worm Wheel
Worm
Section B B
Latch
Section C C
D
℄ Bott. Chord
Wedge
D
A
A
Long't ℄ of Bridge
C
Reduction Gears
Long't Shaft operated from center of draw span.
Worm Wheel
Z
X
Y
B
B
C
PLAN (Ties Removed)
Section D D
CAST GEAR BOX
Key
Rail
Guide
Guide
Section M–M showing rail key and guides
Distance A
B
Rad. R
Rad. R
Rad. R
B
B
Z
Y
X
With Gears in position shown, wedges are driven, bridge locked and rail keys are in place.
INTERMITTENT GEARS
Gear X moves in direction shown by arrow which moves gear Y which operates the rail key. After rail key is fully withdrawn gear Z is then moved in direction of arrow withdrawing the wedge. Reverse motions for closing and locking.

FIG. 10L.

surface v until stop e rests on the latch frame, holding the latch bar in a fixed position, and ready for automatic action when the bridge is closed.

When the bridge is nearly closed, the velocity of rotation is retarded by frictional resistances and the brake. The latch-roller engages surface v, up which it rolls; thence it rolls down u and drops into the pedestal guide by gravity. The latch being nearly balanced, its fall by gravity is slow; hence, if the end of the span approaches the pier too rapidly, the latch-roller passes across the guide and engages surface u_1, and the span passes on without shock. The latch must be counterpoised by

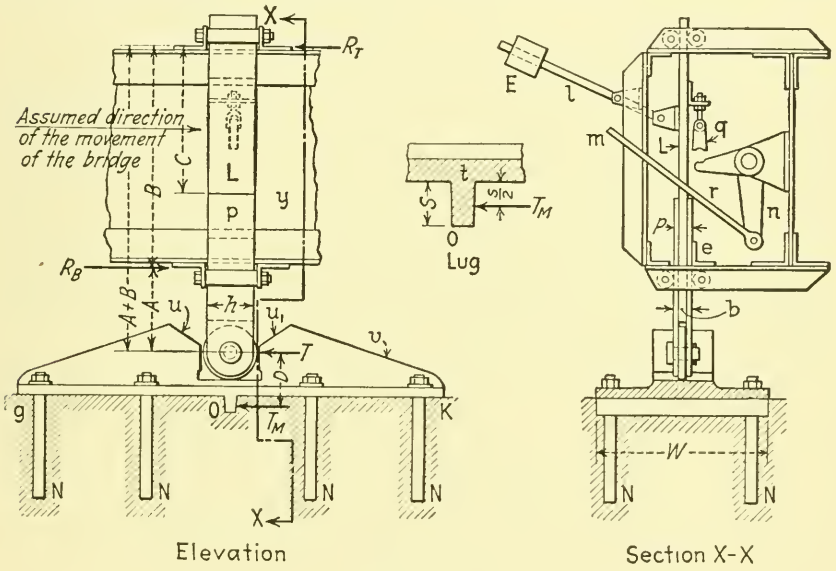

FIG. 10M.

moving E until the latch will drop into its guide without shock and stop the span in central position.

Design:

Let X = maximum rack force due to the motor or the brake;

r = radius of the pitch circle of the rack;

R = distance from the center of the rack to the center of the latch;

T = thrust on the latch and its catch.

The latch, catch, and connections should be designed for a thrust of not less than $T = 2\frac{Xr}{R}$. The reactions are:

$$R_B = T\frac{A+B}{B} \quad \text{and} \quad R_T = T\frac{A}{B}.$$

The bending moment on the latch bar at $R_B = TA$.

The moment of resistance of the latch bar at $R_B = \frac{bh^2}{6}$.

The section of the latch bar must be such that $15{,}000\ \frac{bh^2}{6} \gtrless TA$, or $2500bh^2 \gtrless TA$.

The bending moment at the end of plates p at $y = R_T C = T\frac{A}{B}C$.

The section at y must be such that $2500\ b_0h^2 \gtrless T\frac{A}{B}C$.

The anchor bolts N should be considered as holding-down bolts only, except in small manually operated bridges. Their number and size should be ample to resist the moment TD, which is the measure of the tendency to lift the catch from the point g and to cause it to turn about point k. They should never be less than $1\frac{1}{2}$ in. in diameter, and should be of ample length to provide the necessary anchorage.

On all but small, manually operated bridges, the anchor bolts should not be depended upon to resist the shear due to the thrust T, but a lug of ample size, as shown at O, should be used. The pressure of this lug against the masonry, due to thrust $T_M = T$, should not exceed 600 lbs. per square inch. Assume the dimensions of the lug to be as shown and its length to be w, then,

$$600Sw \gtrless T_M.$$

The bending moment at t is $T_M\frac{S}{2}$.

The moment of resistance of the lug is $\frac{wt^2}{6}$. The minimum thickness where it joins the base of the catch must be such that

$$2500wt^2 \gtrless T_M\frac{S}{2}.$$

The thickness must also be proportioned to the thickness of the base of the catch, in order to avoid excessive cooling stresses in the casting. S should not be less than 2 in., nor t less than $1\frac{1}{4}$ in.

12. Disc center with load applied by hanger bolts.—Figure 10N shows a type of disc center that has been used in many bridges. The weight of the span is delivered to the pivot on the discs by means of two short longitudinal box girders A, framed between two transverse center girders C. Girders A deliver the weight of the span to transverse I-beams B, through hanger bolts E, thence to pivot P, which rests on discs D. The purpose of the design is not only to deliver the load centrally on the discs, but to permit the whole center to be taken

out for repairs from the track level, after the center girders *C* have been blocked up from the pier. This type is satisfactory for bridges of ordinary size and weight, but becomes almost impracticable for the largest modern center-bearing structures. It is an expensive design.

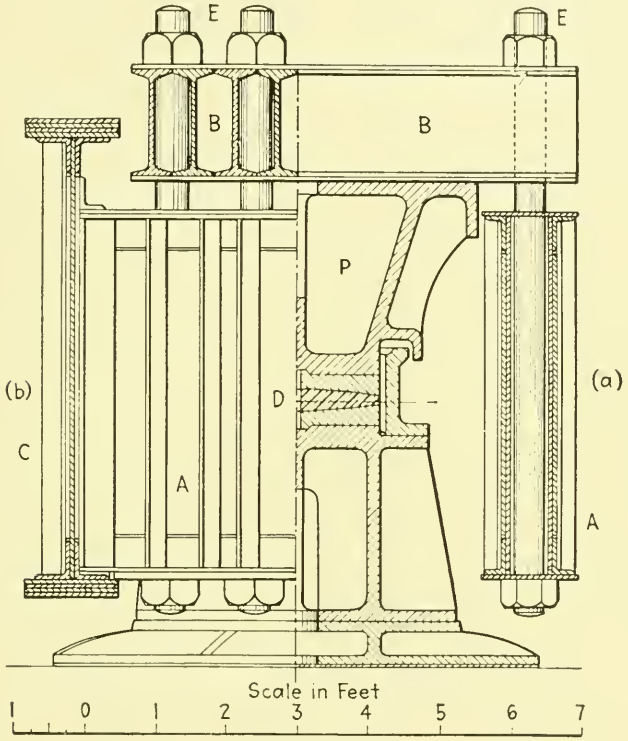

FIG. 10N.

It was supposed that this construction would neutralize the effect of an unequal settlement of the center pier, but it has little advantage over the simpler form where the center transverse girders rest directly on the pivot. The particular design shown was used in a bridge on the Pennsylvania Railroad across the Passaic River, which is a double-track swing span 277 ft. 2 in. between centers of end bearings.

13. Three-disc center with pier not level.—Reference was made in the last article to the effect of unequal settlement of the center pier on a center bearing with three discs. Assume that the pier settles until the discs take the position shown by full lines in Fig. 10 O, which is not to scale but much exaggerated. As the balance wheels clear their track by only a small fraction of an inch at a radius of several feet

from the center, the bottom of the lower and the top of the upper disc will be practically parallel with the top of the pier after settlement. The center of gravity of the span moves over until a vertical through it falls at some point, as d, on the top of the upper disc, and the weight of the span acts at this point. Before the bridge is turned, the discs still contact practically at a and b. When turning begins, the gravity and disc-pressure forces are not in equilibrium and, as the turning continues, the discs will move in a direction that tends to restore equilibrium. The lower disc is practically a part of the pier; hence all adjustment must be made by movements of the middle and upper discs. The upper disc is practically a part of the span and the span must move with it. The new position of equilibrium is roughly indicated by broken lines in the figure. When the unequal settlement is small, the middle and upper discs and the bridge will assume a new center of rotation, but one side of the discs will project beyond the lower disc and impinge against the inside of the oil box, enclosing the discs.

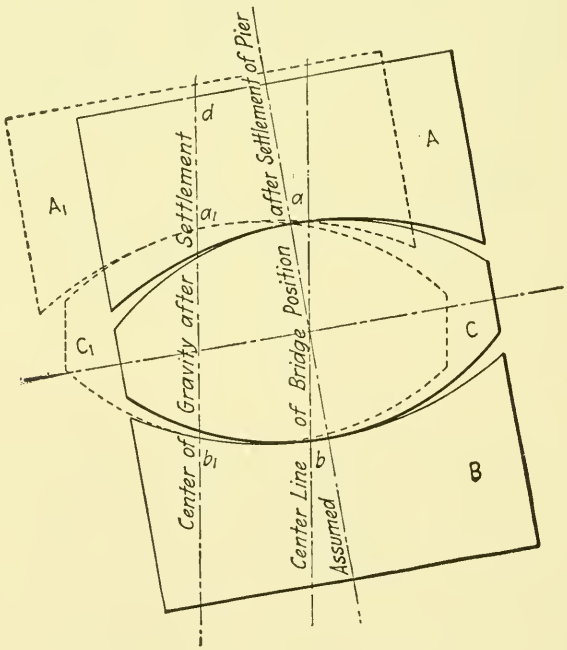

Fig. 10 O.

In one case, observed by the author, the settlement was so small that the new center of rotation was only $\frac{5}{16}$ in. from the original one, and by boring out the inside of the oil box and moving the rails on the

bridge all further trouble was avoided. The discs in this case were 36 in. in diameter. The settlement occurred in 1912 and the bridge is still operating in a satisfactory manner.

In another case, with 31-in. discs, a pier grillage had been set so much out of level that it corresponded to about $\frac{3}{16}$ in. in the diameter of the discs. Large side pressure developed against the inside of the oil box; the bolts coupling its halves together and those attaching it to the lower center pier casting were loosened, and the inside of the oil box was badly abraded. The oil box was removed and the bridge turned several times. The movement of the middle disc and the bridge continued until the discs projected nearly 2 in. and the new center of rotation had not then been found. The experiment was stopped, and the bridge was then jacked back to central position. Beveled shims were placed, bringing the lower center casting and the upper pivot truly level. A new middle disc and oil box were placed, completing the repairs, and the bridge now operates perfectly.

The middle disc of this type of center, being double-convex, is easily displaced and is very sensitive to changes of level of the center pier, and there is no tendency for it to move back into position after displacement. Bridge erectors are well aware of this fact and have nicknamed it a "melon seed." It is the author's opinion that only two discs should be used, the lower being concave upward and the upper convex downward. There is then some tendency for the upper disc to slip back into place after displacement, for the curvature tends toward stability. In European practice this arrangement is commonly used.

14. Disc centers with superposed load.—A simple type of disc center with superposed load is shown by Fig. 10P. This type has been extensively used and has been satisfactory. The transverse center girders of the span rest directly upon the pivot P, and the base G on the masonry. Height adjustment is made by shims between the center girders and the pivot. Many such centers have been built with three discs, but the use of only two is recommended. Circular lugs are formed on both discs to hold them in central position. A small clearance is provided between the circumference of the discs and the inside of the oil box. The latter is in halves, bolted together, so that it can be removed for inspection and repairs. The oil box rests on the top of the lower casting around a lug on its top surface, and is securely bolted down. The oil box must be oil-tight and kept full of oil. Oil is supplied by a pipe screwed into the pivot and extending to the floor level and capped, to exclude dirt and water. An oil hole extends down through the pivot and the upper disc and half-way through the lower

disc. It connects with oil grooves cut in the bottom of the upper disc. These oil grooves constitute one of the most important details of the design. They must not have sharp corners at the bearing surface, which would prevent the entrance of the oil between the discs. These corners should be well rounded, as shown by Fig. 10 Q (*a*), so that the oil will be drawn in between the discs as they rotate.

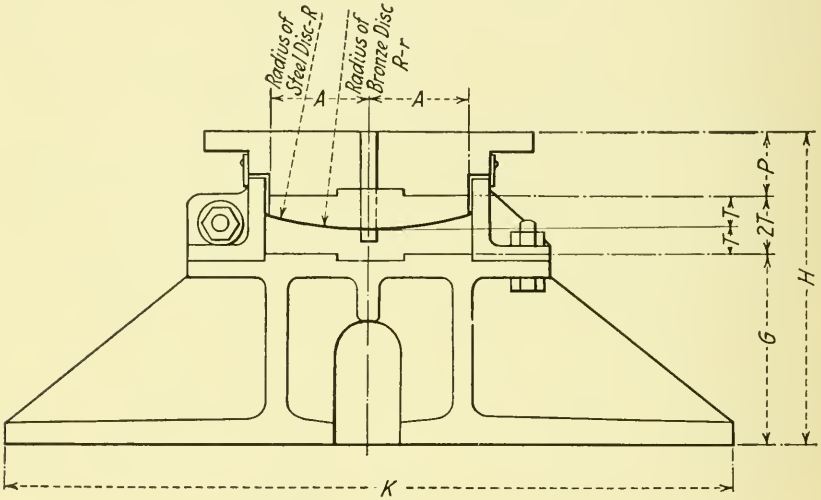

Fig. 10P.

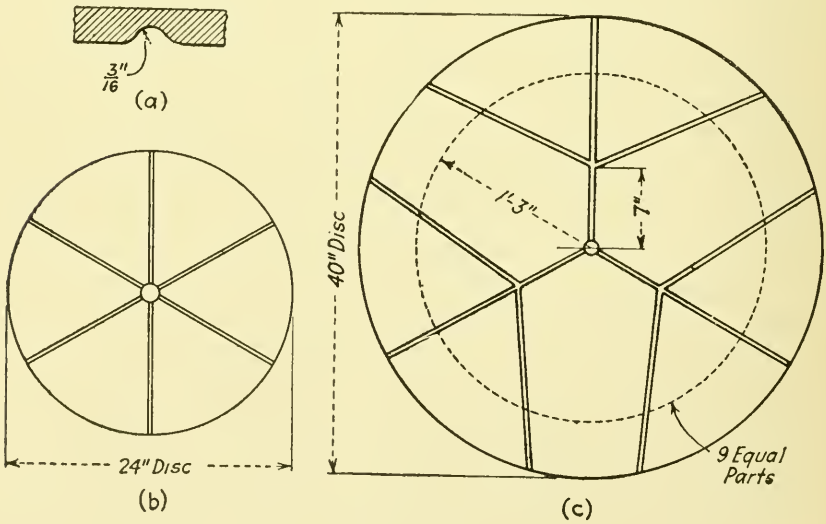

Fig. 10 Q.

For discs of ordinary sizes, the best oil grooves are radial, as shown at (*b*). They should be large enough for the insertion of a wire for cleaning. The number of grooves should be such that the extreme distance between them shall not be more than from 12 to 15 in. When the discs are more than about 36 in. in diameter, the principal grooves should be radial, with straight branches extending to the circumference, as indicated by (*c*). They can then be cleaned out by wires.

The upper disc should be of hard phosphor-bronze, Grade A, in accordance with Arts. 804 and 805 of the Specifications in Volume II, Appendix A, and its lower bearing surface should be polished. The lower disc should be of steel, in accordance with Art. 802 of the Specifications, with its upper bearing surface accurately ground.

Designs in detail.—A series of designs is used by the American Bridge Company, proportioned in accordance with the following requirements:

Two discs are used.

The pressure on the projected net area of the discs is 3000 lbs. per square inch.

The radius of the spherical bearing surface of the upper, or bronze, disc is made smaller than that of the lower, or steel disc, to diminish the radius of friction and the power required for turning. The difference of the radii is such that the maximum pressure, under full load, is 10,000 lbs. per square inch at the edge of the central oil hole. The analysis of the stress distribution in discs of different radii, when under load, is given in Appendix A. An illustrative example is also given.

The maximum unit compression in the central part of the lower center casting is 10,000 lbs. per square inch.

The pressure on the masonry is 400 lbs. per square inch of net area of the base casting.

The maximum fiber stress, due to bending, in the castings, is 16,000 lbs. per square inch, assuming that the center girders are rigid and that the masonry pressure is uniform.

Table 10A gives data for this series of designs.

When local conditions, or specifications, require a pressure on the masonry of less than 400 lbs. per square inch, steel plates, slabs, castings or embedded grillages may be used under the pedestal casting to increase the bearing area upon the masonry.

15. Rack anchors.—When large center-bearing swing bridges are turned rapidly, the mechanism produces large tangential forces on the rack at the main pinions. In former practice the rack was simply bolted to the masonry by dowel bolts, which were usually small, seldom being more than $1\frac{1}{2}$ in. in diameter. The bolts are commonly set in

cement mortar, so that their resistance to a tangential rack force depends on the crushing strength of the mortar around them, in combination with their stiffness as vertical cantilevers. Further, the maximum unit pressure where they enter the mortar is $\frac{4}{\pi}$ times the unit pressure on the projected diameter of the bolts. The result has been that in some cases the racks became loosened and could not be held tight. This difficulty does not occur in rim-bearing bridges, for the rack is a part of the roller track, which is firmly held by the great weight upon it and the resulting friction on the masonry.

TABLE 10A

Diameter 2A	Turning Load W	Fixed Load W_F	Rad. St. R	Rad. Bz. $R-r$	Pivot P	Disc T	Discs $2T$	Pedestal G	Height H	Diameter K
12″	291,000	591,000	2′–6″	2′–5″	4″	1¾″	3½″	1′–0″	1′–7½″	3′–8″
15″	471,000	771,000	3′–0″	2′–11″	4″	2″	4″	1′–1½″	1′–9½″	4′–2″
18″	678,000	978,000	4′–0″	3′–10¾″	4½″	2¼″	4½″	1′–3″	2′–0″	4′–8″
21″	939,000	1,239,000	5′–0″	4′–10″	6″	2½″	5″	1′–5″	2′–4″	5′–3″
24″	1,245,000	1,545,000	6′–0″	5′–9¾″	7½″	2¾″	5½″	1′–6½″	2′–7½″	5′–11″
27″	1,590,000	1,890,000	7′–0″	6′–9½″	7½″	3″	6″	1′–7″	2′–8½″	6′–6″
30″	1,980,000	2,280,000	8′–0″	7′–9″	8½″	3¼″	6½″	1′–9″	3′–0″	7′–2″
33″	2,412,000	2,712,000	9′–0″	8′–8¾″	9″	3½″	7″	1′–11″	3′–3″	7′–9″
36″	2,886,000	3,186,000	10′–0″	9′–8¼″	10½″	3¾″	7½″	2′–2″	3′–8″	8′–5″
40″	3,582,000	3,882,000	11′–0″	10′–8″	11½″	4″	8″	2′–3½″	3′–11″	9′–4″
44″	4,356,000	4,656,000	12′–0″	11′–8″	1′–1″	4¼″	9″	2′–5″	4 –3	10′–2″
48″	5,205,000	5,505,000	13′–9″	13′–4¼″	1′–2½″	5″	10″	2′–6½″	4′–7″	11′–1″
52″	6,129,000	6,429,000	15′–0″	14′–7″	1′–4″	5½″	11″	2′–8″	4′–11″	12′–0″

In order to prevent this difficulty in center-bearing bridges the detail shown in Fig. 10R is recommended. Cast lugs, L, are bolted to the bottom of each rack segment. The bottom of the segment can be planed and the lugs bolted on at any time before erection. A pit is left in the top of the pier for each lug, and, after the rack has been accurately set, the pits are filled with concrete. The anchor bolts are only used as holding-down bolts and might almost be dispensed with.

The following method of proportioning will produce a good design:

Use cast-steel lugs with metal not less than $1\frac{1}{4}$ in. thick.

Assume the rack force from each pinion to be taken by a single lug.

Limit the pressure of the lug against the concrete to 600 lbs. per square inch.

Limit the unit tension at the root of the threads of the lug-attachment bolts to 10,000 lbs. per square inch.

Limit the combined bending and tension in the body of the bolts to 15,000 lbs. per square inch.

Design the lug casting for a maximum fiber stress of 16,000 lbs. per square inch.

Turn the bolts to a light driving fit.

The anchor bolts should not be less than $1\frac{1}{2}$ in. in diameter and should extend at least 12 in. into the masonry.

16. Main pinion bearings of center-bearing swing spans.—One of the most common defects in the design of the turning mechanism of center-bearing swing bridges is the failure to provide adequate stiffness and strength in the main pinion bearings and their attachments. In

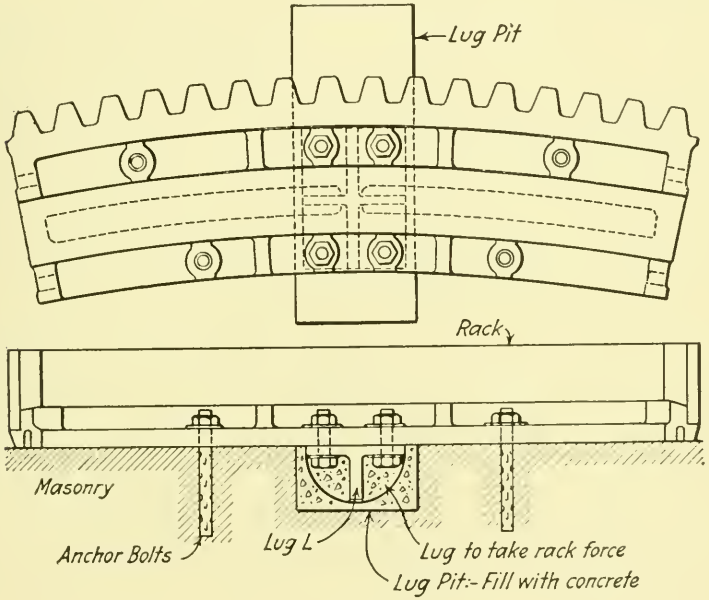

FIG. 10R.

too many cases the structural supports have been designed to support the bearings and withstand the most obvious stresses, and no consideration has been given to the surge due to the sudden application of power during acceleration, or to the couple due to the pinion tooth load, and its parallel reaction, at the center of the shaft within the bearing. The component of the pinion tooth load, tending to separate the pinion from its rack, is also overlooked. There are also vibratory stresses, due to imperfections in the teeth, which cannot be calculated.

Experience proves that it is wise to cast the two bearings of the pinion shaft as a unit, as indicated in Fig. 10S. The bearings may

then be bored out at a single setting of the work, and the shaft accurately and permanently aligned within the bearings. All stresses must be considered, and the bolts attaching the union bearings to the structural supports must be large and numerous enough to withstand them. Some designers have used too few bolts near the lower box and have depended upon structural plates bearing against the sides of the casting. This is not a good detail, for it is almost impossible to do the machine work with such accuracy that the plates and bolts bear at the same time, and the result is that either the one or the other takes all of the stress.

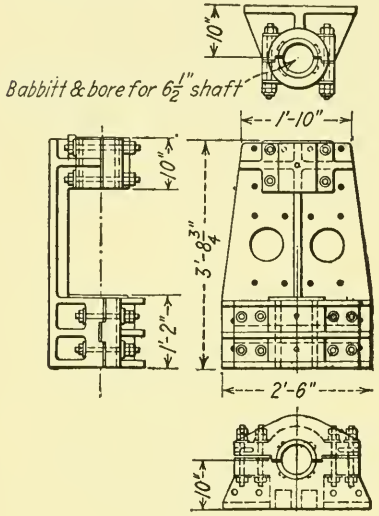

Fig. 10S.

17. Union bearings for bevel gears.—One of the most common defects in movable-bridge mechanism is the use of separate bearings for the two shafts which carry a pair of bevel gears. In many cases the two separate bearings are attached to separate structural supports. The exact position of such supports is, at best, quite uncertain, and it often becomes a difficult matter to align the shafts and secure the exact mesh of the bevel-gear teeth so necessary for satisfactory operation. Further, the structural supports are not always of adequate stiffness and they may deflect from the thrust of the gears. Vibration is set up and the gears are noisy and irregular in operation. Wear ensues, increasing the difficulty.

It is usually feasible to design the supports so that a single casting can be used with a bearing for each shaft. Figure 10T shows a form of union bearing that has been used at the floor level of operators' houses at the points where the transverse horizontal shaft from the motor joins the vertical shaft which extends down outside of the trusses to the turning and end-lifting mechanism of swing bridges. The shaft alignment and gear mesh can be accurately fixed in the shop, and the field work is reduced to properly locating and securing the union bearing. Similar union bearings should be used wherever it is necessary to use bevel gears.

Bevel and miter gears should be avoided when it is feasible to arrange the mechanism for the use of spur gears.

18. Bearing supports.—Ordinary pedestal bearings for shafts are sometimes carried by a single I-beam, or channel, as indicated in Fig. 10U (*a*). Such supports are often unsatisfactory on account of their lack of horizontal stiffness. This is particularly true at points where spur gears transmit stress to the bearings, but is sometimes observed even in cases where the bearing and its support carry only the

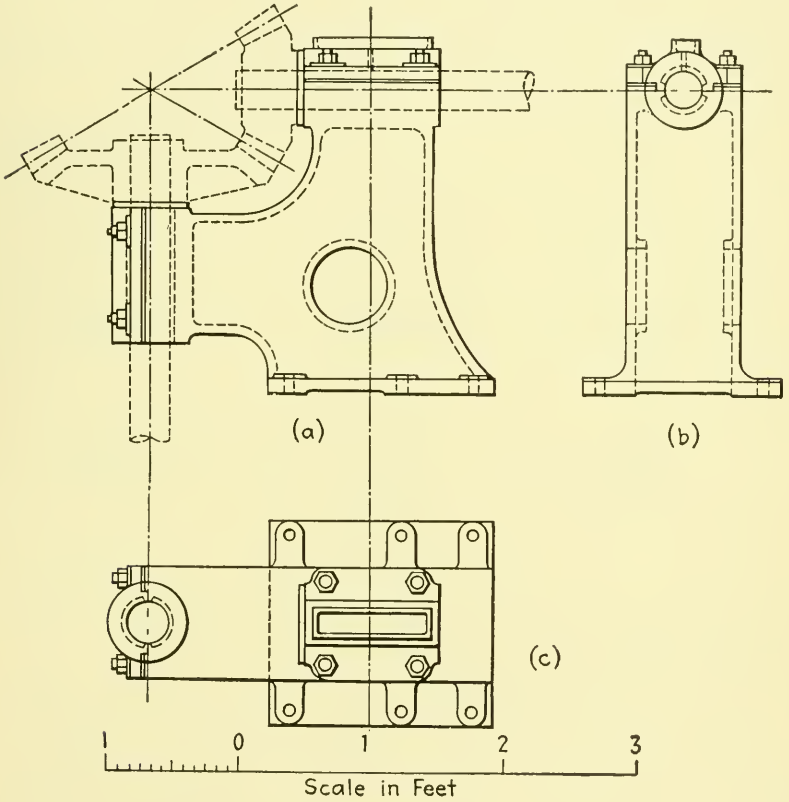

FIG. 10T.

weight of the shaft. For light work, an angle with equal legs will serve in some cases; in others, an I-beam stiffened on top by a channel, or a plate, as indicated at (*b*), may be used. In other places and for heavier loads, two channels with a top plate and bottom lattice, as at (*c*), will be preferred. In most cases of this kind, the determining consideration is stiffness against vibration and not strength to resist calculated stresses.

The type of bearing support must, in general, be determined by good judgment based on experience and the observed performance in actual construction.

19. Turntables of drum-bearing swing spans.—When it is impracticable to use a center-bearing swing span, the next best arrangement is a combination of rim and center bearings, with the center-loading girders arranged so that a definite part of the load is delivered to the center. The load on the center helps to hold it in position and to secure it against overturning forces. The center should also be anchored to the masonry by adequate bolts. Radial struts should connect the center and the track on which the turntable rollers run.

When a purely rim-bearing design is used, the center casting must be rigidly anchored to the masonry by bolts and lugs on the bottom of

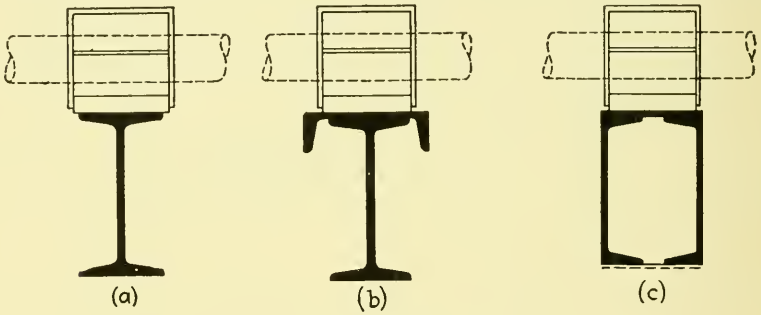

Fig. 10U.

the casting, and radial struts should be used to connect the center with the roller track.

The lower roller track should be high enough to allow the rollers to be taken out and replaced without removing the rack.

There are two general methods of arranging the live rings. The older construction was to provide a ring plate resting and turning on a shoulder on the center pivot. Long radial rods were pin-connected to this ring plate and extended outward through the live-ring members and the rollers, and had nuts at their extreme outer ends. It was customary to make the inner and outer live-ring members independent of each other, on the theory that the rollers and ring members would take equilibrium positions when turning. Whenever workmanship and installation were perfect, this theory was realized. Experience indicated that there was danger of the ring plate on the pivot lagging behind the live ring, skewing the rollers and setting up heavy side stresses on the pivot. In order to avoid this and to force the pivot

ring to move with the live ring, at least four tangential rods should be used, as shown at R in Fig. 10V.

A more recent design, which is well adapted for modern heavy bridges, is shown in Fig. 10W. The inner member of the live ring is a circular girder, strong enough to resist the outward radial thrusts from the rollers. This ring is connected to a central sliding ring plate on the pivot by eight or more struts, each riveted, or bolted, to the ring plate on the center. These connections are sometimes made by a single pin, but a group of several rivets or bolts is better.

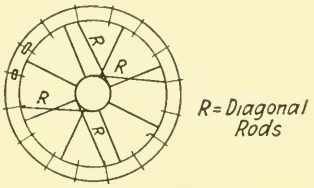

FIG. 10V.

The outer ring is in short pieces, bolted at the joints and also bolted to tie plates, which extend outward from the inner ring girder, so that they can be readily removed for repairs of the rollers and rods. Each roller turns around a short rod. These rods have square heads at the inner end, engaging lugs on the ring girder, to prevent them

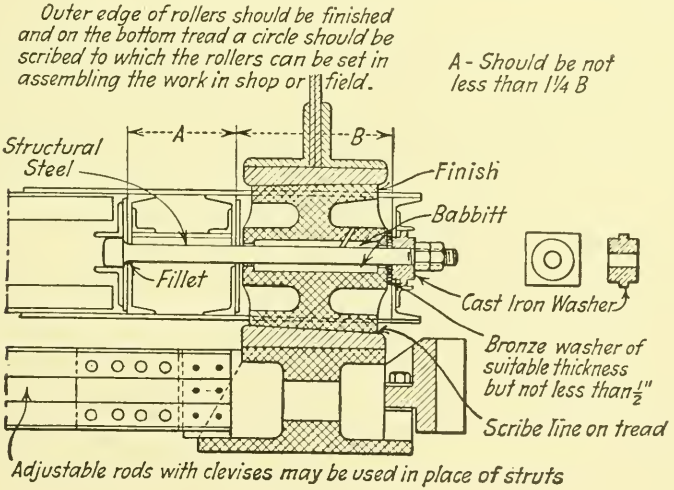
Outer edge of rollers should be finished and on the bottom tread a circle should be scribed to which the rollers can be set in assembling the work in shop or field.
A - Should be not less than 1¼ B
A
B
Structural Steel
Finish
Babbitt
Fillet
Cast Iron Washer
Bronze washer of suitable thickness but not less than ½"
Scribe line on tread
Adjustable rods with clevises may be used in place of struts

FIG. 10W.

from rotating. Bronze thrust washers are placed at the outer end of the hubs of the rollers. An adjusting block bears against the bronze washer. These blocks are inserted in round holes in the outer ring member. A part of their outer ends is made square to engage lugs and prevent rotation. Bearing and lock nuts are used on the outer ends of

the rods. By turning up the nuts, endwise adjustment of the rollers is readily made. Note that all roller thrust is delivered in tension by the rods as outward radial forces on the inside of the ring girder.

In order to allow for inaccuracies in the adjustment of the whole device, the diameter of the holes in the rollers should not be less than $\frac{1}{16}$ in. greater than the diameter of the rods.

When the rollers are of steel they should be babbitted to prevent seizing, should the rollers become far enough out of line to bear against the rods. When cast-iron rollers are used they should be bored throughout their full length.

Accurate installation of the center, track, and live ring is most important. The drum, live ring, and track must be concentric with the center pivot, and their center lines must be vertically above each other.

The outer ends of the treads of the rollers should be accurately finished, and the upper and lower coned tread plates should be lightly scored at the ends of the roller tread while they are still set up on the machine in the shop. The rollers can then be adjusted until their ends match the scores on the treads, insuring proper working relations of the parts.

The main pinion shaft bearing and its supports is one of the most important details of a rim-bearing draw span. The two bearings of each vertical shaft should be made as a union bearing, similar to Fig. 108, but usually much larger and longer. The bearing should be thoroughly bolted to a vertical structural bracket, built as a part of the drum. The cover plates of the drum can be interrupted at the shaft connection and large horizontal sketch plates substituted. These extend radially beyond the drum as far as necessary to connect to the vertical member of the bracket. The edges of these plates should be stiffened by heavy angles, so that the whole shaft-support bracket will be rigidly braced against forces developed by the tractive effort of the machinery.

The shaft-support brackets of many bridges have been made too light and braced inadequately to the drum. The result has been loosened connections and false alignment and mesh of the pinions, with rapid deterioration of all parts, which occasioned troublesome and expensive repairs. It is better to make these details too strong than to slight them and then be forced to patch them up after failure has begun.

CHAPTER XI

RAIL JOINTS

1. General.—The rails at the ends of movable bridges should bridge the joints between the movable span and its approaches. In most cases it is best that the rails on the movable span be extended far enough across the joint to be firmly supported on the approaches. When this is done on a swing span, the end rails must be lifted while the ends of the span are being lowered. The end rails cannot be spiked down, but must be secured in some other way. Safety against derailment when rolling stock crosses the joint is of primary importance, and the design must be such that the rails will surely be bedded on their supports and securely held in position when the bridge is closed and ready for traffic.

2. Lifting rails with square ends.—Figure 11A shows a form of lifting rail that has been successfully used on many swing bridges and is the most common construction. The lifting rails are joined to the fixed rails on the moving span by a joint that permits rotation in a vertical plane. The lifting rails are spaced by ordinary switch rods and are also bolted to the transverse member of the lifting device. Each tie toward the inner end of the rail is fitted with a steel shoe, having sloped rail guides, spiked to the ties. Near the joint a long shoe, of similar construction, extends over several ties. The approach rail is clamped in a fixed shoe having a support, with guides, for the end of the lifting rail. In the figure, the ends of the rails are cut square. An interlocking device should be used, so that the signals cannot indicate a clear track until the lifting rails are seated and the ends of the span have been fully lifted. This is not shown in the figure, but is a vital part of the construction.

3. Lifting rails with scarfed ends.—Figure 11B shows an improvement over Fig. 11A. In this case the ends of the rails are not cut square; instead, the end of the lifting rail overlaps that of the fixed approach rail. The heads and feet of the rails are cut away flush with the webs and bent to form a scarfed joint. The inner projecting portion of the head of the bent lifting rail is planed to the gauge line. Clearances are left between the ends and the scarf for expansion. The joints should be located well back from the break in the floor. The

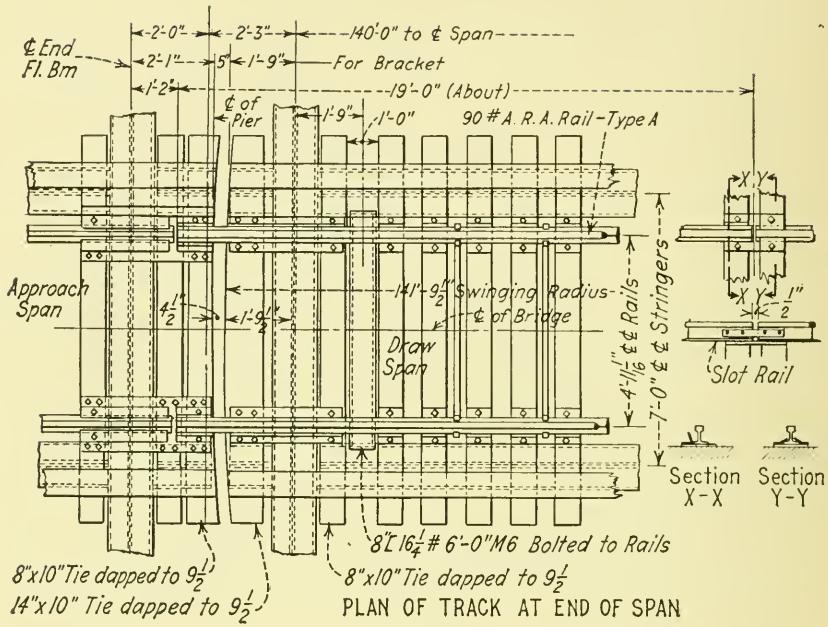

Fig. 11A.

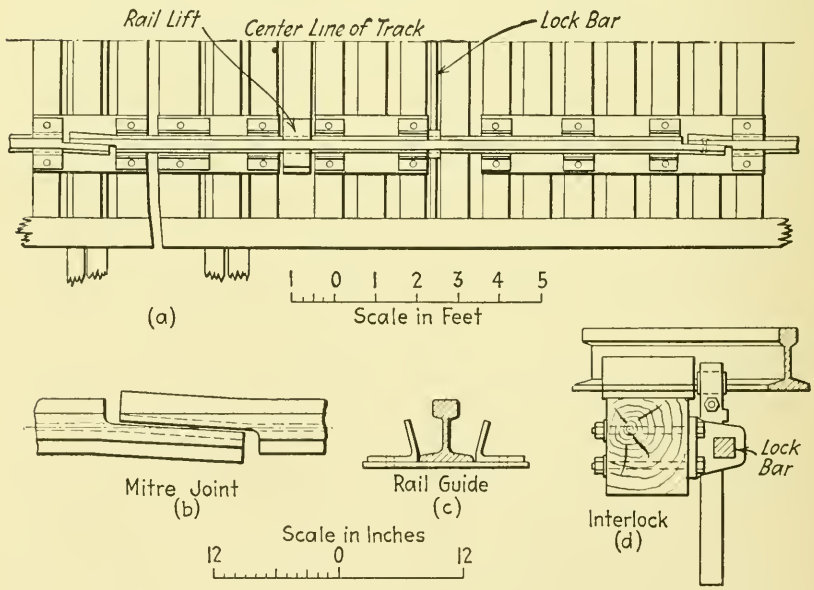

Fig. 11B.

scarfed joint is shown at (*b*), one of the rail shoes, with guides, at (*c*), and the interlocking device at (*d*).

This form of rail joint is also well adapted for use on bascule and vertical-lift bridges. In such cases the rails on the movable span are spiked down and some of the rail shoes may be omitted.

4. Sliding rail joints.—Figure 11C shows a typical rail joint in which the rails on the movable span and approaches are rigidly fastened down and the break in the floor is bridged by a tongue, sliding between

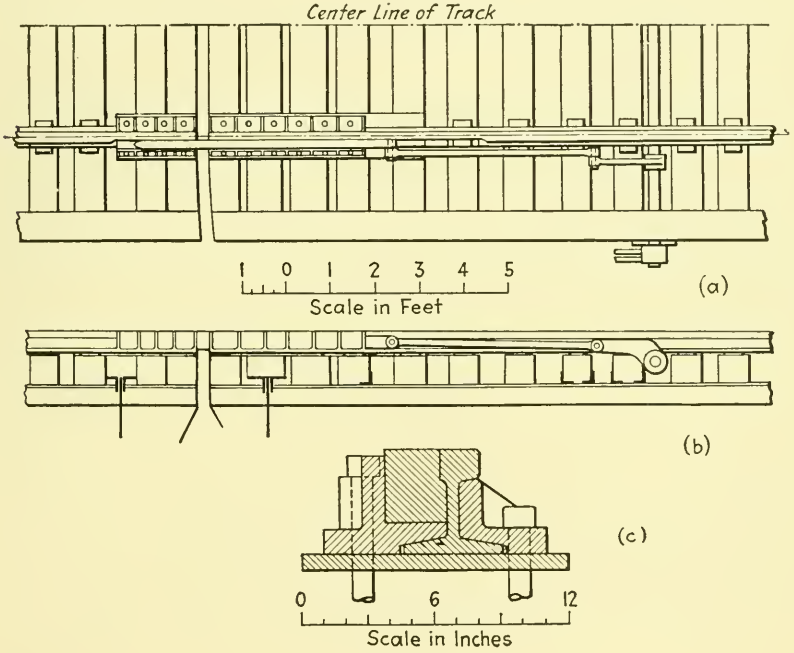

FIG. 11C.

the webs of the rails and fixed guides on the ties. A part of the outside of the rail head should be left to guide the tongue.

Some designers have made the tongue, at and near the joint, higher than the top of the rails, with sloped approaches, the idea having been to lift the wheels across the joint. This should never be done. The tongue should be of the same height throughout its length, and its outer side no higher than the extended coning of the wheel treads. It may even be better to leave it flat and of the same height as the rail, for wheels with worn treads are common, particularly engine drivers. Such wheels impinge violently on the tongues. The extreme ends of the tongues should be tapered below the rail tops.

In one case in which sliding rail joints were observed the tongues were $\frac{7}{16}$ in. above the rail tops at the joints, and the vertical approach slopes were relatively steep. The shock of passing wheels was so violent that slow orders had been issued across the bridge when it was first put in service. A passenger train was observed approaching the draw faster than it was allowed to do by the slow order. The impact of the driving wheels was so great that the tongues were forced back across the joint, which was thus left unprotected. The tongues were driven by a rod and crank, as shown in the figure, and the mechanism was overhauled by the impact so that the bell crank was in open position after the passage of the train.

This form of rail joints should be interlocked with the signals, like the two designs previously mentioned.

When sliding rail joints are used on swing bridges, some form of progressive motion must be provided. The tongues must be withdrawn before the ends of the bridge are lowered, and driven after the ends are lifted. The mechanism shown in Chapter X, Article 10, Fig. 10L was developed for this purpose.

CHAPTER XII

COUNTERWEIGHTS

1. Plain concrete.—Concrete mixtures are usually specified by volumes, based on one volume of cement, as 1 : 3 : 6, meaning one part of cement, three parts of sand, and six parts of broken stone. In order to make sure that the desired mixture is obtained, it is necessary to define both the volume and weight of a unit of cement. The Joint Committee on Standard Specifications for Concrete and Reinforced Concrete, in their report of August 14, 1924, state that "the unit of measure shall be the cubic foot; ninety-four (94) pounds of Portland cement (one bag or $\frac{1}{4}$ bbl.) shall be considered as one (1) cubic foot." This corresponds to a barrel containing 4 cu. ft. of cement weighing 376 lbs., net. The barrel is the unit used in the following discussion.

Problem 1.—Find the weight per cubic foot of a concrete of known proportions and materials, when used in a massive counterweight.

Let N = number of cubic feet of concrete that can be made from one barrel of Portland cement when mixed with sand and broken stone, or gravel, in the known proportions;

W = weight of the materials used for N cubic feet of concrete, not including the mixing water;

then

$\frac{W}{N}$ = weight of the materials per cubic foot of concrete, not including the mixing water.

Experiments show that from 12 to 14 lbs. of water is required to mix a cubic foot of wet concrete; also that from 3 to 5 lbs. of water drips out of the forms, that about 2 lbs. of water evaporates slowly from a mass of concrete, and that the remainder is permanently retained. The probable average weight per cubic foot of concrete then is,

$$\frac{W}{N} + (13 - 4 - 2) = \frac{W}{N} + 7. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let S = number of cubic feet of sand in N cubic feet of concrete;
G = number of cubic feet of broken stone, or gravel, in N cubic feet of concrete;
V_s = percentage of voids of the sand; and
V_g = percentage of voids of the broken stone, or gravel.

The value of N is closely expressed by a formula given by Taylor and Thompson:

$$N = 2.61 + 1.34(1 - V_s)S + 1.08(1 - V_g)G. \quad . \quad . \quad (2)$$

Assume average dry sand to weigh 89 lbs. per cubic foot and its specific gravity to be 2.65; then $V_s = 0.46$, or $(1 - V_s) = 0.54$, hence (2) becomes

$$N = 2.61 + 0.724S + 1.08(1 - V_g)G. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The values of V_s vary somewhat for different sands but not as much as those of V_g for different aggregates.

When V_g is found experimentally from the material to be used, Eq. (3) gives quite accurate results.

To indicate the approximate weights per cubic foot of concrete counterweights of various materials and proportions, Table 12A is given. The weights were calculated from Eqs. (1) and (3). In lines 1, 2, 3, and 4 the aggregate was assumed to be broken stone, screened to $\frac{3}{8}$ in. to $1\frac{1}{2}$ in. in size, with voids of 45 per cent. Lines 6 and 7 are based upon tests made by the Erection Department of the American Bridge Company.

TABLE 12A

Average Weights per Cubic Foot of Concrete in Massive Counterweights

Line	Material of Aggregate	Average Specific Gravity	Assumed Percentage of Voids	Assumed Weight per Cubic Foot of Aggregate	Proportions by Volume			
					1 : 2 : 4	1 : 2½ : 5	1 : 3 : 5	1 : 3 : 6
1	Trap	2.90	45	99.37	157	157	155	157
2	Granite	2.70	45	92.51	150	150	149	150
3	Limestone	2.65	45	90.79	149	149	148	149
4	Sandstone	2.40	45	82.23	141	141	140	141
5	Bituminous cinders	1.525	52.42	45.20	116	115	116	114
6	Coke braise	1.193	50.51	36.80	106	105	106	104
7	Steel furnace slag	2.30	46.61	76.48	137	136	135	134

Concrete mixed in the proportion of 1 part of cement, 3 parts of sand, and 6 parts of broken stone, or gravel, has been found satisfactory

for counterweights. It is rich enough in cement to fill all of the voids and to give a good finished surface. It is economical and dense.

The data given above are intended for office use when calculating the size and shape of counterweights. When a counterweight is to be built, careful field tests should be made with the materials to be used, and the results sent to the engineer responsible for the shop drawings, as the basis of any necessary recalculations of the balancing of the bridge. The field tests should be made on 1-ft. cubes. Experience indicates that the weight of a 1-ft. cube twenty-four hours after molding represents the probable weight per cubic foot of the massive concrete in the counterweight. During construction, the materials should be weighed and frequent checks made, so as to be sure that the actual counterweight is of the correct weight.

Example 1.—Let it be required to find the weight per cubic foot of a plain concrete, in a counterweight, mixed in the proportions of 1 part of Portland cement, 3 parts of sand, and 6 parts of broken trap rock.

Assume dry sand to weigh 89 lbs. per cubic foot.

Assume broken trap rock, screened to $\frac{3}{8}$ in. to $1\frac{1}{2}$ in. in size, of a specific gravity of 2.9, with 45 per cent. of voids. Weight of broken stone per cubic foot $= 2.9 \times 62.3 \times 0.55 = 99.37$ lbs.

VOLUME AND WEIGHT OF MATERIALS

	Volume, Cubic Feet	Weight per Cubic Foot, Pounds	Weight, Pounds
One barrel of Portland cement	4	94	376
Three barrels of sand	12	89	1068
Six barrels of broken trap rock	24	99.37	2385
Total weight of dry materials	.....	.....	3829

Volume of concrete from Eq. (3).

$$N = 2.61 + 0.724S + 0.594G.$$

	Volume, Cubic Feet
Cement	2.61
Sand, 0.724×12	8.688
Broken stone, 0.594×24	14.256
Volume of concrete per barrel of cement $= N =$	25.554

The weight per cubic foot is found from Eq. (1),

$$\frac{W}{N} = \frac{3829}{25.554} = 149.8 \text{ lbs.}$$

Add water, $13 - (4 + 2)$ = 7.0

Probable weight per cubic foot in the finished counterweight = 156.8 lbs.

2. Concrete loaded with steel punchings.—It is sometimes necessary to use counterweights of greater weight per cubic foot than can be made by using broken stone, or gravel, as the aggregate. It is then best and most economical to add clean steel punchings to the mixture, to give it increased weight.

Problem 2.—Find the proportions of a concrete containing clean steel punchings, to produce an assumed weight per cubic foot in the finished counterweight.

Let M = required weight per cubic foot;

V_p = percentage of voids of the punchings; and

x = volume of punchings required for N cubic feet of concrete.

Use all other notations as given for Problem 1.

By analogy, from Eq. (3),

$$N = 2.61 + 0.724S + 1.08(1 - V_g)G + 1.08(1 - V_p)x. \quad . \quad (4)$$

Equation (1) becomes,

$$\frac{W}{N} + 7 = M. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If the proportions of cement, sand, and stone or gravel be assumed, the value of x can readily be found from Eqs. (4) and (5).

The weight of clean steel punchings, shaken in a rectangular box, varies from 275 lbs. per cubic foot, with 43.87 per cent. of voids, to 285 lbs. per cubic foot, with 41.83 per cent. of voids. A fair average value is 280 lbs. per cubic foot with 42.85 per cent of voids.

Experiments with clean steel punchings, rammed in 6-in. layers and grouted with a mortar of 1 part of Portland cement and 2 parts of sand, gave weights of from 315 to 320 lbs. per cubic foot. These results show that the mortar did not fill all of the voids.

Note that these results were obtained with clean steel punchings. When the available punchings are mixed with other material, a test should be made of the actual weight and the actual voids, and these results used in all of the calculations.

When a maximum weight is required and grouted punchings are used, no dependence should be placed on the strength of the mixture as concrete.

Example 2.—Let it be required to proportion a concrete containing clean steel punchings, so that it will weigh 250 lbs. per cubic foot in the finished counterweight. The value of M is 250. From Eq. (5),

$$\frac{W}{N} = 250 - 7, \quad \text{or} \quad \frac{W}{N} = 243. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5a)$$

Assume a mixture in the proportion of 1 part of Portland cement, 3 parts of sand, 2 parts of broken trap rock, and x parts of steel punchings.

Assume the dry sand to weigh 89 lbs. per cubic foot.

Assume trap rock, screened to $\frac{3}{8}$ in. to $1\frac{1}{2}$ in. in size, of a specific gravity of 2.9, with 45 per cent of voids. Weight of broken stone per cubic foot $= 2.9 \times 62.3 \times 0.55 = 99.37$ lbs.

Assume clean steel punchings to weigh 280 lbs. per cubic foot, with 42.85 per cent of voids.

VOLUME AND WEIGHT OF MATERIALS

	Volume, Cubic Feet	Weight per Cubic Foot, Pounds	Weight, Pounds	Weight, Pounds
One barrel of Portland cement	4	94	376	
Three barrels of sand	12	89	1068	
Two barrels of broken stone	8	99.37	795	
x barrels of punchings	$4x$	280		$1120x$
Total weight of dry materials $= W =$			$2339 +$	$1120x$

Using the data above in Eq. (4),

$$N = 2.61 + 0.724S + 0.594G + 0.617x. \quad . \quad . \quad . \quad (4a)$$

Volume of concrete from Eq. (4*a*).

	Volume, Cu. Ft.	
Cement	2.61	
Sand, 0.724×12	8.688	
Broken stone, 0.594×8	4.752	
Punchings, $0.617 \times 4x$	.....	$2.468x$
Total volume of concrete per barrel of cement $= N =$	$16.040 +$	$2.468x$

Substitute in Eq. (5*a*),

$$\frac{W}{N} = \frac{2239 + 1120x}{16.040 + 2.468x} = 243.$$

Solve this for the value of x.

$$x = 3.184.$$

The required proportions then become: 1 part of Portland cement, 3 parts of sand, 2 parts of broken trap rock, and 3.184 parts of clean steel punchings.

3. Pig-iron counterweights.—Experiments show that carefully piled sand-cast pig iron, without sows, weighs about 308 lbs. per cubic foot. The voids then amount to 31.55 per cent, taking the weight of cast iron at 450 lbs. per cubic foot.

If it were possible to fill all of the voids with a mortar grout of 1 part of cement and 2 parts of average sand, the resulting total weight per cubic foot would be 349 lbs. Probably not over two-thirds of the voids would be filled, in which case the total weight per cubic foot would be 335 lbs.

Experiments show that carefully piled machine-cast pig iron weighs from 270 to 290 lbs. per cubic foot. The voids are from 39.99 to 35.55 per cent. If all voids were filled with a mortar grout, of 1 part of cement and 2 parts of average sand, the total weight per cubic foot would be from 322 to 336 lbs. With two-thirds of the voids filled, the weight per cubic foot would be from 305 to 321 lbs.

A grouted pile of pig iron possesses little strength as a mass.

Comparing the weight of grouted pig iron with that of grouted steel punchings, it is evident that the grouted punchings are the cheaper, for the market price of steel scrap is nearly always less than that of pig iron.

CHAPTER XIII

OPERATORS' HOUSES

1. General.—The operator's house should be located at some point on the bridge where the operator may have an unobstructed view of the navigable channel and the ends of the span. The best location on a swing bridge is usually above, or near the top, of the trusses, and directly over the center pier.

In a bascule it will be found expedient to place the house on an extension of the main trunnion pier or on brackets outside of the counterweight span. In some designs it is possible to put it in the counterweight span above the traffic clearance, but in most cases the operator's view is then cut off when the bridge is open, and obstructed by transverse and lateral bracing when it is closed. The best position should be carefully studied when laying out the structure.

When a vertical lift is used, the house may be put on top of the lift span, at its center, or on one of the towers, depending on the character of the design. Sometimes it is convenient to put it on an extension of the front tower pier or on brackets attached to the tower itself. Here again the nature of the design is the determining consideration.

The house should be liberally fitted with large windows, located to give the operator the most direct view of the channel and the ends of the span. It is often desirable to glaze the upper part of the door. Sometimes only the control apparatus is put in the house, the motors being placed on the span, near the points of application of the power. In other structures, the motors may be in the house. The latter is the common arrangement when steam or internal combustion engines supply the power.

A few examples will be given.

2. French Canal bridge.—Figure 13A shows the arrangement that was adopted on the railway bridge crossing the old French Canal, at Cristobal, Canal Zone. The house is at the top of the middle panel of the span. Separate motors were supplied for turning the span and driving the wedges. The interlocking and signal control was also in the same house. The drum-type controllers are near the longitudinal center line of the bridge, with a window in front of the operator, another

behind him, and others on either side. The brake on the turning train is controlled by a pedal near his station. The switchboard is set well away from the wall, to give easy access to the wiring on its back. The front is readily seen.

This house is of reinforced concrete on a concrete platform. The roof, also of concrete, has wide overhanging ends and sides, on account of the climatic conditions. A footwalk, protected by railings, extends around the house, and a stairway gives access to one corner.

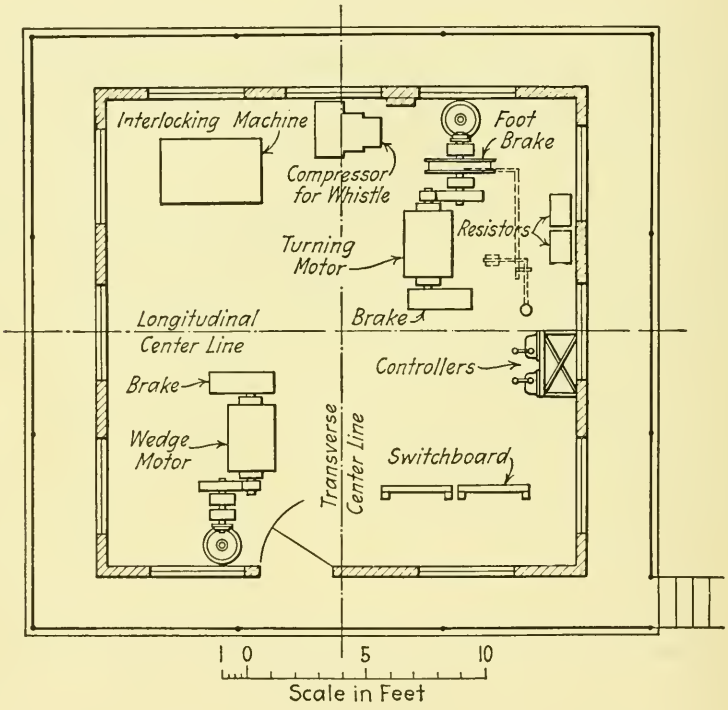

Fig. 13A.

3. Dumbarton bridge.—Figure 13B shows the general arrangement of the house on the Dumbarton bridge across San Francisco Bay, on the line of the Southern Pacific Company. It is at the top of the trusses in the middle panel. The primary source of power is an 80-h.p., 3-cylinder, 4-cycle, gasoline engine, which is on the longitudinal center line of the bridge. As the engine is non-reversible and drives both the turning and wedging gear trains, a self-contained transmission mechanism was placed in front of it. This provides for a forward and reverse drive for both turning and wedging, also for a brake on the turning

train. Provision was made for the installation of an electric motor whenever current becomes available. The motor would drive the two gear trains through the engine transmission machine. All control levers and the brake pedal are brought to the operator's station. A compressed-air self-starter was fitted to the engine. The air tanks for this and the whistle are in one corner of the house, and the cooling-

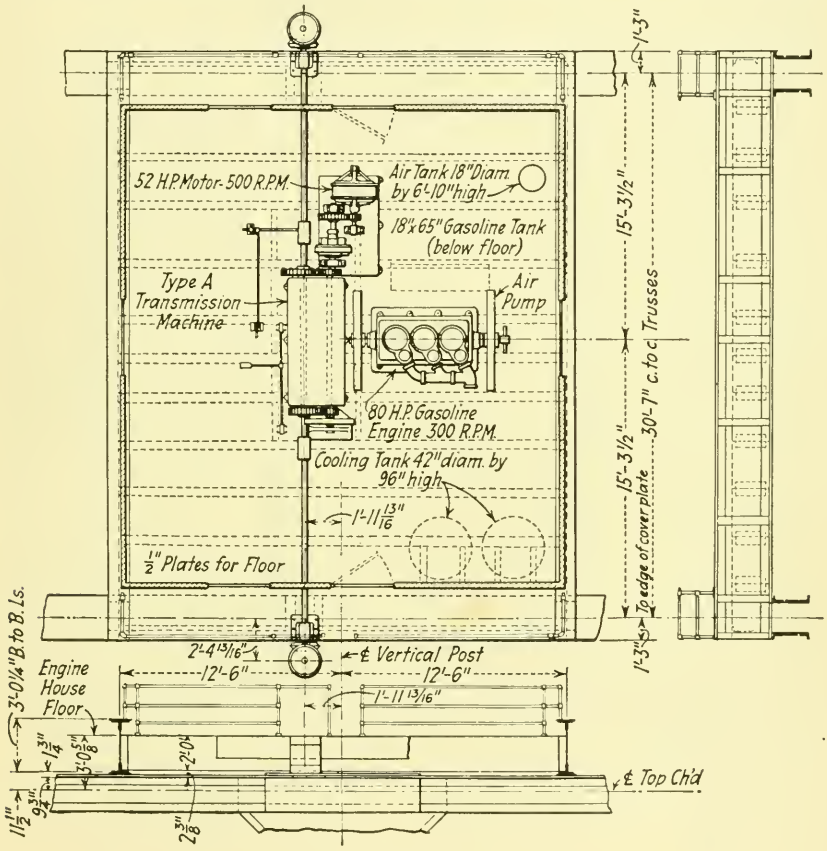

FIG. 13B.

water tank is in another. The gasoline tank is under the floor and the engine is fed by a small pump.

This house is of wood, built upon a steel plate floor. A footwalk, with railings, extends across each end, and a ladder gives access to one of them.

4. Middletown bridge.—Figure 13C shows a house used on the Middletown bridge of the New York, New Haven, and Hartford Rail-

road. It contains only the control mechanism of an electrically driven bridge. The floor is of concrete on I-beams. The frame is built of steel angles, riveted to the supporting steelwork. The outside and

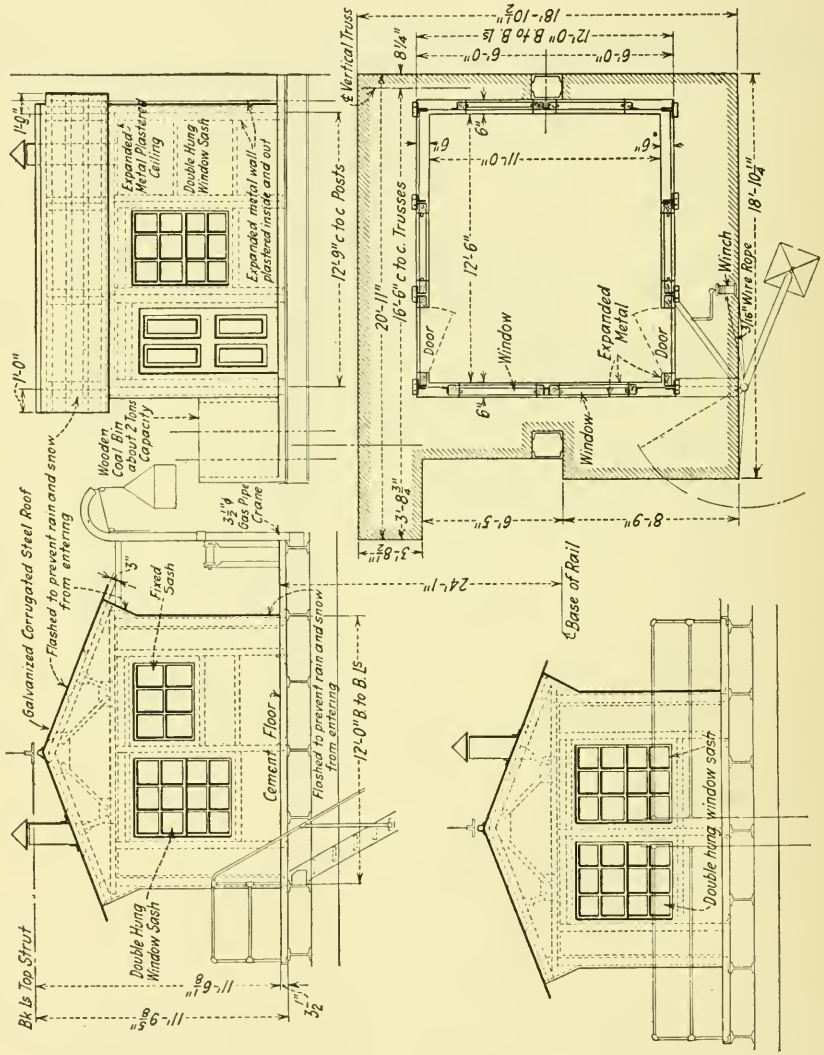

FIG. 13C.

inside are covered with expanded metal, nailed to wooden furring strips bolted to the steel frame, and plastered with cement mortar. The roof is of galvanized corrugated sheet steel flashed to the cement stucco. The ceiling is flat and finished with expanded metal plastered with cement mortar.

A crane, winch, and bucket provide for hoisting coal for the heating stove. The outside platforms are of liberal size and their edges are

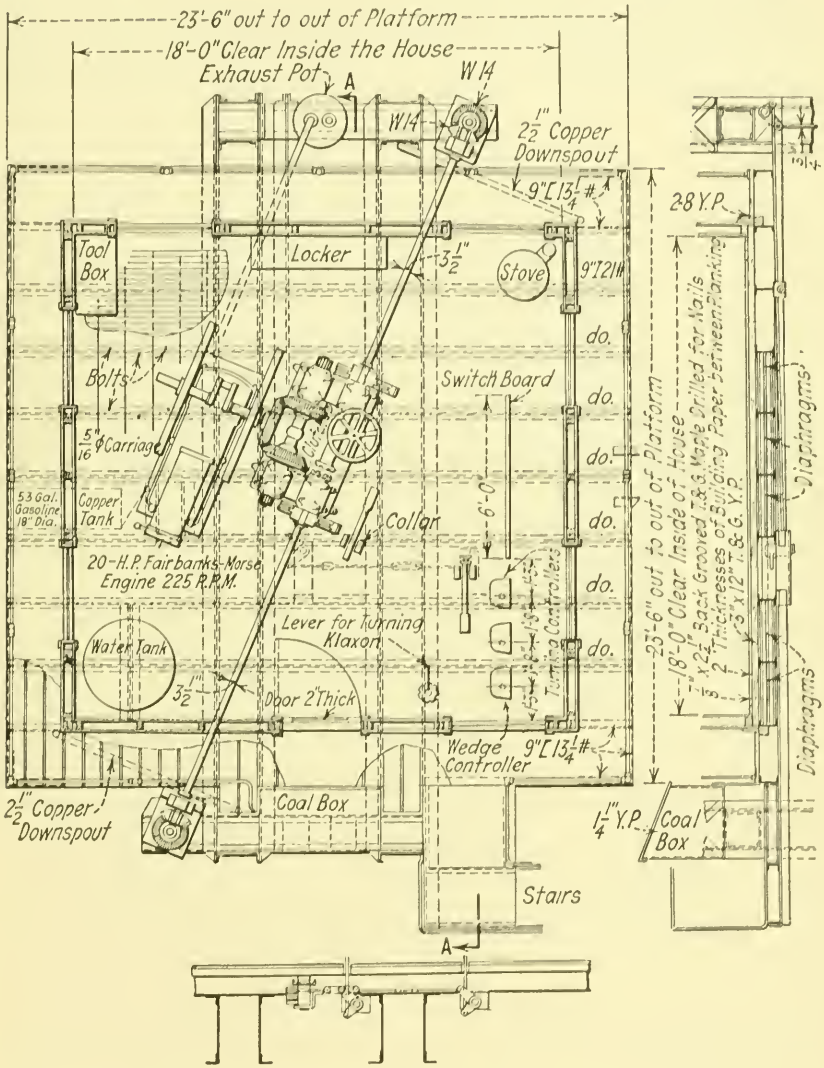

FIG. 13D.

protected by railings. A stairway at one corner affords access to the platform.

5. Congress Street bridge.—Figure 13D shows the arrangement of the control mechanism and gasoline auxiliary engine on the Congress

Street bridge, at Troy, N. Y. The turning and wedging motors are on the steelwork under the bridge floor. The gasoline engine has a single, 4-cycle cylinder, which requires a transmission mechanism similar to that used at Dumbarton, but much smaller, as the engine is only 20-h.p., while that at Dumbarton is 80-h.p. The two 52-h.p. turning motors each have a drum-type controller, and the 20-h.p. wedging

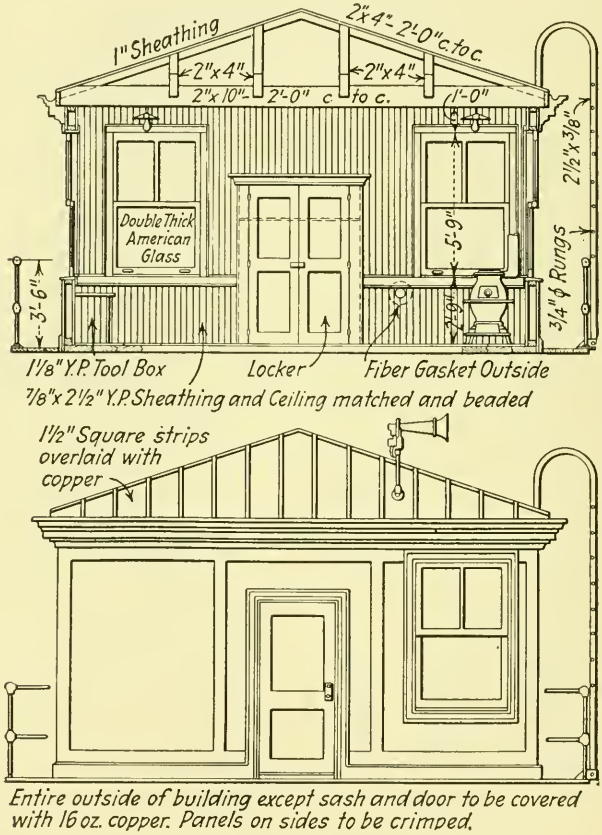

Fig. 13E.

motor is separately controlled. The controllers are in one corner of the house, adjacent to the large windows in front and at the sides of the operator's station. A brake on the turning train is actuated by a pedal adjacent to the controllers. The switchboard is set back from the wall, so that easy access may be had to its back.

The floor of the house is of $\frac{7}{8}$-in. maple laid on 3-in. yellow pine planks, with building paper between, and is supported by I-beams in the

middle panel of the span. The frame is of wood, sheathed outside and inside, as shown in Fig. 13E. The entire outside and the roof are covered with 16-oz. copper, with a gutter, cornice, and down-spouts for drainage of storm water. Foot walks extend around the house, the edges being protected by railings. A steel staircase gives access to the house from the bridge floor below. A steel ladder extends from one of the platforms to the roof.

APPENDICES

APPENDIX A

ANALYSIS OF STRESSES IN DISCS OF CENTER-BEARING SWING SPANS

1. General.—The contact surfaces of the discs on which center-bearing swing spans turn are usually made as spherical segments. As stated in Chapter X, Arts. 13 and 14, the author recommends the use of but two discs, the lower of tempered steel concave upward, and the upper of hard bronze convex downward.

In order to reduce the turning friction, it is best to make the radius of the upper convex spherical segment smaller than that of the lower concave segment. Thus, the effective radius of friction is reduced, but the bearing pressures are increased at the center of the discs.

It is necessary to know the law of the variation of the pressures between the discs, in order to determine the proper difference between the radii of the two discs, and to make sure that no excessive pressure is caused at the center of the discs.

2. Outline of the problem.—Figure AA is a section through a swing-span bridge center, taken transversely with respect to the center line of the bridge.

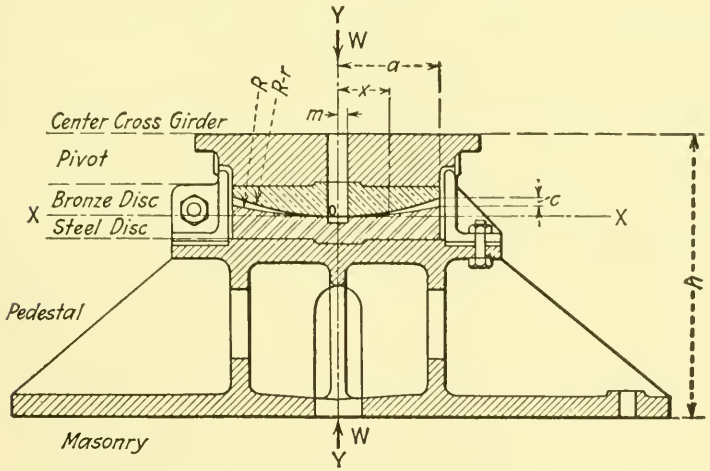

FIG. AA.

(1) *Nomenclature.*

Let R = radius of the lower, or steel, disc surface;
r = difference in radius of steel and bronze disc surfaces;
$R - r$ = the radius of the bronze disc surface;
a = the radius of the discs;
x = the radius of any point on the disc surfaces;

c = the clearance between the disc surfaces at the edge of the discs;
O be the origin of coordinates = the center of the discs;
b = the ordinate at the edge of the steel disc when no load is carried;
$b + c$ = the ordinate at the edge of the bronze disc when no load is carried;
y = the clearance between the discs at radius x when no load is carried;
h = the height of the center from masonry to cross girders;
E = the average modulus of elasticity of the entire center, of height, h;
S = the unit stress at the center of the disc, or at the edge of the center hole, when the full turning load is carried by the center;
S_y = the unit stress on an element distant x from the center;
W = any turning load on the center;
e = the compression of the center element, or the element at the edge of the center hole, when load W is carried;
$e - y$ = the compression of any element distant x from the center of the discs when load W is carried;
L = the lever arm of friction; and
m = the radius of the central hole.

(2) *Form of Disc Surfaces.*

Discs with spherical surfaces.

The ordinate of any point on the steel disc surface,

$$y_s = R - \sqrt{R^2 - x^2}.$$

The ordinate of any point on the bronze disc surface,

$$y_B = (R - r) - \sqrt{(R - r)^2 - x^2}.$$

The clearance between the discs is,

$$y = \sqrt{R^2 - x^2} - \sqrt{(R - r)^2 - x^2} - r. \quad . \quad . \quad . \quad . \quad (1)$$

Discs with parabolic surfaces.

The ordinate of any point on the bronze disc surface is,

$$y_B = \frac{b + c}{a^2} x^2$$

The ordinate of any point on the steel disc surface is,

$$y_s = \frac{b}{a^2} x^2$$

The clearance between the discs is,

$$y = \frac{c}{a^2} x^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Discs are made with spherical contact surfaces.

Calculations have been made of the clearances between spherical discs, of dimensions used in practice, and these have been compared with those between parabolic discs of the same size and the same edge ordinates and clearances as the corresponding spherical discs.

The differences between the spherical and parabolic surfaces are no more than the errors that are likely to occur in finishing the discs with ordinary machine tools; hence, for simplicity, parabolic surfaces will be assumed in this analysis.

(3) *Approximations.*—It will be assumed that the center girders, resting upon the upper pivot casting, and the masonry supporting the lower pedestal do not deflect under loads within the limits of action of the discs.

As the edge clearance, c, between the discs is small when compared with h, it will be assumed that the height h, subject to compression, is constant for varying values of the radius x.

It will be assumed that an average value of E may be found for the actual combinations of metals and forms of the parts of the center which make up the height of the center, h.

It will be assumed that the same value of E applies to any vertical element within the compressed area of the discs.

(4) *Analysis.*—When the discs carry no load they will be in contact at a point, if there is no central oil hole, or on a line at the edge of the hole, when an oil hole is used.

When load is applied to the center, from the center girders, the pivot, discs, and pedestal all compress and a circular area of contact is developed between the discs, which increases as load is added.

The law of the increase in this contact area and the resulting unit pressures on the discs are to be found.

(3) Solution for discs with no central hole.

From the common theory of elasticity,

$$c = \frac{Sh}{E}. \qquad (3)$$

The unit stresses are proportional to resulting distortions, hence,

$$S_y = S\frac{e - y}{e}.$$

Substitute the value of c from (3) and

$$S_y = S - \frac{Ey}{h}. \qquad (4)$$

From (2), $y = \frac{cx^2}{a^2}$, hence,

$$S_y = S - \frac{Ecx^2}{a^2h}. \qquad (5)$$

The unit stress, S_y, acts over an area of $2\pi x dx$.

The sum of all stresses, S_y, within the compressed area must equal the applied load W, then,

$$2\pi \int S_y x dx = W.$$

Substitute the value of S_y from (5), and

$$2\pi \int \left(S - \frac{Ecx^2}{a^2h}\right) x dx = W. \qquad (6)$$

$$2\pi \left(\frac{Sx^2}{2} - \frac{Ecx^4}{4a^2h}\right) + K = W.$$

When $x = 0$, the area taking load is 0, hence $W = 0$ and therefore $K = 0$.

$$2\pi\left(\frac{Sx^2}{2} - \frac{Ecx^4}{4a^2h}\right) = W. \quad \ldots\ldots\ldots \quad (7)$$

Solve (7) for value of S.

$$S = \frac{W}{\pi x^2} + \frac{Ecx^2}{2a^2h}. \quad \ldots\ldots\ldots \quad (8)$$

From (8),

$$S^2 = \frac{W^2}{\pi^2 x^4} + \frac{WEc}{\pi a^2 h} + \frac{E^2c^2x^4}{4a^4h^2}. \quad \ldots\ldots\ldots \quad (9)$$

In (8) the values of S and x are unknown, hence another equation must be found. This can be done from the condition that the total external work of the weight W, in compressing the center, must equal the internal work of compression in the center.

External Work.

Let W = the load that has been applied to the center at any epoch;

c = the resulting compression of the center element.

The differential work is

$$d\ (\text{Work}_{\text{Ext.}}) = dW \cdot dc. \quad \ldots\ldots\ldots \quad (10)$$

From (6)

$$dW = 2\pi\left(S - \frac{Ecx^2}{a^2h}\right)xdx = \left(2\pi S - \frac{2\pi Ecx^2}{a^2h}\right)xdx. \quad \ldots \quad (11)$$

Substitute the value of S from (8) in (11).

$$dW = \left(\frac{2W}{x^2} - \frac{\pi Ecx^2}{a^2h}\right)xdx. \quad \ldots\ldots\ldots \quad (12)$$

From (3), $c = \dfrac{Sh}{E}$. Then

$$dc = d\left(\frac{Sh}{E}\right). \quad \ldots\ldots\ldots \quad (13)$$

Substitute the value of S from (8) in (13).

$$dc = d\left(\frac{Wh}{\pi Ex^2} + \frac{cx^2}{2a^2}\right), \quad \ldots\ldots\ldots \quad (13a)$$

hence,

$$dc = \left(\frac{c}{a^2} - \frac{2Wh}{\pi Ex^4}\right)xdx. \quad \ldots\ldots\ldots \quad (14)$$

Substitute the values of (12) and (14) in (10).

$$d(\text{Work}_{\text{Ext.}}) = \left(\frac{2W}{x^2} - \frac{\pi Ecx^2}{a^2h}\right)\left(\frac{c}{a^2} - \frac{2Wh}{\pi Ex^4}\right)x^2dx^2,$$

or

$$d(\text{Work}_{\text{Ext.}}) = \left(\frac{4Wc}{a^2x^2} - \frac{4W^2h}{\pi Ex^6} - \frac{\pi Ec^2x^2}{a^4h}\right)x^2dx^2. \quad \ldots \quad (15)$$

Internal Work.

Let S_y = the unit stress on any compressed elemental area.

The stress increases from 0 to S_y at any epoch; hence the average stress is $\frac{S_y}{2}$.

The compression of any element due to stress S_y is $S_y \frac{h}{E}$.

The internal work of the element then is

$$\frac{S_y}{2} \times \frac{S_y h}{E} = \frac{S_y^2 h}{2E}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Substitute the value of S_y from (5) in (16).

$$\text{Internal Work} = \frac{h}{2E}\left(S - \frac{Ecx^2}{a^2 h}\right)^2 = \frac{S^2 h}{2E} - \frac{Scx^2}{a^2} + \frac{Ec^2x^4}{2a^4 h}. \quad . \quad . \quad . \quad (17)$$

Substitute the value of S from (8) and S^2 from (9) in (17).

$$(\text{Work}_{\text{Int.}}) = \frac{h}{2E}\left(\frac{W^2}{\pi^2 x^4} + \frac{WEc}{\pi a^2 h} + \frac{E^2c^2x^4}{4a^4 h}\right) - \frac{cx^2}{a^2}\left(\frac{W}{\pi x^2} + \frac{Ecx^2}{2a^2 h}\right) + \frac{Ec^2x^4}{2a^4 h}.$$

This reduces to,

$$(\text{Work}_{\text{Int.}}) = \frac{W^2 h}{2\pi^2 E x^4} - \frac{Wc}{2\pi a^2} + \frac{Ec^2x^4}{8a^4 h}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

The internal work of a circular element is the internal work of an element $\times$ $2\pi x dx$, hence:

$$d(\text{Work}_{\text{Int.}}) = 2\pi\left(\frac{W^2 h}{2\pi^2 E x^4} - \frac{Wc}{2\pi a^2} + \frac{Ec^2x^4}{8a^4 h}\right)x dx,$$

or

$$d(\text{Work}_{\text{Int.}}) = \left(\frac{W^2 h}{\pi E x^4} - \frac{Wc}{a^2} + \frac{\pi Ec^2x^4}{4a^4 h}\right)x dx. \quad . \quad . \quad . \quad . \quad (19)$$

Equating (15) and (19), dividing through by $x dx$ and expressing the integration,

$$\int\left(\frac{4Wc}{a^2x^2} - \frac{4W^2 h}{\pi E x^6} - \frac{\pi Ec^2x^2}{a^4 h}\right)x dx = \frac{W^2 h}{\pi E x^4} - \frac{Wc}{a^2} + \frac{\pi Ec^2x^4}{4a^4 h}. \quad . \quad . \quad (20)$$

$$\frac{4Wc}{a^2}\log_e x + \frac{W^2 h}{\pi E x^4} - \frac{\pi Ec^2x^4}{4a^4 h} + K = \frac{W^2 h}{\pi E x^4} - \frac{Wc}{a^2} + \frac{\pi Ec^2x^4}{4a^4 h},$$

or

$$\frac{Wc}{a^2}(4\log_e x) + K = \frac{\pi Ec^2x^4}{2a^4 h} - \frac{Wc}{a^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

When $x = 0$, area is 0, hence $W = 0$, or

$$\frac{o \cdot c}{a^2}(4\log_e x) + K = 0.$$

The term $\frac{o \cdot c}{a^2}(4\log_e x)$ is indeterminate.

The only value of K which will satisfy this expression will be $-\frac{Wc}{a^2}(4\log_e x)$.

Substitute this value of K in (21).

$$\frac{Wc}{a^2}(4\log_e x) - \frac{Wc}{a^2}(4\log_e x) = \frac{\pi Ec^2x^4}{2a^4h} - \frac{Wc}{a^2},$$

or

$$\frac{\pi Ec^2x^4}{2a^4h} = \frac{Wc}{a^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Solve this for x.

$$x = \sqrt[4]{\frac{2Wa^2h}{\pi Ec}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

Substitute the value of x from (23) in (8).

$$S = \pi\frac{W}{\sqrt{\frac{2Wa^2h}{\pi Ec}}} + Ec\frac{\sqrt{\frac{2Wa^2h}{\pi Ec}}}{2a^2h}.$$

$$S = \sqrt{\frac{WEc}{2\pi^2h}} + \sqrt{\frac{WEc}{2\pi a^2h}} = 2\sqrt{\frac{WEc}{2\pi a^2h}},$$

therefore,

$$S = \sqrt{\frac{2WEc}{\pi a^2h}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

From (24) is found,

$$c = \frac{\pi S^2a^2h}{2WE}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

To find the value of r when R, a, and c are known: Resume equation (1), making $y = c$ and $x = a$.

$$c = \sqrt{R^2 - a^2} - \sqrt{(R - r)^2 - a^2} - r, \quad . \quad . \quad . \quad . \quad . \quad (25a)$$

or

$$c = \sqrt{R^2 - a^2} + r = -\sqrt{(R - r)^2 - a^2}.$$

Let

$$c - \sqrt{R^2 - a^2} = d;$$

then

$$-d - r = \sqrt{(R - r)^2 - a^2}.$$

$$d^2 + 2dr + r^2 = R^2 - 2Rr + r^2 - a^2.$$

$$2r(R + d) = R^2 - d^2 - a^2.$$

$$r = \frac{R^2 - d^2 - a^2}{2(R + d)}.$$

Restore the value of d, and

$$r = \frac{R^2 - c^2 + 2c\sqrt{R^2 - a^2} - R^2 + a^2 - a^2}{2(R + c - \sqrt{R^2 - a^2})}.$$

Therefore,

$$r = c \cdot \frac{2\sqrt{R^2 - a^2} - c}{2(R + c - \sqrt{R^2 - a^2})}. \quad \ldots\ldots\ldots \quad (26)$$

The lever arm of friction is found as follows:

If each element of weight carried by the disc is multiplied by its lever arm the sum of all such products is a moment proportional to the total moment of friction. If this moment is divided by W the quotient is the lever arm of friction L. The differential values of W are given in Eq. (6) and x is the lever arm of any elemental pressure; then,

$$2\pi \int \left(S - \frac{Ecx^2}{a^2h}\right) x^2 dx = W \cdot L. \quad \ldots\ldots\ldots \quad (27)$$

$$2\pi \left[\frac{Sx^3}{3} - \frac{Ecx^5}{5a^2h}\right] = W \cdot L.$$

(For when $x = 0$, $W = 0$, hence $K = 0$), hence,

$$L = \frac{2\pi \left[\frac{Sx^3}{3} - \frac{Ecx^5}{5a^2h}\right]}{W}. \quad \ldots\ldots\ldots \quad (28)$$

(4) **Solution for Discs with Central hole.**—All data will be the same as in the analysis with no central hole.

From (2) we may write $y_1 = \frac{cx^2}{a^2}$ which expresses any clearance between discs with no central hole.

With a central hole, when the center is without load, the upper disc and pivot will be lowered a small amount for contact at the circumference of the hole, and all clearances between the discs will be reduced by the amount lowered.

The amount of the lowering is $y_2 = \frac{cm^2}{a^2}$.

The net clearance at any point then is,

$$y_1 - y_2 = y = \frac{c}{a^2}(x^2 - m^2) \quad \ldots\ldots\ldots \quad (29)$$

y_2 is very small when compared with h; hence the original value of h will be used.

Substitute the value $(x^2 - m^2)$ for x^2 in (6),

$$2\pi \int \left[S - \frac{Ec}{a^2h}(x^2 - m^2)\right] x dx = W. \quad \ldots\ldots\ldots \quad (30)$$

By proceeding with the solution in the same manner as in Art. 3 the expression for the value of x takes the form:

$$(x^2 - m^2) = \sqrt{\frac{2Wa^2h}{\pi Ec}}. \quad \ldots\ldots\ldots \quad (31)$$

This is in the same form as (23), and may also be written,

$$x = \left[m^2 + \sqrt{\frac{2Wa^2h}{\pi Ec}} \right]^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

When m is small, it has very little effect on the value of x.

By proper substitutions it is found that:

The value of S is the same as in (24).
The value of c is the same as in (25).
The value of r is the same as in (26).

Find the radius of friction L.

Proceed in the same manner as in the development of equation (28). From (30),

$$2\pi \int \left[S - \frac{Ec}{a^2h}(x^2 - m^2) \right] x^2 dx = W \cdot L. \quad . \quad . \quad . \quad . \quad (33)$$

$$2\pi \left[\frac{S}{3}x^3 - \frac{Ec}{a^2h}\left(\frac{x^5}{5} - \frac{m^2x^3}{3} \right) \right] + K = WL. \quad . \quad . \quad . \quad . \quad . \quad (34)$$

When $x = m$, area of contact $= 0$, hence, $W = 0$, and K may be found.

$$K = -2\pi \left[\frac{Sm^3}{3} + \frac{Ec}{a^2h}\left(\frac{2m^5}{15} \right) \right].$$

Substitute in (34) and reduce.

$$2\pi \left[\frac{S}{3}(x^3 - m^3) - \frac{Ec}{a^2h}\left(\frac{x^5}{5} - \frac{m^2x^3}{3} + \frac{2m^5}{15} \right) \right] = WL. \quad . \quad . \quad . \quad (35)$$

The lever arm of friction then becomes,

$$L = \frac{2\pi \left[\frac{S}{3}(x^3 - m^3) - \frac{Ec}{a^2h}\left(\frac{x^5}{5} - \frac{m^2x^3}{3} + \frac{2m^5}{15} \right) \right]}{W} \quad . \quad . \quad . \quad . \quad (36)$$

When m is a fraction this may be written approximately:

$$L = \frac{2\pi \left[\frac{Sx^3}{3} - \frac{Ec}{a^2h}\left(\frac{x^5}{5} - \frac{m^2x^3}{3} \right) \right]}{W} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

(5) **Example.**—A study of a large group of centers with three discs, which were known to have operated successfully, indicates that a unit stress of 10,000 pounds per square inch at the edge of the oil hole is conservative.

A study of the two-disc designs given in Chapter X, Art. 14, with discs from 12 up to 52 in. in diameter, indicates that the radii of the steel disc surfaces were well selected.

The radii of the bronze discs were calculated so that the maximum unit stress at the edge of the oil hole is 10,000 lbs. per square inch.

The radii of the bronze discs were fixed to the nearest quarter of an inch from the theoretical radii, and the maximum unit stresses, edge clearances, and radius of friction under full turning loads were calculated.

A sample calculation will be given.

Data for center with 30 in. discs.

Diameter of discs,		30 in.
Radius of discs,	$a =$	15 in.
Radius of oil hole,	$m =$	0.5 in.
Radius of upper surface of steel disc,	$R = 8$ ft. $=$	96 in.
Height of center,	$h = 3$ ft. $=$	36 in.
Capacity of center when turning,	$W =$	1,980,000 lbs.
Maximum unit stress at edge of oil hole,	$S =$	10,000 lbs. per sq. in.
The approximate average value of E,	$E =$	17,240,000

The pivot is a solid steel casting.

The thicknesses of the bronze and steel disc will be taken at about the point of application of their frictional lever arm.

The pedestal is a cored steel casting. It will be assumed that the metal compressed includes the cylindrical body of the casting and the radial ribs inside the cylinder. No deductions will be made for core holes in the walls of the cylinder, nor the lower part of the central radial ribs. It is assumed that the outside ribs will be brought into action enough to make up for the holes not deducted. The height of the resulting cylinder and radials will be taken as the full height of the pedestal. The top and bottom plates between the cylindrical shell and the radials will be considered inactive, for they can readily deflect away from the loads.

	E	h	$E \times h$
Pivot	29,000,000 $\times$	8.5	= 246,500,000
Bronze disc	10,000,000 $\times$	3.05	= 30,500,000
Steel disc	29,000,000 $\times$	3.45	= 100,050,000

Pedestal

Area, 30 in. d. $=$	706.86 sq. in.
Area, 264 in. d. $=$	$-$ 541.10
Area of cylinder,	165.76 sq. in.
$5 \times 12.1875 \times 1.875$ $=$	$+$ 114.25
Center, 1.875 in., $d =$	$+$ 2.76
Area compressed $=$	282.77

$$\text{Equivalent value of } E = 29{,}000{,}000 \times \frac{282.77}{706.86} = 11{,}600{,}000 \times 21 = 243{,}600{,}000$$

$$\text{Average value of } E, \qquad 17{,}240{,}000 \times 36 = 620{,}650{,}000$$

From equation (25)

$$c = \frac{\pi S^2 a^2 h}{2WE}.$$

Substitute above values.

$$c = \frac{\pi \times \overline{10{,}000}^2 \times \overline{15}^2 \times 36}{2 \times 1{,}980{,}000 \times 17{,}240{,}000} = 0.03727 \text{ in.}$$

From equation (26),

$$r = c \cdot \frac{2\sqrt{R^2 - a^2} - c}{2(R + c - \sqrt{R^2 - a^2}}.$$

Substitute above values.

$$r = 0.03727 \cdot \frac{2\sqrt{96^2 - 15^2} - 0.03727}{2(96 + 0.03727) - \sqrt{96^2 - 15^2}}$$

$$= 0.03727 \frac{189.6418}{2.43278} = 2.905 \text{ in.}$$

Use a value of $r = 3$ in., hence,

$$R - r = 96 - 3 = 93 \text{ in.}, = \text{radius of bronze disc bearing surface.}$$

Find edge clearance for these radii from equation (25a).

$$c = \sqrt{R^2 - a^2} - \sqrt{(R - r)^2 - a^2} - r.$$

Substitute values above.

$$c = \sqrt{96^2 - 15^2} - \sqrt{93^2 - 15^2} - 93 = 0.038533 \text{ in.}$$

Find maximum unit stress from equation (24):

$$S = \sqrt{\frac{2WEC}{\pi a^2 h}}.$$

Substitute above values.

$$S = \sqrt{\frac{2 \times 1{,}980{,}000 \times 17{,}240{,}000 \times 0.038533}{\pi \times 225 \times 36}}$$

$$= 10{,}167 \text{ lbs. per sq. in.}$$

Find the radius limiting the area of contact from equation (31).

$$x^2 - m^2 = \sqrt{\frac{2Wa^2h}{\pi Ec}}.$$

Substitute the values above.

$$x^2 - 0.5^2 = \sqrt{\frac{2 \times 1{,}980{,}000 \times 15^2 \times 36}{\pi \times 17{,}240{,}000 \times 0.038533}} = 11.145 \text{ in.}$$

Find the lever arm of friction from equation (37).

$$L = \frac{2\pi\left[\frac{Sx^3}{3} - \frac{Ec}{a^2h}\left(\frac{x^5}{5} - \frac{m^2x^3}{3}\right)\right]}{W}.$$

Substitute the values above.

$$L = \frac{2\pi\left[\frac{10{,}167 \times \overline{11.145}^3}{3} - \frac{17{,}240{,}000 \times 0.038533}{15^2 \times 36}\left(\frac{\overline{11.145}^5}{5} - \frac{\overline{0.5}^2 \times \overline{11.145}^3}{3}\right)\right]}{1{,}980{,}000}$$

$$L = \frac{2\pi[4{,}682{,}273 - 82.01345(34{,}389.88 - 230.72201)]}{1{,}980{,}000}$$

$$L = \frac{2\pi(4{,}682{,}273 - 2{,}801{,}510)}{1{,}980{,}000} = \frac{2\pi \times 1{,}880{,}763}{1{,}980{,}000} = 5.967 \text{ in.}$$

APPENDIX B

DESIGN OF TREADS FOR THE SUPPORTING AND SEGMENTAL GIRDERS OF ROLLING-LIFT BRIDGES

1. General.—Rolling-lift bridges have several advantages, particularly when the spans are not too long and the resulting rolling loads are not too great. Except for the rolling segmental girders, the superstructure is of simple construction, and has no mechanical joints or trunnions. The rolling friction is small and the operating machinery is simple. Rolling-lift bridges require less power than trunnion bascules, and the machinery is correspondingly lighter and more economical.

The treads on the horizontal supporting girders and the rolling segmental girders of many rolling-lift bridges have been made too thin and too weak to distribute the concentrated rolling loads to the girders. The result has been that the tread plates have been stressed far beyond the elastic limit of the material, and have elongated, widened, and bent away from the central planes of contact. Rivets and bolts attaching them to the girders have been sheared and girder flanges and stiffeners wrecked by the intensity of the undistributed rolling loads. These failures have been so serious that extensive repairs have been made on a number of bridges.

For many years, the author has strongly advocated the use of much thicker tread plates, and engineers who have devised methods of making repairs have increased the depth and strength of the treads as much as was feasible under the limiting conditions imposed by other details of the designs. The author believes that the weakness of the tread plates, in the older designs, is the principal reason why the rolling-lift type of bridge has not been more favorably considered during recent years. One difficulty has been the inadequate knowledge of the laws of the distribution of heavy loads on the line contact between the rolling segment and the horizontal track on which it rolls.

An elaborate series of tests to determine these laws is being made by Professor W. M. Wilson for the Engineering Experiment Station of the University of Illionois.[1] These tests have not been completed, but Professor Wilson has drawn the conclusion that the stress relation at the line of contact has been only a minor cause of the many failures that have occurred, and the author fully agrees with this conclusion. The major fault has been the weakness of the treads on the rolling segments and the track girders. The treads not only must be thick and strong enough to resist local distortions beyond the elastic limit of the material, but their strength and stiffness must be great enough to distribute the local concentrated load from the line of rolling contact to the girders to which they are attached.

2. Unit stress on line contacts.—The Engineering Experiment Station has kindly furnished the author with the results of the tests already made, and his

[1] Professor Wilson is working in consultation with a special committee of the American Railway Engineering Association, of which the author is a member. The results of the tests will be published in a forthcoming bulletin of the Engineering Experiment Station of the University of Illinois.

analysis of some of them, made on medium steel castings, indicates that the smallest load per inch of line contact which produces permanent set in thick tread plates from three to four inches wide on the line of contact, may be expressed by

$$P = 900\sqrt[4]{D^3}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In this expression:

P = the load per inch of line contact that produces set, in pounds and

D = the diameter of the rolling segment, in inches.

Some of the tests, in which the specimens were rolled when under load, seem to indicate little, if any, additional effect due to movement.

Probably no danger to the surfaces would be caused if the loads were limited to three-fourths of the minimum static load that produces permanent set, or

$$P = 675\sqrt[4]{D^3}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

These expressions must be considered as only tentative, pending the completion of the tests and their analysis, but will serve as a basis for a study of the design of treads to distribute the rolling loads to the girders and segments to which they are attached, which the author conceives to be the major problem.

3. Design of the treads.—When a rolling-lift bridge is relatively long and heavy, the radius of the rolling segment must be made proportionately large, which results in a supporting girder of considerable length. In such a track girder, the load concentrated at the line of rolling contact must be distributed so that by means of the direct support of the tread by the girder web, and the rivets attaching the vertical parts of the flange to the web, all of the stress is delivered to the girder within a reasonable distance along its length. In the case of a bridge of ordinary span, the treads usually become too long to be made as a single piece, and joints are necessary. When the rolling load is near a joint, or near the end of the tread, the tread must act practically as a cantilever on the side of the load adjacent to the joint, or to the end of the tread. The ideal distribution would be one in which the reciprocal pressures between the girder and the treads were constant per unit of the length through which the distribution of pressure was made.

The supporting girder will deflect downward as a simple supported beam when the bridge rolls along it. The cantilever end of the tread, near a joint or end, will tend to deflect upward as a cantilever uniformly loaded along its bottom. The laws of these two deflections are mutually incompatible, and no arrangement will give a perfectly uniform distribution, but an approximation within reasonable limits may be found.

The object of the following discussion is to propose a method of proportioning the treads that is founded on a rational theory that takes into account the elastic action of the supporting girder and the treads; and which enables the designer to make his calculations from data normally developed during the design of the supporting girder.

4. Radii of curvature of deflected beams.—Let Figure $B\ A$ represent a supporting girder of a rolling-lift bridge, carrying a concentrated load W, assumed to be uniformly distriubted along a length b by means of a thick, stiff tread resting on the girder.

In order that the load on the tread be uniformly distributed along the length b of the supporting girder $A\ B$, the radii of curvature of the deflected girder and the tread must be the same. This is not attainable. If the radii were made the same at a point e, one-fourth of b from the point of application of W, a

close approximation to the ideal condition would be made. The tread would have full contact at W, but, theoretically, would be slightly above the girder at other points in the length $\frac{b}{2}$. However, the load W would cause compression within the portion of the tread $f\,g$, so that instead of the cantilever end of the tread separating from the girder, the reciprocal pressures between the tread and the girder would become variable along the length $f\,g$.

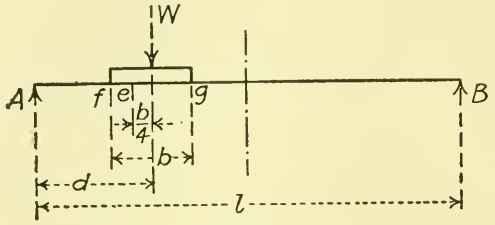

Fig. BA.

The radii of curvature of the girder at various points along its length are different and also vary when the load is moved along it. The stiffness of the tread cannot be made variable without serious complications and unwarranted expense, but should bear some relation to the stiffness of the girder. It is well known that the radius of curvature of any deflected beam, when the stresses are within the elastic limit, is closely expressed by

$$\frac{1}{R} = \frac{d^2y}{dx^2} = \frac{M}{EI}, \quad \text{or} \quad R = \frac{EI}{M}. \qquad (3)$$

Let W = rolling load in pounds;
l = length of the girder in inches;
b = length of the portion of the tread necessary to distribute the rolling load to the web of the girder;
d = distance of the rolling load from the end of the girder;
I_g = moment of inertia of the girder, and
I_t = moment of inertia of the tread.

Consider the load, W, to be placed at various points along the girder. At each of these points calculate the moments for point e, both in the girder and the tread, considering that the girder is a simple span and that the tread is a cantilever of length, $\frac{b}{2}$. Substitute the results in equation (3), equate the expressions and solve them for the values of I_t. The values thus found for I_t are shown in Table B A.

It will be noted that I_t is very large when $d = \frac{b}{2}$, diminishes rapidly to $d = \frac{l}{4}$, and reaches its minimum at $d = \frac{l}{2}$. Assume that the tread is proportioned for the value of I_t when $d = \frac{l}{2}$. Having designed the tread for this value of I_t, it can be tested for the variation in the unit pressures between its bottom and the top of the

girder. When W is placed at $d = \frac{b}{2}$ from the extreme end of the girder, the difference between the radius of deflected curvature of the girder and that of the tread, considered as a cantilever, is a maximum. If the inequalities in the reciprocal unit pressures are not too great, the tread will still be in contact with the girder.

TABLE B A

VALUES OF I_t

Position of W on the Girder. Distance, d	Values of I_t in Terms of l, b, and I_g
$\frac{b}{2}$	$\frac{I_g l}{7l - 4b}$
$\frac{l}{4}$	$\frac{I_g b}{6l - 7b}$
$\frac{l}{3}$	$\frac{9I_g b}{64l - 57b}$
$\frac{l}{2}$	$\frac{I_g b}{8l - 5b}$

The expressions in the table show that I_t is a direct function of I_g and l, or b, and an inverse function of multiples of l and b, hence, a short and deep supporting girder will require a deeper and stiffer tread than a long and shallow one.

The tread on the rolling segment may safely be made of the same section as for that on the supporting girder.

5. Example.—Assume a double track rolling-lift bridge weighing 3,000,000 pounds. Let the radius of the rolling segment be 28 feet and the angle of opening be 85 degrees. The length of the track along which the bridge rolls becomes

$$\frac{56\pi}{4} \times \frac{85}{90} = 41.5 \text{ ft.}$$

Assume that the treads are ordinary steel castings with carbon of about 0.40 per cent. Let the permissible rolling line contact load be as given by equation (2), or

$$P = 675\sqrt[4]{D^3},$$

then

$$P = 675\sqrt[4]{(672)^3} = 675 \times 132 = 89{,}100 \text{ lbs. per linear inch.}$$

Assume an impact of 20 per cent, then $\frac{1{,}500{,}000 \times 1.2}{89{,}100} = 20.2$ linear inches required for line contact across the width of the tread. Assume the weight of the supporting girder to be 100,000 lbs., then:

	Shears	Moments
The girder, dead.....	50,000 lbs.	518,750 ft.-lbs.
Rolling load, live.....	1,500,000	15,562,500
Impact.....	300,000	3,112,500
Totals.....	1,850,000	19,193,750

The web area required is 1,850,000 ÷ 10,000 = 185 sq. in.

This can be made up as follows:

2 web plates, 108 in. × $\frac{7}{8}$ in. = 189 sq. in.

The girder may be made up as follows:

2 webs, 108 in. × $\frac{7}{8}$ in.	= 189 sq. in.
8 side plates, 16 in. × $\frac{7}{8}$ in.	= 112
4 side plates, 8 in. × $\frac{7}{8}$ in.	= 28
4 angles, 8 in. × 6 in. × $\frac{7}{8}$ in.	= 46
4 cover plates, 30 in. × $\frac{7}{8}$ in.	= 105
Total area	= 480 sq. in.

This section has a gross moment of inertia of 930,987 in.^{4}, or a net moment of inertia of 805,665 in.^{4}

The extreme fiber stress on net area is

$$\frac{19{,}193{,}750 \times 12 \times 55.75}{805{,}665} = 15{,}940 \text{ lbs. per sq. in.}$$

The load which must be distributed to the girder web from the tread is

$$1{,}500{,}000 \times 1.20 = 1{,}800{,}000 \text{ lbs.}$$

Refer to Fig. B B, and assume one inch rivets.

Direct bearing on 1 web in $22\frac{1}{2}$ in. = $22\frac{1}{2} \times \frac{7}{8} \times 24{,}000$	= 472,500 lbs.
21 rivets, 1 in. in diameter, in double shear at 18,850	= 395,850
Available in $22\frac{1}{2}$ in., on 1 web	= 868,350 lbs.
Available on 2 webs	= 1,736,700 lbs.

Length required for distribution, $22.5 \times \frac{1{,}800{,}000}{1{,}736{,}700} = 23.32$ in.

Assume the rolling load to be placed at $d = \frac{l}{2}$ from the end of the girder, then,

using the gross moment of inertia,

$$I_t = \frac{I_g b}{8l - 5b} = \frac{980{,}987 \times 23.32}{8 \times 498 - 5 \times 23.32} = 5613 \text{ in.}^4\text{, required.}$$

The tread section shown in Fig. B C gives $I_t = 5612$ in.^{4}

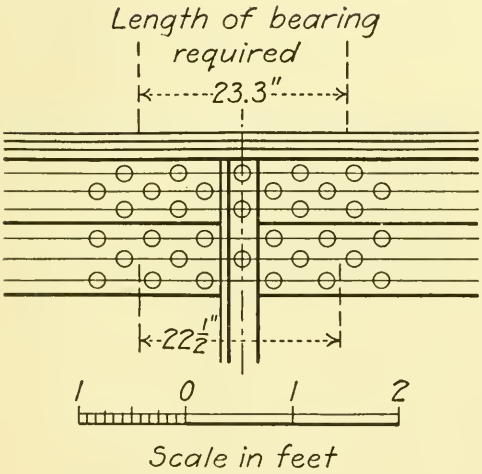

FIG. BB.

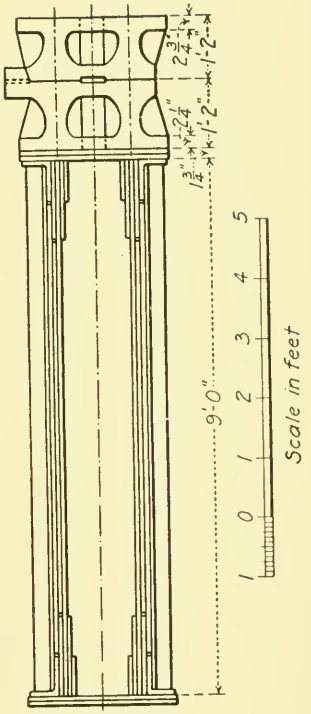

FIG. BC.

Now place the load W so that the distance d, Fig. B A, is $\frac{23.32}{2} = 11.66$ in. from the end support of the girder.

Load W will not act at a point, but will be distributed somewhat by the elastic distortions of the two treads at their line of contact. W is assumed as concentrated on a line on account of simplicity in the analysis. The calculated deflections will be a little too large, but will be on the safe side.

Calculate the deflections of the girder downward from the end and that of the the cantilever end of the tread upward from a tangent to the deflection curve of the girder at point d, 11.66 inches from the end. Combine the results and find the theoretical distances between the points on the deflected girder and the corresponding points on the tread. Table B B gives these distances.

TABLE B B

Distance of Points from the End of the Girder, in Inches	Theoretical Distances between the Top of the Girder and the Bottom of the Tread, in Inches
0	0.000974
2.915	0.000600
5.830	0.000253
8.745	0.000028
11.660	0.0

Figure B D illustrates the theoretical condition, the vertical distances having been greatly exaggerated.

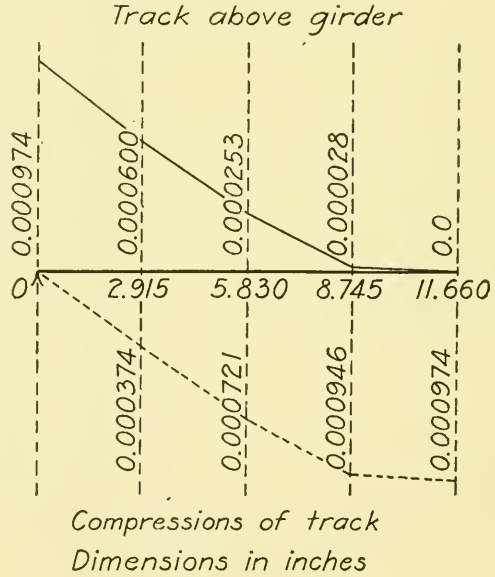

FIG. BD.

Now conceive the girder to be incompressible and that the tread is brought into contact with it by elastic compression of the tread by the superposed load, which will now be assumed as uniformly distributed along the portion of the tread that is required to distribute the load to the web of the girder. Under this assumption, it is found that about 137,300 lbs. on 11.66 in. of tread is required to cause contact at point *O*. There then remains 900,000 − 137,300 = 762,700 lbs. to be taken uniformly by the two webs of the tread, the section of which is shown in Fig. B C. This corresponds to 762,700 ÷ 104.94 = 7268 lbs. per sq. in. in the webs.

Had the distribution been uniform, the unit compression in the webs of the tread would have been, 900,000 ÷ 104.94 = 8576 lbs. per sq. in.

Table B C shows the various unit stresses and their combinations.

TABLE B C

Distance from End of Girder, in Inches	Unit Stress if Uniformly Distributed, Lbs. per Sq. In.	Unit Stress to Cause Contact at O, Lbs. per Sq. In.	Total Unit Stress, Lbs. per Sq. In.	Per Cent of 8576 Lbs. per Sq. In.
0	7268	0	7268	84.7
2.915	7268	775	8042	93.8
5.830	7268	1494	8762	102.2
8.745	7268	1960	9228	107.6
11.660	7268	2018	9286	108.3

The maximum unit stress is only 8.3 per cent more than for a uniform distribution.

It is recognized that this approximate method does not give the actual variations in the unit pressures between the tread and the girder; but the results are close enough to indicate that a tread designed in this manner will be satisfactory in service.

Table B D gives a comparison of tread plates to carry a load of 1,800,000 pounds, including 20 per cent impact, in accordance with several methods that have been proposed for their design.

TABLE B D

Designation	Theory	Type of Construction	Depth of Tread, Inches	Sectional Area of Tread, Sq. In.	Per Cent
1	Elastic stress directly under load W at bottom of tread. No adequate provision for distributing the load to the girder	Solid with attachment flanges	6.6	163	66
2	A.R.E.A. Spec. Pressure of 13,000 lbs. per sq. in. at bottom of tread. Distribution 45 deg. each side of vertical through load W.	Solid with attachment flanges.	7.9	221	89
3	J. E. Greiner's Spec. for Movable Bridges.	Ribbed castings	22.6	325	131
4	The author's theory as developed in this article.	Ribbed castings.	14	248	100

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